Indian Journal of Mathematics and Mathematical Sciences Vol. 12, No. 1, (January. 2016)

HIGHER DIMENSIONAL PLANE SYMMETRICAL SPACE-TIME IN POLAR COORDINATE SYSTEM

S. S. Pokley & K. T. Thomas*

ABSTRACT

The general form of the plane symmetric line element of V_5 in spherical polar coordinates have been derived through the study of killing vectors

$$\xi^{1i} = (0, \sin\phi, \cot\theta\cos\phi, 0, 0),$$

$$\xi^{2i} = (\sin\theta\sin\phi, \frac{1}{r}\cos\theta\sin\phi, \frac{1}{r}\frac{\cos\phi}{\sin\theta}, 0, 0),$$

$$\xi^{3i} = (\cos\theta, -\frac{1}{r}\sin\theta, 0, 0, 0),$$

admitted by the plane symmetric space-time V_5. Here we obtained most general form of the plane symmetric line element of V_5 in spherical polar coordinates (r, θ, ϕ, t, u) as

$$ds^2 = f_1[dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2] + f_2[\cot^2\theta\, d\theta^2 - d\theta^2 + \tan^2\phi d\phi^2 - \sin^2\theta d\phi^2$$

$$+\frac{2\cot\theta}{r}drd\theta - \frac{2\tan\phi}{r}drd\phi - 2\cot\theta\tan\phi\, d\theta\, d\phi]$$

$$+ f_3[\frac{2}{r}drdt + 2\cot\theta\, d\theta dt - 2\tan\phi d\phi dt]$$

$$+ f_4[\frac{2}{r}drdu + 2\cot\theta d\theta du - 2\tan\phi d\phi du] + f_5 dt^2 + f_6 du^2 + 2f_7 dtdu$$

where $f_1 = -A$, $f_2 = Br\tan\theta$, $f_3 = C\tan\theta$, $f_4 = E\tan\theta$, $f_5 = D$, $f_6 = -G$, $f_7 = F$ *and A, B, C, D, E, F, G are the functions of* (r, θ, ϕ, t, u).

Key Words: *Killing Vectors, Plane Symmetry.*

* Department of Mathematics, Kavikulguru Institute of Technology and Science, Ramtek, M.S. India.

INTRODUCTION

Plane symmetric (PS) space-time have been studied by Takeno (1966). Takeno defines a four dimensional Riemannian space of signature –2 is plane symmetric if it admits the three parameter group of transformations

$$R_1 = z\partial_y - y\partial_z,\ R_2 = \partial_y,\ R_3 = \partial_z, \tag{1.1}$$

where $\partial_y = \dfrac{\partial}{\partial y}$ etc. as a subgroup of motions. Here the coordinates are (x, y, z, t) and t corresponds to time. If we transform the coordinates (x, y, z) by

$$x = r\sin\theta\cos\phi,\quad y = r\sin\theta\sin\phi,\quad z = r\cos\theta$$

then we obtain polar coordinates (r, θ, ϕ) and the expressions of R_1, R_2 and R_3 become

$$\left.\begin{aligned} R_1 &= \sin\phi\,\partial_\theta + \cot\theta\cos\phi\,\partial_\phi, \\ R_2 &= \sin\theta\sin\phi\,\partial_r + \frac{\cos\theta\sin\phi}{r}\partial_\theta + \frac{\cos\phi}{r\sin\theta}\partial_\phi, \\ \text{and}\quad R_3 &= \cos\theta\,\partial_r - \frac{\sin\theta}{r}\partial_\theta, \end{aligned}\right\} \tag{1.2}$$

where $\partial_\theta \equiv \dfrac{\partial}{\partial\theta}$ etc.

Here extending Takeno's work on V_4 to V_5. We define a five dimensional space-time V_5 is PS if it admits

$$\left.\begin{aligned} \xi^{1i} &= (0, \sin\phi, \cot\theta\cos\phi, 0, 0), \\ \xi^{2i} &= \left(\sin\theta\sin\phi,\ \frac{1}{r}\cos\theta\sin\phi,\ \frac{1}{r}\frac{\cos\phi}{\sin\theta},\ 0,\ 0\right), \\ \xi^{3i} &= \left(\cos\theta,\ -\frac{1}{r}\sin\theta,\ 0,\ 0,\ 0\right) \end{aligned}\right\} \tag{1.3}$$

as the killing vectors.

THE PLANE SYMMETRIC LINE ELEMENT OF V_5 IN SPHERICAL POLAR COORDINATES

Theorem: The general form of the line element of the plane symmetrical space-time in polar coordinates system is given by

$$ds^2 = f_1 [dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2] + f_2[\cot^2\theta \, d\theta^2 - d\theta^2 + \tan^2\phi d\phi^2$$

$$- \sin^2\theta d\phi^2 + 2\frac{\cot\theta}{r} drd\theta - \frac{2\tan\phi}{r} drd\phi - 2\cot\theta \tan\phi \, d\theta d\phi]$$

$$+ f_3 [\frac{2}{r} drdt + 2\cot\theta \, d\theta dt - 2\tan\phi \, d\phi dt]$$

$$+ f_4[\frac{2}{r} drdu + 2\cot\theta d\theta du - 2\tan\phi d\phi du]$$

$$+ f_5 dt^2 + f_6 du^2 + 2 f_7 dtdu \quad (2.1)$$

where $f_1 = -A$, $f_2 = Br\tan\theta$, $f_3 = C\tan\theta$, $f_4 = E\tan\theta$, $f_5 = D$, $f_6 = -G$, $f_7 = F$ and A, B, C, D, E, F, G are the functions of (r, θ, ϕ, t, u).

Proof : We define a space-time

$$ds^2 = g_{ij} \, dx^i dx^j, (i, j = 1, 2, 3, 4, 5) \quad (2.2)$$

is said to be PS if it admits the three parameter group of transformations given in (1.2) with killing vectors (1.3). Here the Killing vectors ξ^σ satisfy the Killing equation

$$\xi^\sigma \frac{\partial g_{ij}}{\partial x^\sigma} + g_{\sigma j} \frac{\partial \xi^\sigma}{\partial x^i} + g_{i\sigma} \frac{\partial \xi^\sigma}{\partial x^j} = 0$$

or

$$\xi^1 \frac{\partial g_{ij}}{\partial x^1} + \xi^2 \frac{\partial g_{ij}}{\partial x^2} + \xi^3 \frac{\partial g_{ij}}{\partial x^3} + \xi^4 \frac{\partial g_{ij}}{\partial x^4} + \xi^5 \frac{\partial g_{ij}}{\partial x^5}$$

$$+ g_{1j} \frac{\partial \xi^1}{\partial x^i} + g_{2j} \frac{\partial \xi^2}{\partial x^i} + g_{3j} \frac{\partial \xi^3}{\partial x^i} + g_{4j} \frac{\partial \xi^4}{\partial x^i} + g_{5j} \frac{\partial \xi^5}{\partial x^i}$$

$$+ g_{i1} \frac{\partial \xi^1}{\partial x^j} + g_{i2} \frac{\partial \xi^2}{\partial x^j} + g_{i3} \frac{\partial \xi^3}{\partial x^j} + g_{i4} \frac{\partial \xi^4}{\partial x^j} + g_{i5} \frac{\partial \xi^5}{\partial x^j} = 0 \quad (2.3)$$

where ξ^{1i}, ξ^{2i} and ξ^{3i} are given by (1.3). Inserting the values of ξ^{1i} in (2.3) and varying i and j from 1 to 5, we obtain the following set of equations

$$\sin\phi\frac{\partial g_{11}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{11}}{\partial\phi}=0 \tag{2.4a$_1$}$$

$$\sin\phi\frac{\partial g_{12}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{12}}{\partial\phi}-g_{13}\cos\phi\cos ec^2\theta=0 \tag{2.4a$_2$}$$

$$\sin\phi\frac{\partial g_{13}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{13}}{\partial\phi}+g_{12}\cos\phi-g_{13}\sin\phi\cot\theta=0 \tag{2.4a$_3$}$$

$$\sin\phi\frac{\partial g_{14}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{14}}{\partial\phi}=0 \tag{2.4a$_4$}$$

$$\sin\phi\frac{\partial g_{15}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{15}}{\partial\phi}=0 \tag{2.4a$_5$}$$

$$\sin\phi\frac{\partial g_{22}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{22}}{\partial\phi}-2g_{23}\cos\phi\mathrm{cosec}^2\theta=0 \tag{2.4a$_6$}$$

$$\sin\phi\frac{\partial g_{23}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{23}}{\partial\phi}-g_{33}\cos\phi\mathrm{cosec}^2\theta+g_{22}\cos\phi-g_{23}\cot\theta\sin\phi=0 \tag{2.4a$_7$}$$

$$\sin\phi\frac{\partial g_{24}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{24}}{\partial\phi}-g_{34}\cos\phi\mathrm{cosec}^2\theta=0 \tag{2.4a$_8$}$$

$$\sin\phi\frac{\partial g_{25}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{25}}{\partial\phi}-g_{35}\cos\phi\mathrm{cosec}^2\theta=0 \tag{2.4a$_9$}$$

$$\sin\phi\frac{\partial g_{33}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{33}}{\partial\phi}+2g_{23}\cos\phi-2g_{33}\cot\theta\sin\phi=0 \tag{2.4a$_{10}$}$$

$$\sin\phi\frac{\partial g_{34}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{34}}{\partial\phi}+g_{24}\cos\phi-g_{34}\cot\theta\sin\phi=0 \tag{2.4a$_{11}$}$$

$$\sin\phi\frac{\partial g_{35}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{35}}{\partial\phi}+g_{25}\cos\phi-g_{35}\cot\theta\sin\phi=0 \tag{2.4a$_{12}$}$$

$$\sin\phi\frac{\partial g_{44}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{44}}{\partial\phi}=0 \tag{2.4a$_{13}$}$$

$$\sin\phi\frac{\partial g_{45}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{45}}{\partial\phi}=0 \qquad (2.4a_{14})$$

$$\sin\phi\frac{\partial g_{55}}{\partial\theta}+\cot\theta\cos\phi\frac{\partial g_{55}}{\partial\phi}=0\cdot \qquad (2.4a_{15})$$

Similarly substituting the values of ξ^{2i} and ξ^{3i} in (2.3), we obtain

$$\sin\theta\sin\phi\frac{\partial g_{11}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{11}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{11}}{\partial\phi}-2g_{12}\frac{\cos\theta\sin\phi}{r^2}-2g_{13}\frac{\cos\phi}{r^2\sin\theta}=0 \qquad (2.5a_1)$$

$$\sin\theta\sin\phi\frac{\partial g_{12}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{12}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{12}}{\partial\phi}-g_{22}\frac{\cos\theta\sin\phi}{r^2}-g_{23}\frac{\cos\phi}{r^2\sin\theta}$$

$$-g_{13}\frac{\cos\phi\operatorname{cosec}\theta\cot\theta}{r}+g_{11}\cos\theta\sin\phi-g_{12}\frac{\sin\theta\sin\phi}{r}=0 \qquad (2.5a_2)$$

$$\sin\theta\sin\phi\frac{\partial g_{13}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{13}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{13}}{\partial\phi}-g_{23}\frac{\cos\theta\sin\phi}{r^2}-g_{33}\frac{\cos\phi}{r^2\sin\theta}$$

$$+g_{11}\sin\theta\cos\phi+g_{12}\frac{\cos\theta\cos\phi}{r}-g_{13}\frac{\sin\phi}{r\sin\theta}=0 \qquad (2.5a_3)$$

$$\sin\theta\sin\phi\frac{\partial g_{14}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{14}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{14}}{\partial\phi}-g_{24}\frac{\cos\theta\sin\phi}{r^2}-g_{34}\frac{\cos\phi}{r^2\sin\theta}=0 \qquad (2.5a_4)$$

$$\sin\theta\sin\phi\frac{\partial g_{15}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{15}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{15}}{\partial\phi}-g_{25}\frac{\cos\theta\sin\phi}{r^2}-g_{35}\frac{\cos\phi}{r^2\sin\theta}=0 \qquad (2.5a_5)$$

$$\sin\theta\sin\phi\frac{\partial g_{22}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{22}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{22}}{\partial\phi}+2g_{12}\cos\theta\sin\phi$$

$$-2g_{22}\frac{\sin\theta\sin\phi}{r}-2g_{23}\frac{\cos\phi\operatorname{cosec}\theta\cot\theta}{r}=0 \qquad (2.5a_6)$$

$$\sin\theta\sin\phi\frac{\partial g_{23}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{23}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{23}}{\partial\phi}+g_{13}\cos\theta\sin\phi-g_{23}\frac{\sin\theta\sin\phi}{r}$$

$$-g_{33}\frac{\cos\phi\operatorname{cosec}\theta\cot\theta}{r}+g_{21}\sin\theta\cos\phi+g_{22}\frac{\cos\theta\cos\phi}{r}-g_{23}\frac{\sin\phi}{r\sin\theta}=0 \qquad (2.5a_7)$$

$$\sin\theta\sin\phi\frac{\partial g_{24}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{24}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{24}}{\partial\phi}+g_{14}\cos\theta\sin\phi-g_{24}\frac{\sin\theta\sin\phi}{r}$$

$$-g_{34}\frac{\cos\phi\operatorname{cosec}\theta\cot\theta}{r}=0 \qquad (2.5a_8)$$

$$\sin\theta\sin\phi\frac{\partial g_{25}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{25}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{25}}{\partial\phi}+g_{15}\cos\theta\sin\phi-g_{25}\frac{\sin\theta\sin\phi}{r}$$

$$-g_{35}\frac{\cos\phi\operatorname{cosec}\theta\cot\theta}{r}=0 \qquad (2.5a_9)$$

$$\sin\theta\sin\phi\frac{\partial g_{33}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{33}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{33}}{\partial\phi}+2g_{13}\sin\theta\cos\phi$$

$$+2g_{23}\frac{\cos\theta\cos\phi}{r}-2g_{33}\frac{\sin\phi}{r\sin\theta}=0 \qquad (2.5a_{10})$$

$$\sin\theta\sin\phi\frac{\partial g_{34}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{34}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{34}}{\partial\phi}+g_{14}\sin\theta\cos\phi$$

$$+g_{24}\frac{\cos\theta\cos\phi}{r}-g_{34}\frac{\sin\phi}{r\sin\theta}=0 \qquad (2.5a_{11})$$

$$\sin\theta\sin\phi\frac{\partial g_{35}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{35}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{35}}{\partial\phi}+g_{15}\sin\theta\cos\phi$$

$$+g_{25}\frac{\cos\theta\cos\phi}{r}-g_{35}\frac{\sin\phi}{r\sin\theta}=0 \qquad (2.5a_{12})$$

$$\sin\theta\sin\phi\frac{\partial g_{44}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{44}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{44}}{\partial\phi}=0 \qquad (2.5a_{13})$$

$$\sin\theta\sin\phi\frac{\partial g_{45}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{45}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{45}}{\partial\phi}=0 \qquad (2.5a_{14})$$

$$\sin\theta\sin\phi\frac{\partial g_{55}}{\partial r}+\frac{\cos\theta\sin\phi}{r}\frac{\partial g_{55}}{\partial\theta}+\frac{\cos\phi}{r\sin\theta}\frac{\partial g_{55}}{\partial\phi}=0 \qquad (2.5a_{15})$$

and

$$\cos\theta \frac{\partial g_{11}}{\partial r} - \frac{\sin\theta}{r}\frac{\partial g_{11}}{\partial \theta} + 2g_{12}\frac{\sin\theta}{r^2} = 0 \qquad (2.6a_1)$$

$$\cos\theta \frac{\partial g_{12}}{\partial r} - \frac{\sin\theta}{r}\frac{\partial g_{12}}{\partial \theta} + g_{22}\frac{\sin\theta}{r^2} - g_{11}\sin\theta - g_{12}\frac{\cos\theta}{r} = 0 \qquad (2.6a_2)$$

$$\cos\theta \frac{\partial g_{13}}{\partial r} - \frac{\sin\theta}{r}\frac{\partial g_{13}}{\partial \theta} + g_{23}\frac{\sin\theta}{r^2} = 0 \qquad (2.6a_3)$$

$$\cos\theta \frac{\partial g_{14}}{\partial r} - \frac{\sin\theta}{r}\frac{\partial g_{14}}{\partial \theta} + g_{24}\frac{\sin\theta}{r^2} = 0 \qquad (2.6a_4)$$

$$\cos\theta \frac{\partial g_{15}}{\partial r} - \frac{\sin\theta}{r}\frac{\partial g_{15}}{\partial \theta} + g_{25}\frac{\sin\theta}{r^2} = 0 \qquad (2.6a_5)$$

$$\cos\theta \frac{\partial g_{22}}{\partial r} - \frac{\sin\theta}{r}\frac{\partial g_{22}}{\partial \theta} - 2g_{12}\sin\theta - 2g_{22}\frac{\cos\theta}{r} = 0 \qquad (2.6a_6)$$

$$\cos\theta \frac{\partial g_{23}}{\partial r} - \frac{\sin\theta}{r}\frac{\partial g_{23}}{\partial \theta} - g_{13}\sin\theta - g_{23}\frac{\cos\theta}{r} = 0 \qquad (2.6a_7)$$

$$\cos\theta \frac{\partial g_{24}}{\partial r} - \frac{\sin\theta}{r}\frac{\partial g_{24}}{\partial \theta} - g_{14}\sin\theta - g_{24}\frac{\cos\theta}{r} = 0 \qquad (2.6a_8)$$

$$\cos\theta \frac{\partial g_{25}}{\partial r} - \frac{\sin\theta}{r}\frac{\partial g_{25}}{\partial \theta} - g_{15}\sin\theta - g_{25}\frac{\cos\theta}{r} = 0 \qquad (2.6a_9)$$

$$\cos\theta \frac{\partial g_{33}}{\partial r} - \frac{\sin\theta}{r}\frac{\partial g_{33}}{\partial \theta} = 0 \qquad (2.6a_{10})$$

$$\cos\theta \frac{\partial g_{34}}{\partial r} - \frac{\sin\theta}{r}\frac{\partial g_{34}}{\partial \theta} = 0 \qquad (2.6a_{11})$$

$$\cos\theta \frac{\partial g_{35}}{\partial r} - \frac{\sin\theta}{r}\frac{\partial g_{35}}{\partial \theta} = 0 \qquad (2.6a_{12})$$

$$\cos\theta\frac{\partial g_{44}}{\partial r}-\frac{\sin\theta}{r}\frac{\partial g_{44}}{\partial\theta}=0 \tag{2.6a$_{13}$}$$

$$\cos\theta\frac{\partial g_{45}}{\partial r}-\frac{\sin\theta}{r}\frac{\partial g_{45}}{\partial\theta}=0 \tag{2.6a$_{14}$}$$

$$\cos\theta\frac{\partial g_{55}}{\partial r}-\frac{\sin\theta}{r}\frac{\partial g_{55}}{\partial\theta}=0\cdot \tag{2.6a$_{15}$}$$

The equations (2.4a$_1$ to 2.4a$_{15}$), (2.5a$_1$ to 2.5a$_{15}$) and (2.6a$_1$ to 2.6a$_{15}$) can be solved for ten g_{ij}, from the equations (2.4a$_1$ to 2.4a$_{15}$) and (2.6a$_1$ to 2.6a$_{15}$), we get

$$\sin\phi\cos\theta\frac{\partial g_{11}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{11}}{\partial\phi}+2g_{12}\frac{\sin\theta\sin\phi}{r^2}=0 \tag{2.7a$_1$}$$

$$\sin\phi\cos\theta\frac{\partial g_{12}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{12}}{\partial\phi}-g_{13}\frac{\cos\phi}{r\sin\theta}+g_{22}\frac{\sin\theta\sin\phi}{r^2}$$

$$-g_{11}\sin\theta\sin\phi-g_{12}\frac{\sin\phi\cos\theta}{r}=0 \tag{2.7a$_2$}$$

$$\sin\phi\cos\theta\frac{\partial g_{13}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{13}}{\partial\phi}+g_{12}\frac{\sin\theta\cos\phi}{r}-g_{13}\frac{\sin\phi\cos\theta}{r}+g_{23}\frac{\sin\theta\sin\phi}{r^2}=0 \tag{2.7a$_3$}$$

$$\sin\phi\cos\theta\frac{\partial g_{14}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{14}}{\partial\phi}+g_{24}\frac{\sin\theta\sin\phi}{r^2}=0 \tag{2.7a$_4$}$$

$$\sin\phi\cos\theta\frac{\partial g_{15}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{15}}{\partial\phi}+g_{25}\frac{\sin\theta\sin\phi}{r^2}=0 \tag{2.7a$_5$}$$

$$\sin\phi\cos\theta\frac{\partial g_{22}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{22}}{\partial\phi}-2g_{23}\frac{\cos\phi}{r\sin\theta}-2g_{12}\sin\theta\sin\phi-2g_{22}\frac{\sin\phi\cos\theta}{r}=0 \tag{2.7a$_6$}$$

$$\sin\phi\cos\theta\frac{\partial g_{23}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{23}}{\partial\phi}-g_{33}\frac{\cos\phi}{r\sin\theta}+g_{22}\frac{\sin\theta\cos\phi}{r}$$

$$-g_{13}\sin\theta\sin\phi-2g_{23}\frac{\sin\phi\cos\theta}{r}=0 \tag{2.7a$_7$}$$

$$\sin\phi\cos\theta\frac{\partial g_{24}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{24}}{\partial\phi}-g_{34}\frac{\cos\phi}{r\sin\theta}-g_{14}\sin\theta\sin\phi-g_{24}\frac{\sin\phi\cos\theta}{r}=0 \tag{2.7a$_8$}$$

$$\sin\phi\cos\theta\frac{\partial g_{25}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{25}}{\partial\phi}-g_{35}\frac{\cos\phi}{r\sin\theta}-g_{15}\sin\theta\sin\phi-g_{25}\frac{\sin\phi\cos\theta}{r}=0 \quad (2.7a_9)$$

$$\sin\phi\cos\theta\frac{\partial g_{33}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{33}}{\partial\phi}+2g_{23}\frac{\sin\theta\cos\phi}{r}-2g_{33}\frac{\cos\theta\sin\phi}{r}=0 \quad (2.7a_{10})$$

$$\sin\phi\cos\theta\frac{\partial g_{34}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{34}}{\partial\phi}+g_{24}\frac{\sin\theta\cos\phi}{r}-g_{34}\frac{\cos\theta\sin\phi}{r}=0 \quad (2.7a_{11})$$

$$\sin\phi\cos\theta\frac{\partial g_{35}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{35}}{\partial\phi}+g_{25}\frac{\sin\theta\cos\phi}{r}-g_{35}\frac{\cos\theta\sin\phi}{r}=0 \quad (2.7a_{12})$$

$$\sin\phi\cos\theta\frac{\partial g_{44}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{44}}{\partial\phi}=0 \quad (2.7a_{13})$$

$$\sin\phi\cos\theta\frac{\partial g_{45}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{45}}{\partial\phi}=0 \quad (2.7a_{14})$$

$$\sin\phi\cos\theta\frac{\partial g_{55}}{\partial r}+\frac{\cos\theta\cos\phi}{r}\frac{\partial g_{55}}{\partial\phi}=0\cdot \quad (2.7a_{15})$$

Similarly from equations ($2.4a_1$ to $2.4a_{15}$) and ($2.5a_1$ to $2.5a_{15}$), we have

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{11}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{11}}{\partial r}+2g_{12}\frac{\cos^2\theta\sin\phi}{r^2}+2g_{13}\frac{\cot\theta\cos\phi}{r^2}=0 \quad (2.8a_1)$$

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{12}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{12}}{\partial r}+g_{22}\frac{\cos^2\theta\sin\phi}{r^2}-g_{13}\frac{\cos\phi}{r}$$
$$+g_{23}\frac{\cot\theta\cos\phi}{r^2}-g_{11}\cos^2\theta\sin\phi+g_{12}\frac{\cos\theta\sin\theta\sin\phi}{r}=0 \quad (2.8a_2)$$

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{13}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{13}}{\partial r}+g_{23}\frac{\cos^2\theta\sin\phi}{r^2}+g_{12}\frac{\sin^2\theta\cos\phi}{r}$$
$$+g_{33}\frac{\cot\theta\cos\phi}{r^2}-g_{11}\cos\theta\sin\theta\cos\phi=0 \quad (2.8a_3)$$

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{14}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{14}}{\partial r}+g_{24}\frac{\cos^2\theta\sin\phi}{r^2}+g_{34}\frac{\cot\theta\cos\phi}{r^2}=0 \quad (2.8a_4)$$

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{15}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{15}}{\partial r}+g_{25}\frac{\cos^2\theta\sin\phi}{r^2}+g_{35}\frac{\cot\theta\cos\phi}{r^2}=0 \quad (2.8a_5)$$

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{22}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{22}}{\partial r}-2g_{23}\frac{\cos\phi}{r}-2g_{12}\cos^2\theta\sin\phi$$
$$+2g_{22}\frac{\cos\theta\sin\theta\sin\phi}{r}=0 \quad (2.8a_6)$$

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{23}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{23}}{\partial r}-g_{33}\frac{\cos\phi}{r}+g_{22}\frac{\sin^2\theta\cos\phi}{r}$$

$$-g_{13}\cos^2\theta\sin\phi+g_{23}\frac{\cos\theta\sin\theta\sin\phi}{r}-g_{12}\cos\theta\sin\theta\cos\phi=0 \qquad (2.8a_7)$$

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{24}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{24}}{\partial r}-g_{34}\frac{\cos\phi}{r}-g_{14}\cos^2\theta\sin\phi$$

$$+g_{24}\frac{\cos\theta\sin\theta\sin\phi}{r}=0 \qquad (2.8a_8)$$

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{25}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{25}}{\partial r}-g_{35}\frac{\cos\phi}{r}-g_{15}\cos^2\theta\sin\phi$$

$$+g_{25}\frac{\cos\theta\sin\theta\sin\phi}{r}=0 \qquad (2.8a_9)$$

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{33}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{33}}{\partial r}+2g_{23}\frac{\sin^2\theta\cos\phi}{r}-2g_{13}\cos\theta\sin\theta\cos\phi=0 \qquad (2.8a_{10})$$

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{34}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{34}}{\partial r}+g_{24}\frac{\sin^2\theta\cos\phi}{r}-g_{14}\cos\theta\sin\theta\cos\phi=0 \qquad (2.8a_{11})$$

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{35}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{35}}{\partial r}+g_{25}\frac{\sin^2\theta\cos\phi}{r}-g_{15}\cos\theta\sin\theta\cos\phi=0 \qquad (2.8a_{12})$$

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{44}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{44}}{\partial r}=0 \qquad (2.8a_{13})$$

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{45}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{45}}{\partial r}=0 \qquad (2.8a_{14})$$

$$\frac{\sin^2\theta\sin\phi}{r}\frac{\partial g_{55}}{\partial\theta}-\cos\theta\sin\theta\sin\phi\frac{\partial g_{55}}{\partial r}=0\cdot \qquad (2.8a_{15})$$

Similarly the set of equations ($2.5a_1$ to $2.5a_{15}$) and ($2.6a_1$ to $2.6a_{15}$), gives

$$\frac{\sin\phi}{r}\frac{\partial g_{11}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{11}}{\partial\phi}-2g_{12}\frac{\sin\phi}{r^2}-2g_{13}\frac{\cot\theta\cos\phi}{r^2}=0 \qquad (2.9a_1)$$

$$\frac{\sin\phi}{r}\frac{\partial g_{12}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{12}}{\partial\phi}-g_{22}\frac{\sin\phi}{r^2}-g_{13}\frac{\cot^2\theta\cos\phi}{r}$$

$$-g_{23}\frac{\cot\theta\cos\phi}{r^2}+g_{11}\sin\phi=0 \qquad (2.9a_2)$$

$$\frac{\sin\phi}{r}\frac{\partial g_{13}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{13}}{\partial\phi}-g_{23}\frac{\sin\phi}{r^2}-g_{13}\frac{\cot\theta\sin\phi}{r}$$

$$-g_{33}\frac{\cot\theta\cos\phi}{r^2}+g_{11}\cos\theta\sin\theta\cos\phi+g_{12}\frac{\cos^2\theta\cos\phi}{r}=0 \qquad (2.9a_3)$$

$$\frac{\sin\phi}{r}\frac{\partial g_{14}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{14}}{\partial\phi}-g_{24}\frac{\sin\phi}{r^2}-g_{34}\frac{\cot\theta\cos\phi}{r^2}=0 \qquad (2.9a_4)$$

$$\frac{\sin\phi}{r}\frac{\partial g_{15}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{15}}{\partial\phi}-g_{25}\frac{\sin\phi}{r^2}-g_{35}\frac{\cot\theta\cos\phi}{r^2}=0 \qquad (2.9a_5)$$

$$\frac{\sin\phi}{r}\frac{\partial g_{22}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{22}}{\partial\phi}+2g_{12}\sin\phi-2g_{23}\frac{\cot^2\theta\cos\phi}{r}=0 \qquad (2.9a_6)$$

$$\frac{\sin\phi}{r}\frac{\partial g_{23}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{23}}{\partial\phi}+g_{13}\sin\phi-g_{33}\frac{\cot^2\theta\cos\phi}{r}$$

$$+g_{12}\cos\theta\sin\theta\cos\phi+g_{22}\frac{\cos^2\theta\cos\phi}{r}-g_{23}\frac{\cot\theta\sin\phi}{r}=0 \qquad (2.9a_7)$$

$$\frac{\sin\phi}{r}\frac{\partial g_{24}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{24}}{\partial\phi}+g_{14}\sin\phi-g_{34}\frac{\cot^2\theta\cos\phi}{r}=0 \qquad (2.9a_8)$$

$$\sin\phi\frac{\partial g_{25}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{15}}{\partial\phi}+g_{15}\sin\phi-g_{35}\frac{\cot^2\theta\cos\phi}{r}=0 \qquad (2.9a_9)$$

$$\frac{\sin\phi}{r}\frac{\partial g_{33}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{33}}{\partial\phi}+2g_{13}\cos\theta\sin\theta\cos\phi-2g_{33}\frac{\cot\theta\sin\phi}{r}$$

$$+2g_{23}\frac{\cos^2\theta\cos\phi}{r}=0 \qquad (2.9a_{10})$$

$$\frac{\sin\phi}{r}\frac{\partial g_{34}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{34}}{\partial\phi}+g_{14}\cos\theta\sin\theta\cos\phi+g_{24}\frac{\cos^2\theta\cos\phi}{r}$$

$$-g_{34}\frac{\cot\theta\sin\phi}{r}=0 \qquad (2.9a_{11})$$

$$\frac{\sin\phi}{r}\frac{\partial g_{35}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{35}}{\partial\phi}+g_{15}\cos\theta\sin\theta\cos\phi+g_{25}\frac{\cos^2\theta\cos\phi}{r}$$

$$-g_{35}\frac{\cot\theta\sin\phi}{r}=0 \qquad (2.9a_{12})$$

$$\sin\phi\frac{\partial g_{44}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{44}}{\partial\phi}=0 \qquad (2.9a_{13})$$

$$\sin\phi\frac{\partial g_{45}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{45}}{\partial\phi}=0 \qquad (2.9a_{14})$$

$$\sin\phi\frac{\partial g_{55}}{\partial\theta}+\frac{\cot\theta\cos\phi}{r}\frac{\partial g_{55}}{\partial\phi}=0\cdot \qquad (2.9a_{15})$$

Solving the equations ($2.7a_1$ to $2.7a_{15}$), ($2.8a_1$ to $2.8a_{15}$) and ($2.9a_1$ to $2.9a_{15}$) gives the relations

$$g_{12}\sin\theta\sin\phi+g_{13}\cos\theta\cos\phi=0 \qquad (2.10a_1)$$

$$g_{23}\cos\phi+g_{12}r\sin\phi=0 \qquad (2.10a_2)$$

$$-g_{11}\sin\phi+g_{22}\frac{\sin\phi}{r^2}+g_{23}\frac{\cot\theta\cos\phi}{r^2}-g_{13}\frac{\cos\phi}{r}=0 \qquad (2.10a_3)$$

$$g_{12}\frac{\sin^2\theta}{r}+g_{23}\frac{\tan\phi}{r^2}+g_{33}\frac{\cot\theta}{r^2}-g_{11}\sin\theta\cos\theta=0 \qquad (2.10a_4)$$

$$g_{24}\sin\theta\sin\phi+g_{34}\cos\theta\cos\phi=0 \qquad (2.10a_5)$$

$$g_{24}\sin\theta-g_{14}r\cos\theta=0 \qquad (2.10a_6)$$

$$g_{25}\sin\theta\sin\phi+g_{35}\cos\theta\cos\phi=0 \qquad (2.10a_7)$$

$$g_{35}\cos\phi+g_{15}\,r\sin\phi=0\cdot \qquad (2.10a_8)$$

These are eight equations in fifteen unknowns (g_{ij}). So we choose seven of them as independent parameters i.e. $g_{11} = -A, \ g_{12} = B, \ g_{24} = C, g_{44} = D, \ g_{25} = E, g_{45} = F$ and $g_{55} = -G$ (2.11)

Other g_{ij} can be expressed in terms of them as follows

$$\left.\begin{aligned} g_{13} &= -B\tan\theta\tan\phi, \\ g_{23} &= -Br\tan\phi, \\ g_{14} &= \frac{C\tan\theta}{r}, \\ g_{34} &= -C\tan\theta\tan\phi, \\ g_{22} &= -Ar^2 + Br\cot\theta - Br\tan\theta, \\ g_{33} &= -Ar^2\sin^2\theta + Br\tan\theta\tan^2\phi - Br\tan\theta\sin^2\theta, \\ g_{15} &= \frac{E\tan\theta}{r}, \\ g_{35} &= -E\tan\theta\tan\phi\cdot \end{aligned}\right\} \qquad (2.12)$$

Then the line element in spherical polor coordinate is ds_5^2 i.e. equation (2.1). Hence follows the theorem.

PLANE SYMMETRIC SPACE-TIME IN POLAR COORDINATES SYSTEM FOR V_6 AND V_7

By straight forward calculations the respective forms of PS space-time in Polar coordinates for six and seven dimensions are

$$\begin{aligned} ds_6^2 = {} & f_1[dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2] \\ & + f_2[\cot^2\theta d\theta^2 - d\theta^2 + \tan^2\phi d\phi^2 - \sin^2\theta d\phi^2 + \frac{2\cot\theta}{r} dr d\theta \\ & - \frac{2\tan\phi}{r} dr d\phi - 2\cot\theta\tan\phi d\theta d\phi] + f_3[\frac{2}{r} dr dt + 2\cot\theta d\theta dt - 2\tan\phi d\phi dt] \\ & + f_4[\frac{2}{r} dr du + 2\cot\theta d\theta du - 2\tan\phi d\phi du] + f_8[\frac{2}{r} dr dv + 2\cot\theta d\theta dv - 2\tan\phi d\phi dv] \\ & + f_5 dt^2 + f_6 du^2 + 2 f_7 dt du + f_9 dv^2 + 2 f_{10} dt dv + 2 f_{11} du dv \end{aligned}$$

and

$$
\begin{aligned}
ds_7^2 = {} & f_1[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \\
& + f_2[\cot^2\theta d\theta^2 - d\theta^2 + \tan^2\phi d\phi^2 - \sin^2\theta d\phi^2 + \frac{2\cot\theta}{r} dr d\theta \\
& - \frac{2\tan\phi}{r} dr d\phi - 2\cot\theta\tan\phi d\theta d\phi] + f_3[\frac{2}{r} dr dt + 2\cot\theta d\theta dt - 2\tan\phi d\phi dt] \\
& + f_4[\frac{2}{r} dr du + 2\cot\theta d\theta du - 2\tan\phi d\phi du] + f_8[\frac{2}{r} dr dv + 2\cot\theta d\theta dv - 2\tan\phi d\phi dv] \\
& + f_{12}[\frac{2}{r} dr dw + 2\cot\theta d\theta dw - 2\tan\phi d\phi dw] \\
& + f_5 dt^2 + f_6 du^2 + 2 f_7 dt du + f_9 dv^2 + 2 f_{10} dt dv \\
& + 2 f_{11} du dv + f_{13} dw^2 + 2 f_{14} dt dw + 2 f_{15} du dw + 2 f_{16} dv dw.
\end{aligned}
$$

REDUCTION IN NUMBER OF g_{ij}

For V_n the number of independent g_{ij} is given by $m(m+1)/2$ and for V_5 it is fifteen. In V_5 the imposition of the PS space time in polar coordinates reduces the number to seven only, which is an advantage. Hence for V_5 the reduction in number of g_{ij} is 15–7 = 8, similarly for V_6 the reduction is 21–11 = 10 and for V_7 the reduction is 28–16 = 12 etc.

Therefore we observe that for V_n, the reduction in the number of independent components of g_{ij} is

$$\frac{m(m+1)}{2} - \frac{(m^2 - 3m + 4)}{2} = \frac{m(m+1)}{2} - \frac{[(m-2)^2 + m]}{2} = 2(m-1)\,;\ (m = n)$$

by mathematical induction.

ACKNOWLEDGEMENTS

The authors express their sincere thanks to Dr. T. M. Karade, former Dean, Faculty of Science, Nagpur University, Nagpur for his constant encouragements.

REFERENCES

[1] Takeno H., “The theory of spherically symmetric space time”, Research Institute of Theoretical Physics, Hiroshima University, Japan, (1966).

[2] Gaikwad Vidya, “Plane symmetric higher dimensional space-times”, Acta Physica Hungarica, Vol. 65, No. 4, (1989).

[3] Khadekar G. S., Gaikwad Vidya and Mishra Kalpana, “Higher dimensional spherically symmetric space-times”, Proceedings of Einstein Foundation International, Vol. 4, No. 2, pp. 27-30, (1987).

[4] Karade T. M. and Tulsani V., “On spherical symmetry in General Relativity”, Acta Ciencia Indica, Vol. 4, No. 2, 215, (1978).

[5] Thomas K. T. and Pokley S. S., “Plane symmetrical space-time in polar coordinate system”, *Journal of Science and Techn.*, Sambalpur University (SUJST), Vol. XVI(B): 61-69, (2004).