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Identification of Oscillatory System using Relay Feed Back Test

P. Blessy Hepsiba*, S. Lourdu Jame and K. Sharmila Devi*****

ABSTRACT

In this paper, a systematic approach is proposed for the structure identification and parameter estimation of Oscillatory System using relay feedback test. Exact analytical expressions are derived for the relay response of oscillatory system. The parameters are estimated by applying boundary conditions obtained from the limit cycle data on the derived expressions. Then, the closed loop analysis is carried out using the estimated model to study the controller performance.

Keywords: Relay feedback; Biased relay; parameter estimation; System Identification; oscillatory system.

1. INTRODUCTION

Chemical processes are complex and non-linear in nature due to interactions between inputs and outputs. With increase in complexity of the process, it becomes difficult to control the loops properly. Most of the chemical industries use PID controllers. Proper tuning of PID needs proper model structure. Relay feedback is one promising tool for identification of process models.Astrom and Hagglund [1] have suggested the relay feedback test to generate sustained oscillations of the controlled variable. It is based on the observation that when the output lags behind the input by π radians, closed loop system may oscillate with a period of P_u. A relay of magnitude 'h' is inserted in the feedback loop as shown in Fig. 1 and hence the name relay feedback test.

Initially, the input u(t) is increased by "h". Once the output y(t) starts increasing after a time delay (D), the relay switches to opposite direction. Since, there is a phase lag of -π, a limit cycle of amplitude "a" is generated as shown in Fig. 2. The period of limit cycle is ultimate periodP_u.The approximate ultimate gain K_u and the ultimate frequency ω_u are

$$k_u = \frac{4h}{\pi a} \omega_u = \frac{2\pi}{P_u} \tag{1}$$

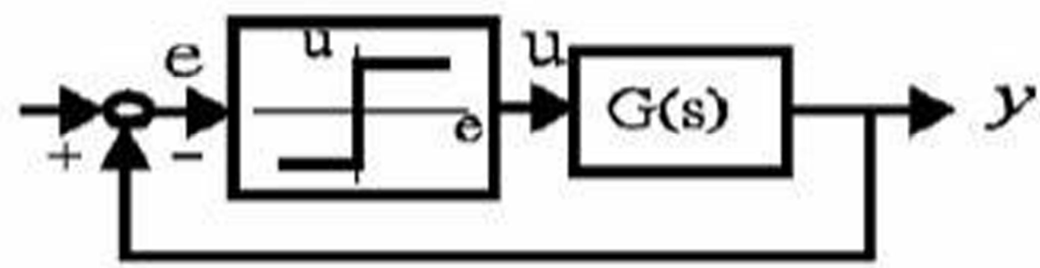

Figure 1: Block diagram of relay feedback test

* Assistant Professor, Department of EEE, SRM University, Chennai, *Email: blessyhepsiba@yahoo.co.in*

** Assistant Professor, Department of EEE, SRM University, Chennai, *Email: lourdujame@gmail.com*

*** Assistant Professor Department of EEE, SRM University, Chennai, *Email: info25j@gmail.com*

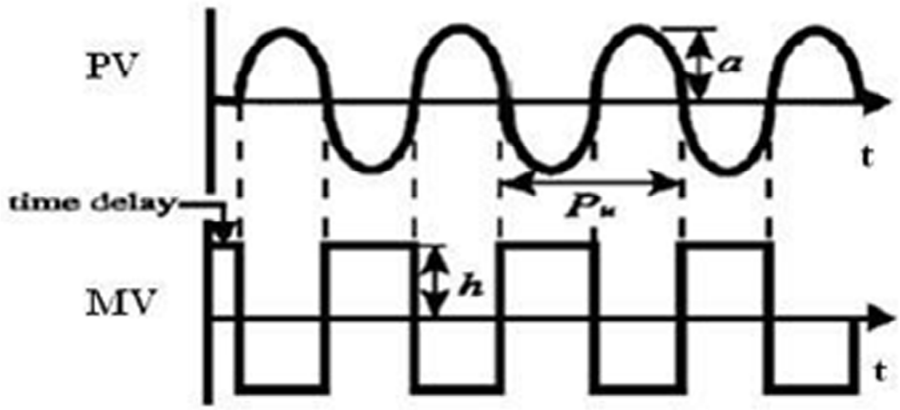

Figure 2: Relay test response

Many commercial auto tuners are designed based on ultimate gain and ultimate period obtained from relay test. Chang et al [2] derived transfer functions from relay feedback tests with increased accuracy. The progress in relay feedback auto tuning up to 1999 is comprehensively documented in the book by Yu [3]. Yu [4] proposed methods for extracting process information relevant to PID controller tuning by means of relay feedback. Luyben [5] is among the first to employ the relay feedback test to get additional information about the process dynamics using the so called shape factor. Thyagarajan and Yu [6] presented the identification procedure for different processes such as FOPDT, SOPDT and HO systems. Panda and Yu [7] have derived time domain model equations for first, second, third and higher order processes. Thyagarajan and Yu [8] have presented a method for performance monitoring and assessment of single loop control system using shape factor from relay feedback. Panda and Yu [9] had presented a method in which the shapes of relay response were studied to get a guess of order and type of unknown process. Panda [10] had estimated the parameters of higher order oscillatory system by using limit cycle data of relay responses obtained from single relay feedback test. Tao Liu and FurongGao [11] have presented a method for Identification of integrating and unstable processes from the biased relay feedback.

In this paper, a systematic identification method using single relay feedback test is proposed to obtain oscillatory system parameters. The paper is organized as follows: Section II briefs the parameter estimation of oscillatory System. The significances of the proposed estimation are discussed using closed loop analysis in section III.Closed loop simulations are carried out using PI controller for set point change and performance of the controller is evaluated using time domain specification and performance indices. Finally, conclusions are given in section IV.

2. PARAMETER ESTIMATION

2.1. Oscillatory System

Its transfer function model can be written as

$$G(S)=\frac{\omega_O^{\,2}}{(\tau S+1)(S^2+2\xi\omega_o S+\omega_o^{\,2})}$$

•• - Natural Resonant frequency, • - Time constant of the process, • -Damping ratio

2.2. Biased relay

In the proposed work relay test is conducted using biased relay because, biased relay is needed for practical application where the environment is noisy. It also rejects the static load disturbances and reduces the time required to estimate the process parameter. The basic structure of biased relay is shown in Fig. 3.

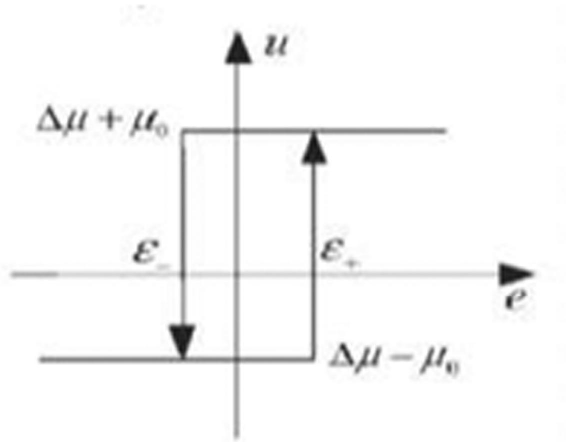

Figure 3: Biased relay

2.3. Analytical expressions

Analytical expressions are mathematical expressions for the stabilized relay feedback output responses. The shape information is unique for oscillatory system and can be used for the identification of model structure. The relay output response is the response of series of step changes in the manipulated variable. Hence, the response at stable oscillating condition is the sum of infinite terms of step responses due to those step changes. The shifted version of a typical relay feedback response provides the basis for the derivation as shown in Fig. 4.

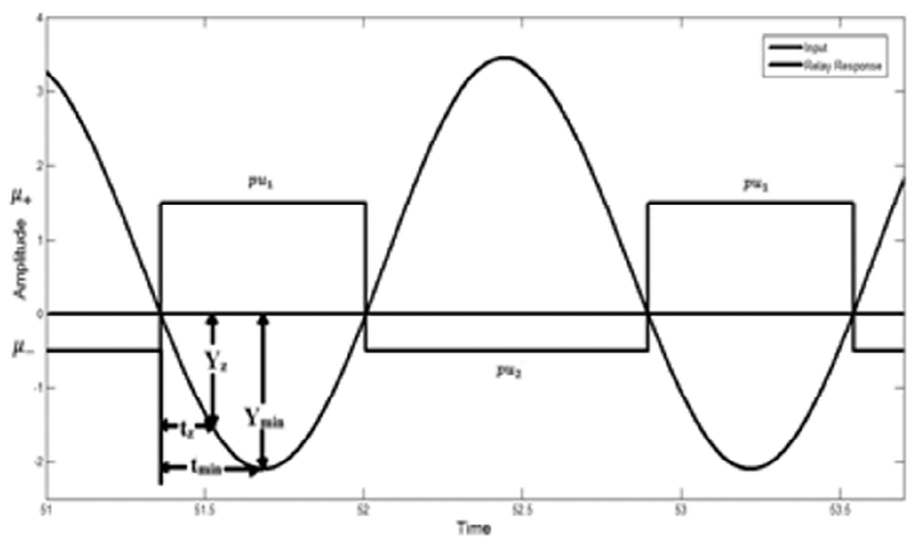

Figure 4: Biased relay response of Oscillatory system

$$G(S) = \frac{k_p \omega_o^2}{(\tau S+1)(S^2+2\xi\omega_o S+\omega_o^2)}$$

$$Y(t) = K_p \omega_0^2 U\left[\frac{1}{P_1+P_2} + ae\frac{t}{\tau} + be^{-P_1 t} + ce^{-P_2 t}\right]$$

$$a = \frac{1}{\left[\frac{-1}{\tau^2} + \frac{P_1+P_2}{\tau} - P_1 P_2\right]}$$

$$b = \frac{-1}{P_1(\tau P_1 - 1)(P_1 - P_2)}$$

$$c = \frac{-1}{P_2(\tau P_2 - 1)(P_2 - P_1)}$$

$$Y_4(t) = K_p \omega_0^2 \mu_- \left[\frac{1}{P_1 + P_2} + ae \frac{(t + \tau + pu_1 + pu_2)}{\tau} + be^{-P_1(t+T+pu_1+pu_2)} + ce^{-P_2(t+\tau+)} \right]$$

$$+K_p \omega_0^2 (\mu_+ - \mu_-) \left[\frac{1}{P_1 + P_2} + ae \frac{(t + pu_1 + pu_2)}{\tau} + be^{-P_1(t+pu_1+pu_2)} + ce^{-P_2(t+\tau)} \right]$$

$$+K_p \omega_0^2 (\mu_- - \mu_+) \left[\frac{1}{P_1 + P_2} + ae^{\frac{(t+pu_2)}{\tau}} + be^{-P_1(t+pu_2)} + ce^{-P_2(t+pu_2)} \right]$$

$$+K_p \omega_0^2 (\mu_+ - \mu_-) \left[\frac{1}{P_1 + P_2} + ae^{\frac{t}{\tau}} + be^{-P_1 t} + ce^{-P_2 t} \right]$$

i. If n is even: (positive step change)

- FIRST TERM:

$$\frac{K_p \omega_0^2}{P_1 P_2} (\mu_- + \mu_+ - \mu_- + -\mu_+ + \mu_+ - \mu_-)$$

$$\frac{K_p \omega_0^2}{P_1 P_2} (\mu_+)$$

- SECOND TERM:

$$K_p \omega_0^2 ae^{-\frac{t}{\tau}} \left\{ \mu_. \left(e \frac{-(T + Pu_1 + Pu_2)}{\tau} - e \frac{-(Pu_1 + Pu_2)}{\tau} + e \frac{-Pu_2}{\tau} - 1 \right) + \mu_+ \left(e \left(\frac{-(Pu_1 + Pu_2)}{\tau} - e \frac{-Pu_2}{\tau} + 1 \right) \right) \right\}$$

Let $x_1 = \left(e \frac{Pu_1}{\tau} \right)$ and $x_2 = \left(e \frac{Pu_2}{\tau} \right)$

The second term can represented as follows

$$\left[K_p \omega_0^2 ae^{-\frac{t}{\tau}} (\mu_+ - \mu_-) \frac{1 - x_2}{1 - x_1 x_2} \right]$$

- THIRD TERM:

The third term can be represented as follows

$$\left[K_p \omega_0^2 be^{-P_1 t} (\mu_+ - \mu_-) \frac{1 - y_2}{1 - y_1 y_2} \right]$$

where $y_1 = \left(e^{-P_1 pu_1}\right)$ and $y_2 = \left(e^{-P_1 pu_2}\right)$

- FOURTH TERM:

$$\left[K_p \omega_0^2 c e^{-P_2 t}\left(\mu_+ - \mu_-\right)\frac{1-z_2}{1-z_1 z_2}\right]$$

where $z_1 = \left(e^{-P_2 pu_1}\right)$ and $z_2 = \left(e^{-P_2 pu_2}\right)$

GENERALISED EXPRESSION:

The generalised expression when n (number of limit cycles) is even is given as follows

$$Y_n(t) = K_p \omega_0^2\left[\frac{(\mu_+)}{P_1 P_2} + \left(ae^{\frac{t}{\tau}}\left(\mu_+ - \mu_-\right)\frac{1-x_2}{1-x_1 x_2}\right) + \left(be^{-P_1 t}\left(\mu_+ - \mu_-\right)\frac{1-y_2}{1-y_1 y_2}\right) + ce^{-P_2 t}\left(\mu_+ - \mu_-\right)\frac{1-z_2}{1-z_1 z_2}\right]$$

If n is odd: (Negative step change) The response can be expressed analytically as

$$Y_n(t) = K_p \omega_0^2\left[\frac{(\mu_-)}{P_1 P_2} + \left(ae^{\frac{t}{\tau}}\left(\mu_- - \mu_+\right)\frac{1-x_2}{1-x_1 x_2}\right) + \left(be^{-P_1 t}\left(\mu_- - \mu_+\right)\frac{1-y_2}{1-y_1 y_2}\right) + ce^{-P_2 t}\left(\mu_+ - \mu_-\right)\frac{1-z_2}{1-z_1 z_2}\right]$$

This equation can be used for the parameter estimation by applying the limit cycle data obtained from the stable oscillating condition.

2.4. Parameter estimation

There are six parameters to be estimated in the Oscillatory system. The parameters can be estimated by applying the following four different boundary conditions and solving Boundary conditions. The boundary conditions are

At 1) t = 0, Y = 0; 2) t = pu1, Y = 0; 3) t = tmin, Y = Ymin 4) t = tz, Y = Yz

Using this information, the process parameters are estimated as given in the estimation procedure. The comparison between the true and estimated process is given in Table II. It is found that, the estimated parameters using the proposed method are very close to the true process parameters

Table 1
Measured Values From Relay Response

True Process	*Measured Values*					
	t min	*Ymin*	*tz*	*Yz*	*Pu1*	*Pu2*
$\frac{32}{(S+1)(S^2+0.8S+16)}$	0.322	-2.103	0.17	-1.572	0.645	0.889

Table 2
Comparison of True and Estimated Parameters

Parameters	kp	ωo	ξ	•
True	2	4	0.1	1
Estimated	2.0072	4.0132	0.0934	1.08
% Error	0.3	0.33	6.6	8

3. CLOSED LOOP ANALYSIS

In the conventional methodology, the controller is tuned based on the limit cycle data K_u and P_u obtained from the relay response. But, this general ZN tuning methodology which is independent of model structure and associated model parameters is not suitable for many applications. To improve the performance, the controller must be tuned based on the model parameters.In this section, the performance of controller, tuned based on conventional ZN method is compared with the controller, tuned based on the estimated model parameters for Oscillatory System. The numerical example considered in section II is used to carry out the closed loop analysis.

[10] tuning methodology is chosen to tune the controller parameters based on the estimated model parameters of oscillatory system. The tuning parameters of PI controller for both ZN method and Proposed model based tuning method are given in Table III.

Table 3
PI Controller Parameters

Tuning Rule	*Controller Parameters*	
	K_i	T_i
ZN Method	0.2081	0.1628
Proposed Model Based Tuning	0.0075	0.1609

The closed loop set point response of the oscillatory system for two different control methods is shown in Fig. 5. From the response, it is clear that in the proposed model based tuning method, the oscillation and over shoot is minimum compare to the conventional method. This would reduce the wear and tear of final control element and hence would increase the life of the controller.

The controller performance is compared quantitatively using performance indices namely settling time and Integral Absolute Error (IAE) for the two different methods. Proposed model based Tuning method provides less settling time and gives minimum IAE values compare to conventional ZN method as shown in Table V.

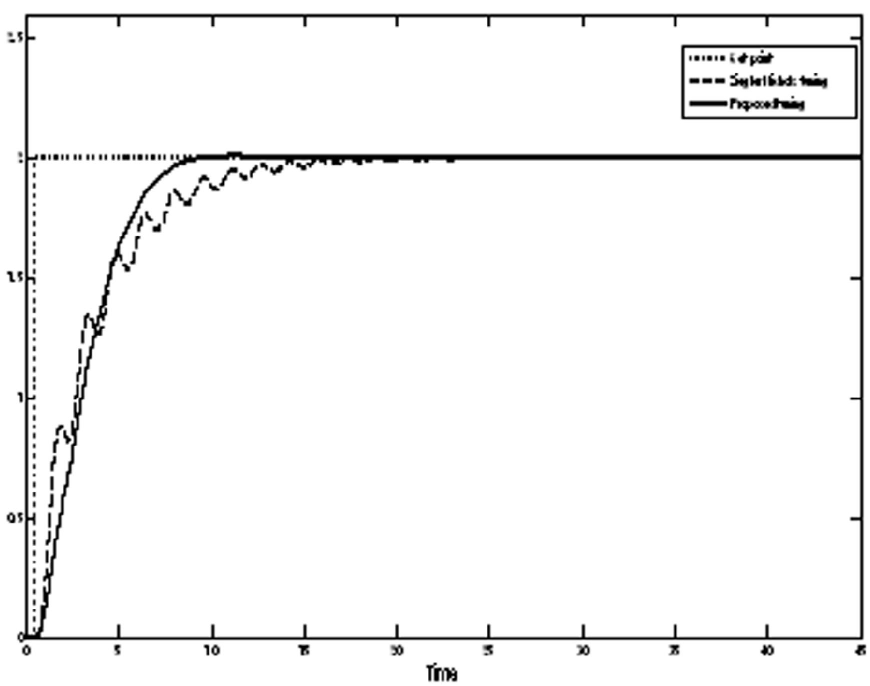

Figure 5: Closed loop set point response of system considered

Table 4
Quantitative Comparison of Controller Performance in Real Time Setup

	Performance Indices	
Tuning Rule	*Settling Time (sec)*	*IAE*
ZN Method	19.5	6.3
Proposed Model Based Tuning	9.5	5.8

4. CONCLUSION

In the present work,the shape information of the relay response was used to identify the model structure of oscillatory system. A systematic approach was proposed to derive the exact analytical expressions for stable relay response. The process parameters were estimated by applying the boundary conditions on the derived analytical expression and it was found that the estimated parameters were very close to the true values. Then, the closed loop studies were carried out by tuning the PI controller using conventional ZN method and model based tuning method. Based on the qualitative and quantitative comparisons, it was found that the model based tuning methodology performs better than the conventional ZN method. It was proved that the proposed method able to identify the process parameters accurately because, the PI controller tuned using the estimated model based tuning method out performs the conventional ZN method tuned controller. Hence, the proposed method can be used for the auto tuning of PI/PID controller for oscillatory system.

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