International Journal of Mathematical Sciences
Vol. 17, Nos. 1-2, January-June 2018, pp. 35-48

AN ANALYTICAL APPROACH OF THREE DIMENSIONAL STRATIFIED LAMINAR GAS-LIQUID TWO PHASE PIPE FLOW MODELS

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Abstract: In this paper an analytical model is obtained for stratified laminar gas liquid two-phase flow in pipes with different pressure drops in each phase, and compared with both the one-dimensional cross sectional average model and experiment. The present model is based on the modification of Ranger's and David model to use the bipolar coordinate for stratified laminar flow analyses. Experimental confirmation has been conducted in air-water flow with two different tube sizes (I.D 1.5 and 2 (cm)). The liquid level and void fraction were measured by ultrasonic pulse echo and capacitance transducer techniques. Pressure drop of each phase is also measured. The results show: (a) the friction factor decreases with increasing liquid phase Reynolds numbers with power law factor of ... 1.32, and no tube size effect is observed with gas flow rate is zero, (b) the present three dimensional theory predicts larger void fraction than that of the one-dimension cross sectional averaged model for lower superficial gas and liquid velocities and agree well experiments; (c) the present three dimension theory agree well with experiments for the prediction of the phase pressure drop in the gas phase. However, for the liquid phase, only qualitative behaviour with order of magnitude agreement is obtained.

Keywords: Two-phase stratified laminar flow, LAMNAR2, fluid-fluid interface, volume flow ratio, stratified laminar flow without gas flow, bipolar coordinates.

1. INTRODUCTION

Two-phase stratified laminar motion has been observed in a variety of situations most notably in pipeline transport. However, there is some confusion as to the transition criterion for laminar to turbulent stratified flow. The authors [1-3] have demonstrated that the Reynolds number, based on superficial velocity, should be less than 1000. Spriggs [4] considered analyzing the two-phase situations using Hack's criteria [5] and found a poor correlation between theoretical and experiments results. Spriggs demonstrated this was due to stabilizing force in one phase which act to dissipate disturbances in the other before they grow into turbulent patches. In general, it has been assumed that, as with single-phase flow, transition to turbulent occurs when the Reynolds number exceeds about 2000. This paper is concerned with an exact description of stratified laminar flows. The procedure flowed will be extend the method of Ranger and David [6] assuming that there is a different pressure drop in each phase. An analytical solution is presented.

2. TWO PHASE LAMINAR VELOCITY PROFILES

The pressure driven laminar flow of two stratified incompressible Newtonian fluids, 1 and 2 in a pipe of circular cross section as in Fig. (1). The fluid-fluid interface is smooth and horizontal subtending an angle of 2α at the axis of the pipe. Now, Cartesian coordinates are

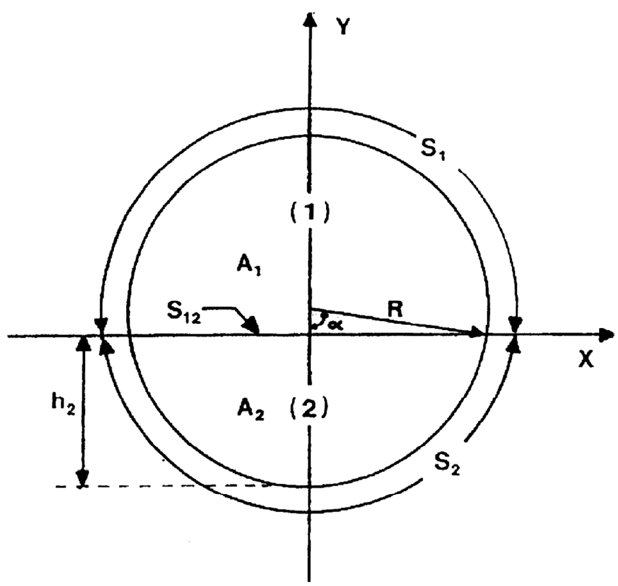

Figure 1: Fluid Configuration in Two-Phase Stratified Smooth Flow

chosen so that *OZ* along the axis of the pipe (parallel to the direction of the fluid flow), *OX* lies on the interface between the two liquids and *OY* passes through the axis of the pipe. The coordinates are then non-dimensionalized by the pipe radius *R* such that

$$X = xR, \qquad Y = yR \tag{1}$$

where x and y represents the dimensionless Cartesian coordinate system. On the other hand, for the steady laminar motion of each phase, j with fluid flow in the z direction only, the Navier Stokes equation becomes

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)_j u_j = \frac{R^2}{\mu_j}\left(\frac{dP}{dz}\right)_j = -\frac{R^2}{\mu_j} G_j \quad (j = 1, 2) \tag{2}$$

The boundary conditions that we apply to u_j are listed in Table 1. Eq. (2) Poisson's equation, may be transformed into Laplace's equation through the introduction of the reduced velocity u_j^* as

$$u_j = -\frac{R^2}{\mu_j} G_j \left\{ \frac{1}{4}[x^2 + (y - \cos\alpha)^2 - 1] - u_{jj}^* \right\} \qquad (j = 1, 2) \tag{3}$$

Substitution of this expression for u_j into Eq.(2) we obtain

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial 2}{\partial y^2} \right)_j u_j^* = 0 \qquad (j = 1, 2) \tag{4}$$

In order to simplify application of the boundary conditions bipolar coordinates (ξ, η) are introduced and the coordinate transformation can be obtained

Table 1

Boundary Condition on for Two-Phase Stratified Laminar – Laminar flow

	Boundary conditions	*Explanation*
[a]	$u_j(R) = 0 \quad (j = 1, 2)$	no slip condition at the wall
[b]	$\frac{\partial u_j}{\partial x} = 0$ at $x = 0 \quad (j = 1, 2)$	symmetry
[c]	$\mu_1 \frac{\partial u_1}{\partial x} = \mu_2 \frac{\partial u_2}{\partial x}$ at $x = 0$	continuity of shear stress at the fluid-fluid interface.
[d]	$\mu_1 = \mu_2$ at $y = 0$	no slip condition at fluid-fluid interface.

Table 2

Boundary Condition on for Two-Phase Stratified Laminar – Laminar Flow

	Boundary conditions	*Explanation*
[a]	$\mu_1^* =$ at $\eta = \alpha$ $\mu_2^* =$ at $\eta = \alpha + \pi$	no slip condition at the wall
[b]	$\frac{\partial u_j^*}{\partial x} = 0$ at $\xi = 0 \quad (j = 1, 2)$	symmetry
[c]	$\mu_1 \frac{\partial u_1}{\partial \eta} = \mu_2 \frac{\partial u_2}{\partial \eta}$ at $\eta = \pi$	continuity of shear stress at the fluid-fluid interface
[d]	$\mu_1 = \mu_2$ at $\eta = \pi$	no slip condition at fluid-fluid interface.
[e]	$\lim_{\xi \to \infty} u_j = \text{finite} \quad (j = 1, 2)$	reasonable physical condition

Again we obtain

$$x = \frac{\sin\alpha \sin\xi}{\cosh\xi - \cos\eta} \tag{5}$$

$$y = \frac{\sin\alpha \sin\eta}{\cosh\xi - \cos\eta} \tag{6}$$

$$x = z \tag{7}$$

In Fig. (2) a diagram showing bipolar coordinates. By comparison of Fig. 1 and Fig. 2 it can be seen in the fluid 1 occupies the region $-\infty < \xi < \infty^*$, $\alpha < \eta < \pi$ and fluid 2 occupies

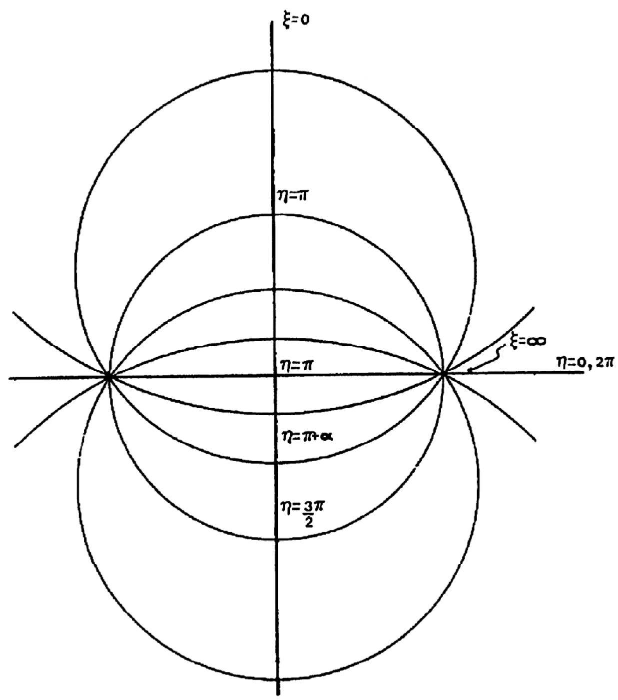

Figure 2: The Bipolar Coordinate System

the region $-\infty < \xi < \infty$, $\pi \le \eta \le \pm\alpha$, Laplace's. equation is invariant under the above the transformation, hence the reduced velocities satisfy

$$\left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}\right) u_j^* = 0 \qquad (j = 1, 2) \tag{8}$$

with the boundary condition listed in Table 2.

Applying separability to Eq. (8) leads to the following expressions

$$u_j^* = E_j(\xi) N_J(\eta) \qquad (j = 1, 2) \tag{9}$$

$$\frac{d^2}{d\xi^2} E_j - k^2 E_j = 0, \qquad (j = 1, 2) \tag{10}$$

$$\frac{d^2}{d\eta^2} E_j + k^2 N_j = 0, \qquad (j = 1, 2) \tag{11}$$

where k represents the separation constant. Solving Eq. (10)-(11) we obtain representation of u_j^*

$$u_j^* = [C \cos k(\xi - \delta) \sinh k(\varepsilon - \eta)]_j \qquad (j = 1, 2) \tag{12}$$

Now C, δ and ε represents the constant of integration. Boundary conditions 1 and 2 may be satisfied by choosing $\delta_1 = 0$, $\varepsilon_1 = \alpha$ and $\varepsilon_2 = \alpha + \pi$. Boundary condition 5 is automatically satisfied by the structure of the equation. Since any value of the separation constant will satisfy the above mentioned boundary conditions, k must be represented as a continuous valued function and so the reduced velocities may be represented as

$$u_1^* = \int_0^\infty C_1(k) \sinh k(\eta - \alpha) \cos k\xi \, dk \tag{13}$$

$$u_2^* = \int_0^\infty C_2(k) \sinh k(\pi + \alpha - \eta) \cos k\xi \, dk \tag{14}$$

substituting Eqs. (5)-(6) into Eq. (3) the velocity profile for each phase may be represented as

$$u_1 = \frac{R^2}{\mu_1} G_1 \left\{ \frac{\sin\alpha \sin(\eta - \alpha)}{2(\cosh\xi - \cos\eta)} + u_1^* \right\} \tag{15}$$

$$_2 = \frac{R^2}{\mu_2} G_2 \left\{ \frac{\sin\alpha \ \ \sin(\eta - \alpha)}{2(\cosh\xi - \cos\eta)} + u_2^* \right\} \tag{16}$$

Furthermore, the constants C_j are determined by imposing the remaining boundary conditions on the system and from boundary condition 3

$$u \int_0^\infty C_1 k \cosh k(\pi - \alpha) \cos k\xi \, dk + \beta \int_0^\infty C_2 k \cosh\alpha \cos k\xi \, dk = (1 - \beta) \frac{\sin\alpha \cos\alpha}{2(\cosh\xi + 1)} \tag{17}$$

where $\beta = G_1/G_g$, the ratio of the pressure drops in each phase. From the integrations, we obtain from Eq. (17)

$$C_1 \cosh k(\pi - \alpha) + \beta C_2 \cosh k\alpha = (1-\beta) \frac{\sin\alpha \cos\alpha}{\sinh k\pi} \tag{18}$$

Similarly, by applying boundary condition 4 to the system we obtain as

$$-C_1\mu_2 \sinh k(\pi-\alpha) + \beta C_2 \mu_1 \sinh k\alpha = (\mu_2 - \beta\mu_1) \frac{k \sin^2\alpha}{\sinh k\pi} \tag{19}$$

solving Eqs. (18)-(19), C_j may be expressed as a function of k

$$C_1 = \frac{-2(\mu_2 - \beta\mu_1) k \sin^2\alpha \cosh k\alpha + (1-\beta)\mu_1 \sinh k\alpha \sin 2\alpha}{(\mu_2 - \mu_1)[\sinh k(\pi - 2\alpha) + \lambda \sinh k\pi] \sinh k\pi} \tag{20}$$

$$\begin{aligned} C_2 = {} & \frac{(1-\beta)}{\beta[\sinh k\pi \cosh k\alpha]} \left\{ \frac{1}{2} - \frac{\mu_1}{(\mu_2 - \mu_1)} \frac{\sinh k\alpha \cosh k(\pi-\alpha)}{[\sinh k(\pi - 2\alpha) + \lambda \sinh k\pi]} \right\} \\ & + \frac{2(\mu_2 - \beta\mu_1)}{\beta \ (\mu_2 - \mu_1)} \ \frac{\sin^2\alpha \ \ k \ \ \cosh k(\pi - \alpha)}{[\sinh k(\pi - 2\alpha) + \lambda \sinh k\pi] \sinh k\pi} \end{aligned} \tag{21}$$

where

$$\lambda = \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} \tag{22}$$

3. DETERMINATION OF VOLUME FLOW RATES

The fluid 2 has been considered

$$Q_2 = \int\int_{A_2} u_2 \, dA \tag{23}$$

In case of bipolar coordinates an elemental area may be expressed as

$$dA = \frac{\sin^2\alpha}{(\cosh\xi - \cos\eta)^2} d\xi d\eta \tag{24}$$

$$Q_2 = \ 2\sin^2\alpha R^2 \frac{G_2}{\mu_2} \int_0^{\infty} \int_{\pi}^{\pi+\alpha} \left\{ \frac{\sin\alpha \sin(\eta - \alpha)}{2(\cosh\xi - \cos\eta)} + u_2^* \right\} \frac{d\eta d\xi}{(\cosh\xi - \cos\eta)^2} \tag{25}$$

The first term in the integrand is considered; the transformation $\eta = \omega + \pi$ is applied and with the integration over ξ is

$$\int_0^{\alpha}\int_0^{\infty}\frac{\sin\alpha\sin(\alpha-\omega)}{2(\cosh\xi-\cos\eta)^2}\,d\omega\,d\xi=\int_0^{\alpha}\frac{\sin\alpha\sin(\alpha-\omega)}{2}\,\frac{(\omega+3\omega\cot^2\omega-3\cot\omega)}{2\sin^3\omega}\,d\omega \qquad (26)$$

Eq. (26) is integrated ω term-by-term which gives

$$\frac{\sin\alpha}{4}\left[-\cos\alpha\left(\frac{\omega\cos\omega}{\sin^3\omega}-\frac{3\cos^2\omega}{\sin^4\omega}+\frac{3\omega\cos^3\omega}{\sin^5\omega}\right)+\sin\alpha\left(\begin{array}{l}\dfrac{\omega}{\sin^2\omega}+\dfrac{1}{2}\cot\omega+\cot^3\omega\\ -\dfrac{3}{4}\dfrac{\omega}{\sin^4\omega}-\dfrac{1}{4}\dfrac{\cos\omega}{\sin^3\omega}\end{array}\right)\right]_0^{\alpha} \qquad (27)$$

Now, the final expression for the first term in the volume flow rate integral is

$$\frac{R^2}{8\mu_2}G_2\left(\alpha-\frac{2}{3}\sin 2\alpha+\frac{1}{12}\sin 4\alpha\right) \qquad (28)$$

The second term in the volume flow rate integral is now considered

$$\int_0^{\infty}\int_{\pi}^{\pi+\alpha}u_2^*\,dA=\int_0^{\infty}\int_{\pi}^{\pi+\alpha}\int_0^{\infty}C_2\,\frac{\sinh k(\pi+\alpha-\eta)\cos k\xi}{(\cosh\xi-\cos\eta)^2}\,dk\,d\eta\,d\xi \qquad (29)$$

The transformation η = ω + π is applied. The integral over ξ is obtained as

$$\int_0^{\infty}\int_{\pi}^{\alpha}\int_0^{\infty}C_2\,\frac{\sinh k(\alpha-\omega)\cos k\xi}{\left(\cosh\xi+\cos\eta\right)^2}\,dk\,d\eta\,d\xi$$

$$=\pi\int_0^{\infty}\frac{C_2}{\sinh k\pi}\int_0^{\alpha}\sinh k(\alpha-\omega)\left\{\frac{k\cosh k\omega}{\sin^2\omega}-\frac{\sinh k\omega\cos\omega}{\sin^3\omega}\right\}dk\,d\omega \qquad (30)$$

The term to be integrated over ω is manipulated into the following form

$$\int_0^{\alpha}\sinh k(\alpha-\omega)\left\{\frac{k\cosh\omega}{\sin^2\omega}-\frac{\sinh k\omega\cos\omega}{\sin^3\omega}\right\}d\omega$$

$$=\int_0^{\alpha}\left\{k\frac{[\sinh k\alpha+\sinh k(\alpha-2\omega)]}{\sin^2\omega}+\cos\omega\frac{[\cosh k\alpha-\cosh k(\alpha-2\omega)]}{\sin^3\omega}\right\}d\omega \qquad (31)$$

This may be integrated by inspection

$$\frac{1}{2}\left[\frac{\cosh k\alpha}{\sin^2\omega}-\frac{\cosh k(\alpha-2\omega)}{\sin^2\omega}-2k\cot\omega\sinh k\alpha\right]_0^{\alpha} \qquad (32)$$

Finally Eq. (24) may be written as

$$\int_0^{\infty}\int_{\pi}^{\pi+\alpha}u_2^*\quad dA=\frac{\pi}{2}\int_0^{\infty}\frac{C_2}{\sinh k\pi}(k^2\cosh k\alpha-k\cot\alpha\sinh k\alpha)\,dk \qquad (33)$$

Combining Eqs. (21), (28) and (33), the volume flow rate of fluid 2 may be expressed as

$$Q_2 = \frac{\pi R^2 G_2}{\mu_2}\sin\alpha\left\{\begin{aligned}&\frac{(1-\beta)}{2\beta}\sin 2\alpha\int_0^\infty \frac{\psi(k,\alpha)}{\cosh k\alpha}\,dk - \frac{(1-\beta)}{\beta}\frac{\mu_1}{(\mu_2-\mu_1)}\sin 2\alpha\\ &\qquad\times\int_0^\infty \frac{\psi(k,\alpha)\sinh k\alpha\cosh k(\pi-\alpha)}{\Gamma(k\alpha,\lambda)\cosh k\alpha}\,dk\\ &\qquad+\frac{R^2G_2}{8\mu_2}\left(\alpha-\frac{2}{3}\sin 2\alpha+\frac{1}{12}\sin 4\alpha\right)\end{aligned}\right\} \tag{34}$$

Where

$$\psi(k,\alpha) = \frac{k^2\sin\alpha\cosh k\alpha - k\cos\alpha\sinh k\alpha}{\sin^2 k\pi} \tag{35}$$

and

$$\Gamma(k,\alpha,\lambda) = \sinh k(\pi-2\alpha) + \lambda\sinh k\pi \tag{36}$$

If we assumed that the pressure drops are the same in each phase $\beta \to 1$, Ranger and Davis' original expression [6] is obtained and if we consider that liquid 2 occupies the entire pipe, $\alpha \to \pi$, and for Pouseullie flow results gives

$$Q_2 = \frac{\pi R^2}{8\mu_2}G_2\,. \tag{37}$$

4. STRATIFIED LAMINAR FLOW WITHOUT GAS FLOW

We are mainly concerned with only *one* fluid (liquid flow) and the velocity distribution of the liquid phase as a function of void fraction and imposed pressure drop is obtained

$$u = \frac{GR^2}{\mu}\left\{\frac{\sin\alpha\sin(\eta-\alpha)}{2(\cosh\xi-\cos\eta)} + \int_0^\infty C\sinh k(\varepsilon-\eta)\cos k(\xi-\delta)\,dk\right\} \tag{38}$$

Now, the constants ε and δ are set to $\pi + \alpha$ and 0 respectively to satisfy boundary conditions 1 and 2 in Table (3). Furthermore, boundary condition 4 is automatically satisfied by the structure of the above equation. The constant C is determined by application of boundary condition 3 gives

$$C = -\frac{\sin\alpha\cos\alpha}{\sinh k\pi\cosh k\alpha} \tag{39}$$

The volume flow rate is obtained

$$Q = \frac{GR^2}{\mu} \left[\begin{array}{l} \frac{1}{8}\left(\alpha - \frac{2}{3}\sin 2\alpha + \frac{1}{12}\sin 4\alpha \right) \\ -\pi \sin^2\alpha \cos\alpha \int_0^\infty \frac{k(k\sin\alpha \cosh k\alpha - \cos\alpha \sinh k\alpha)}{\sin^2 k\pi \cosh k\alpha} dk \end{array} \right] \tag{40}$$

Eq. (40) also illustrates an important aspect relating to the pressure drop. For single phase Poiseuille flow the pressure drop varies linearly with the flow rate and indicates that , again , the pressure drop varies linearly with the flow rate. If is assumed that the Blasius equation, with C_1 = 16 and X_1 = 1.0 applies then the terms in the square brackets of Eq. (40) must always be equal to a simple geometric expressions. This is clearly not the case thus, for stratified laminar flow without gas flow, the constant which appear in the Blasius equation are expected to be different from the Poiseuille flow situation.

The reason for the above is intuitively obvious. The void fraction is dependent on the volume flow rate in the pipe and thus the term in the square brackets in Eq. (40) is dependent on the volume flow rate of the liquid.

Table 3
Boundary Conditions on *u* for Two–Phase Stratified Laminar Flow without Gas Flow

	Boundary conditions	*Explanation*
[1]	$u^* =$ at $\eta = \alpha + \pi$	no slip condition at the wall
[2]	$\frac{\partial u}{\partial x} = 0$ at $\xi = 0$	symmetry
[3]	$\frac{\partial u}{\partial \eta} = 0$ at $\eta = \pi$	continuity of shear stress at the fluid-fluid interface
[4]	$\lim_{\xi \to \infty} u = \text{finite}$	reasonable physical condition

5. CLOSURE OF THE VOLUME FLOW RATE EQUATIONS

The equations of concern are evaluated in computer code LAMNAR2. Fig. (3) shows a flow diagram for LAMNAR2. The structure of the equations makes it easiest to follow the route of assuming a void fraction and a gas flow rate and then determining the required liquid flow rate to maintain these imposed conditions. In addition, Eq. (34) may be separated into two parts, the geometrical and physical dependent terms and the pressure drop dependent terms. The first of these are evaluated numerically, given the required information and stored then, a gas flow is assumed and the equations evaluated, for each pre-set void fraction, using experimentally determined Blasius constants. The correct value for the liquid volume flow rate is determined iteratively.

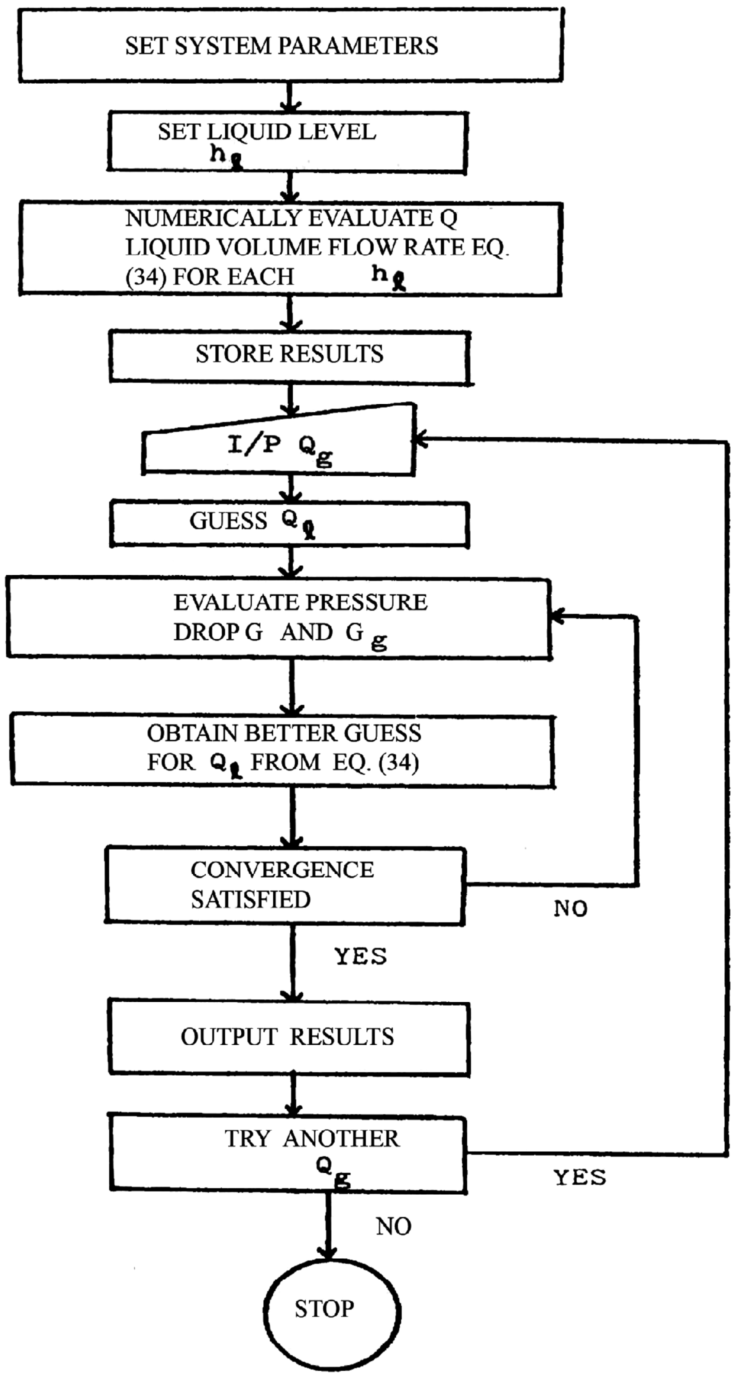

Figure 3: Flow Chart for Routine LAMNAR2

6. EXPERIMENTAL APPARATUS

The lucite tubes of inner diameters 2.0 and 1.50 *cm* are used as shown in Fig. (4). Water taken from the laboratory supply was introduced as one end of the pipe. The water exits from the other end of the pipe at atmospherics conditions and is calculated in a weighing tank for flow rate measurement. Liquid film measurements are made using an ultrasonic pulse echo method (Cheng *et al.*, [7]).

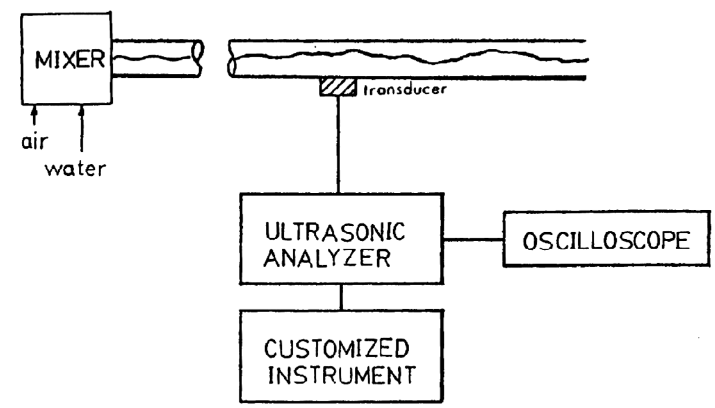

Figure 4: Schematic Experimental Apparatus for Two-Phase Stratified Flows

7. EXPERIMENTAL RESULTS

Fig.(5) represents a log-log plot of the experimentally deduced friction factor *vs*. Reynolds number for stratified single-phase flow of water (zero gas flow) in pipes of inner diameters 12.0 and 1.5 cm. The pressure drop is assumed to be solely due to wall shear. From this figure the Blasius constants are determined as

$$C = 24.0 \pm 1.0 \quad \text{and} \quad X = 1.06, \left(f_l = C_l \left(\rho_l \, \frac{D_l u_l}{\mu_l} \right)^{-x_l} \right)$$

The above results are used to determine the pressure drop and void fraction for stratified two-phase laminar-laminar flow are demonstrated. Moreover, Fig. (6) depicts a comparison of void fraction prediction for the one-dimensional model of Lightstone and Cheng [8] and the present three dimensional model for an air-water system. On the other hand, for low liquid flow rates the three dimensional model predicts higher void fraction in agreement with experiment (as shown isolated points on Fig. (6)).

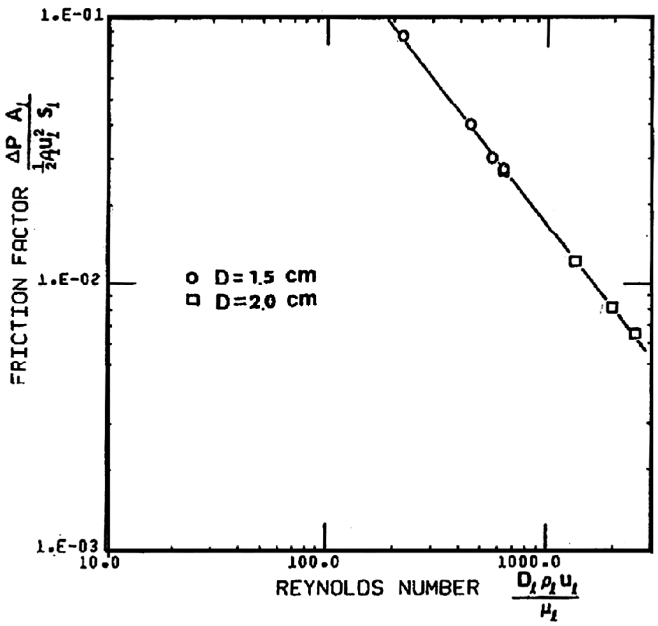

Figure 5: Friction Fraction *versus* Reynolds Number for Two-Phase Stratified Flows

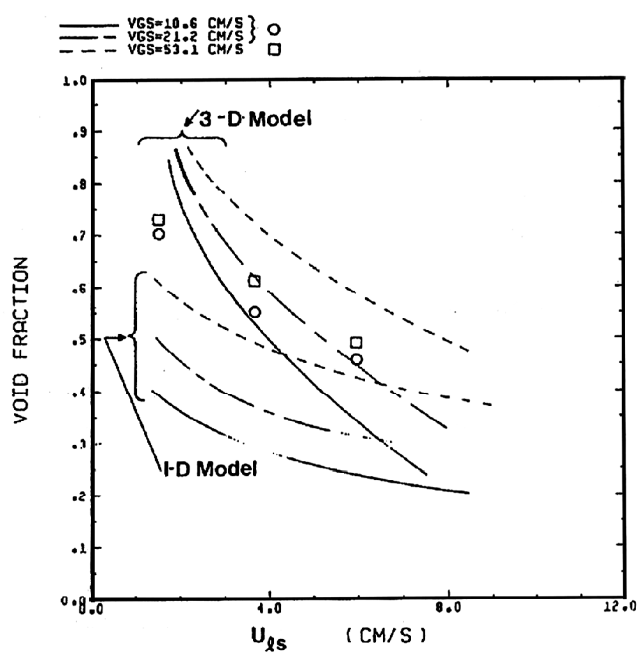

Figure 6: Comparison of Present Theory with 1-D Theory for Prediction of Void Fraction in Laminar-Laminar Flow

Table 4, compares the observed and predicted pressure drop for the two phases. Now experimental errors are quite large however, the experimental results indicate order of magnitude values. It is clear that the theory's ability to predict liquid pressure drop is quite poor. For low liquid flow rates, very high pressure drops are predicted with pressure drop decreasing with increasing liquid flow rate. Although this trends reverses for higher liquid flow rates, it contradicts the experimentally observed behaviour in the region tested.

Table 4
Comparision of Observed Pressure Drop, *G*, with the Three-Dimensional Theory for Stratified Laminar-Laminar Flow

u_{gs} (cm/s)	u_{ls} (cm/s)	G_l (Pa/m)	G_g	G_l (Pa/m)	G_g
10.6	1.4	4±5	0.2±1	40.5	0.18
10.6	3.7	6±5	0.4±1	5.7	1.4
10.6	5.9	7±5	0.7±1	5.1	3.6
21.2	1.4	4±5	0.5±1	45.7	0.35
21.2	3.7	6±5	0.9±1	9.2	0.83
21.2	5.9	8±5	1.4±1	7.0	2.9
53.1	3.76	6±5	1.9±1	18.0	1.5
53.1	5.9	7±5	2.8±1	12.0	2.5

However, when considering the pressure drop in the gas phase the experiment and the theory agree quite well. The obvious explanation for the behaviour observed is the method used to predict pressure drop. The results indicate that the one-dimensional pressure drop simplification undermines the work done in determining the volume flow rates.

8. DISCUSSION AND FURTHER RESEARCH

From the right point of view further research work should concentrate on improved methods for analytical determination of the *pressure* drops in each phase.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the financial support from the Council of Scientific and Industrial Research (EMR-II) Division, New Delhi, under Grant No. **25(0190)/10/EMR-II.** The authors also acknowledge the help from IIT Guwahati particularly Dr. A. Saikia, Department of Chemistry for using instrument measuring pressure drops in each phase**.**

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