EXISTENCE THEORY FOR FUNCTIONAL RANDOM DIFFERENTIAL INCLUSIONS

S. B. Patil and D.S. Palimkar

ABSTRACT: *In this paper, we prove the existence result for perturbed first order neutral functional random differential inclusion under the mixed Lipschitz, compactness and monotonicity conditions.*

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1. STATEMENT OF THE PROBLEM

Consider the neutral functional first order random differential inclusions

$$\begin{gathered}[u(t,\omega)-f(t,\omega,u_t)]' \in G(t,\omega,u_t)+H(t,\omega,u_t)+T(t,\omega,u_t),\ a.e.\ t\in J,\\ u(0,\omega)=\phi,\end{gathered} \tag{1.1}$$

Where, a function $\phi\in C,\ f: I\times\Omega\times C\to R, G, H, T: I\times\Omega\times C\to \rho_p(R), \omega\in\Omega$.

By a solution of the equation(1.1),we mean a function $u\in C(J,R)\cap AC(I,R)\cap\Omega$ such that

(i) The mapping $(t,\omega)\to[u(t,\omega)-f(t,\omega,u)]$ is a absolutely continuous on *I*, and

(ii) There exists a $\upsilon\in L^1(I,R)$ such that

$$w(t,\omega)\in G(t,\omega,u_t)+H(t,\omega,u_t)+T(t,\omega,u_t) \text{ a.e. } t\in I, \text{ Satisfying}$$

$$[u(t,\omega)-f(t,\omega,u_t)]'=w(t,\omega), \text{ for all } t\in I \text{ and } u_0=\phi\in C,\ \omega\in\Omega.$$

Where $AC(I,R)$ is the space of all absolutely continuous real-valued functions on *I*. Let *R* denote the real line. Let $I_0=[-\delta,0],\delta>0$ and $I=[0,T]$ be two closed and bounded intervals in *R*. Let $C=C(I_0,R)$ denote the Banach space of all continuous *R*-valued functions on I_0 with the usual supremum norm $\|.\|_C$ given by

$$\|\phi\|_C=\sup\{|\phi(\theta)|:-\delta\le\theta\le 0\}.$$

For any continuous R-valued function u defined on the interval J, where $J=[-\delta,T]=I_0\bigcup I$, and for any $t\in I$, we denote by u_t the element of C defined by

$$u_t(\theta)=u(t+\theta),\ -\delta\le\theta\le 0\}.$$

The problem(1.1) have been studied in the literature for different aspects of the solutions under different continuity conditions in Dhage,Ntouyas [7], Deimling [9], Dhage,Ntouyas and Palimkar[8,11,12] .In this paper we will prove the existence theorem for problem(1.1) under the mixed Lipschitz, compactness and right monotonic conditions.

2. PRELIMINARIES

We quoted some lemmas which are useful for proving main result.

Lemma 2.1. Let E be a Banach space. If $\dim(E)<\infty$ and $\beta: J\times E\to\rho_{cp}(E)$ is L^1-Caratheodory, then $S^1_\beta(x)\neq\emptyset$ for each

Lemma 2.2. Let E be a Banach space, $\beta: J\times E\to\rho_{cp}(E)$ an L^1 Caratheodory multi-valued mapping with $S^1_\beta(x)\neq\emptyset$ and let $K: L^1(I,R)\to C(I,E)$ be a linear continuous mapping. Then the composition operator $K\,0S^1_\beta: C(I,E)\to\rho_{cp}(C(I,E))$ is a closed graph operator in

We need the following definitions.

Definition 2.1. A multi- valued mapping $\beta: I\times C\to\rho_{cp}(R)$ is called L^1-random caratheodory if

(i) $(t,\omega)\to\beta(t,\omega,u_t)$ is Lebesgue measurable for each $x\in C(J,R)$

(ii) $u\to\beta(t,\omega,u_t)$ is right monotone increasing almost everywhere for $t\in I$, $\omega\in\Omega$ and

(iii) For each real number $r>0$ there exists a function $h_r\in L^1(I,R)$ such that

$\|\beta(t,\omega,u)\|_\rho=\sup\{|x|: x\in\beta(t,\omega,u)\}\le h_r(t,\omega)$ a.e. $t\in I$,$\omega\in\Omega$ for all $u\in C$ with $\|u\|_C\le r$.

Definition 2.2. A function $\alpha\in C(J,R)\cap AC(I,R)$ is called a strict lower random solution of problem(1.1) if $(t,\omega)\to[\alpha(t,\omega)-f(t,\omega,a_t)]$ is absolutely continuous on I and for all $v_1\in S^1_G(\alpha)$ and $v_2\in S^1_H(\alpha)$ we have that $[\alpha(t,\omega)-f(t,\omega,\alpha_t)]'\le v_1(t,\omega)+v_2(t,\omega)$ for all $t\in I$ and $\alpha_0\le\phi$. Similarly, a function $\beta \in C(J,R) \cap AC(I,R)$ is called a strict upper random solution of problem(1.1) if $(t,\omega) \to [\beta(t,\omega) - f(t,\omega,\beta_t)]$ is absolutely continuous on I and for all $v_1 \in S_{\frac{1}{G}}(\beta)$ and $v_2 \in S_{\frac{1}{H}}(\beta)$ we have that $[\beta(t,\omega) - f(t,\omega,\beta_t)]' \geq v_1(t,\omega) + v_2(t,\omega)$ for all $t \in I$ and

Lemma2.3.[1]., If S is a bounded set in the Banach space X, then $\alpha(S) \leq 2\beta(S)$.

Lemma 2.4. [1]. If $A : X \to X$ is a single-valued D-Lipschitz mapping with the D-function ψ, that is, $\| Au - Av \| \leq \psi(\| u - v \|)$ for all $u, v \in X$, then we have $\alpha(A(S)) \leq \psi(\alpha(s))$ for any bounded subset S of X.

A random fixed point theorem for right monotone increasing multi-valued countably condensing operators is

Theorem 2.1. Let $[\alpha, \beta]$ be a norm-bounded order interval in the ordered normed linear space X and let $T : \Omega \times [\alpha, \beta] \to \rho_{cl}([\alpha, \beta])$ be a upper semi- continuous and countably condensing. Furthermore, if T is right monotone increasing, then T has a random fixed point in $[\alpha, \beta]$.

An improvement upon the multi-valued analogue of Tarski's fixed point theorem is the following fixed point theorem.

Theorem 2.2. Let $[\alpha , \beta]$ be an order interval in a subset Y of an ordered Banach space X and let $Q : \Omega \times [\alpha, \beta] \to \rho_{cp}([\alpha, \beta])$ be a right monotone increasing (resp. left monotone increasing) multi-valued random operator. If every monotone increasing(resp. decreasing) sequence $\{y_n\} \subset \bigcup Q([a,b])$ defined by $v_n \in Q(\omega)u_n, n \in N$ converges in Y, whenever $\{u_n\}$ is a monotone increasing (resp. decreasing) sequence in then has a random fixed point.

3. EXISTENCE RESULTS

To prove the main result. we need some of the following theorems and lemma.

Theorem3.1. Let $[\alpha , \beta]$ be a norm-bounded random order interval in a subset Y of an ordered Banach space X and let $T : \Omega \times [\alpha , \beta] \times [\alpha , \beta] \to \rho_{cp}([\alpha , \beta])$ be a multi- valued mapping satisfying the following conditions.

(a) .The multi-valued mapping $u \to T(\omega, u, v)$ is upper semi- continuous uniformly for $v \in [\alpha, \beta]$.

(b) .The multi-valued mapping $u \to T(\omega, u, v)$ is countably condensing and right monotone increasing for all $v \in X$.

(c) . $v \to T(\omega,u,v)$ is right monotone increasing for all $u \in [\alpha,\beta]$, $\omega \in \Omega$ and

(d) .Every monotone increasing sequence $\{z_n\} \subset \cup T(\Omega \times [\alpha,\beta] \times [\alpha,\beta])$ defined by $z_n \in T(\omega,u,v_n), n \in N$, $\omega \in \Omega$ converges for each $\omega \in \Omega$, $u \in [\alpha,\beta]$ whenever $\{v_n\}$ is a monotone increasing sequence in $[\alpha,\beta]$.

Then the inclusion $u \in T(\omega,u,u)$ has a random solution in $u \in [\alpha,\beta]$.

Lemma 3.1. Let $A,B : X \to \rho_{cp}(X)$ be two multivalued operators satisfying

(a) .A is *a* multi- valued *D*- contraction, and

(b) .B is completely continuous.

Then the mutlivalued operator $T : X \to \rho_{cp}(X)$ defined by $Tu = Au + Bu$ is upper semi- continuous and β- condensing on *X*.

Theorem 3.2. Let $[\alpha,\beta]$ be an order interval in the ordered Banach space *X* and let $A,B,C : \Omega \times [\alpha,\beta] \to \rho_{cp}(X)$ be three right monotone increasing multi-valued operators satisfying

(a) . $A(\omega)$ is a multi-valued *D*-contraction.

(b) . $B(\omega)$ is completely continuous,

(c) .Every monotone increasing sequence $\{z_n\} \subset \cup C(\Omega,[\alpha,\beta])$ defined by $z_n \in C(\omega,v_n), n \in N$ converges, whenever $\{v_n\}$ is a monotone increasing sequence in $u \in [\alpha,\beta]$, and

(d) .The elements α and β satisfy $\alpha \le A(\omega)\alpha + B(\omega)\alpha + C(\omega)\alpha$ and $A(\omega)\beta + B(\omega)\beta + C(\omega)\beta \le \beta$. Furthermore, if the cone *K* in *X* is normal then the random operator inclusion $u \in A(\omega)u + B(\omega)u + C(\omega)u$ hasa random solution in $[\alpha,\beta]$.

Corollary 3.1. Let $[\alpha,\beta]$ be an order interval in the ordered Banach space *X*. Let $A,B : \Omega \times [\alpha,\beta] \to \rho_{cp}(X)$ be two right monotone increasing and $C : [\alpha,\beta] \to X$ be a nondecreasing operator satisfying

(a) $A(\omega)$ is *a* single-valued contraction,

(b) $B(\omega)$ is completely continuous,

(c) Every sequence $\{z_n\} \subset \cup C([\alpha,\beta])$ defined by $z_n \in C(v_n), n \in N$ has a cluster point, whenever $\{v_n\}$ is a monotone increasing sequence in $u \in [\alpha,\beta]$, and

(d) The elements α and β satisfy $\alpha \le A(\omega)\alpha + B(\omega)\alpha + C(\omega)\alpha$ and

$A(\omega)\beta + B(\omega)\beta + C(\omega)\beta \leq \beta.$ Furthermore, if the cone K in X is normal, then the operator inclusion $u \in A(\omega)u + B(\omega)u + C(\omega)u$ has a solution in $u \in [\alpha, \beta]$.

4. MAIN RESULT

We consider the following assumptions.

(A_1). $f(0, \omega, u) = 0$ for each $\omega \in \Omega, u \in C$.

(A_2). The mapping f is continuous on $I \times \Omega \times C$ and there exists a real-valued bounded function ℓ on I such that

$|f(t,\omega,u) - f(t,\omega,v)| \leq \ell(t,\omega) \| u - v \|_C,$ for all $(t,\omega,u), (t,\omega,v) \in I \times \Omega \times C$.

(A_3). The mapping $f(t,\omega,u)$ is nondecreasing in u for almost everywhere $\omega \in \Omega, t \in I$.

(A_4). $G(t,\omega,u)$ is compact subset of R for each $t \in I$, and $u \in C$.

(A_5). G is L^1-random caratheodory.

(A_6). The multi- valued mapping $G(t,\omega,u)$ is right monotone increasing in u for almost everywhere $t \in I$, $\omega \in \Omega$.

(A_7). The multi- valued $(u,\omega) \to S_G^1(u,\omega)$ is right monotone increasing in $C(J,\Omega,R)$.

(A_8). $H(t,\omega,u)$, $T(t,\omega,u)$ are compact subset of R for each $t \in I$, $\omega \in \Omega$ and $u \in C$.

(A_9). *H,T are* L^1-random caratheodory.

(A_{10}). The multi- valued $(u,\omega) \to S_H^1(u,\omega)$ is right monotone increasing in $C(J,\Omega,R)$.

(A_{11}). The problem (1.1) has a strict lower random solution α and a strict upper random solution β with $\alpha \leq \beta$.

Theorem 4.1. Suppose that the assumptions $(A_1) - (A_{11})$ holds. Furthermore, if $\| l \| < 1$, then the problem (1.1) has a random solution in $[\alpha,\beta]$ on *J*.

Proof. Let $X = C(J,R)$ and define an order interval $[\alpha,\beta]$ in $C(J,R)$ which does exist from assumtion (A_{11}). Note that the cone *K* is normal in *X*, and therefore, the order interval $[\alpha,\beta]$ is norm bounded in *X*. As a result, there is a constant $r>0$ such that $\|x\|\le r$ for all $u\in[\alpha,\beta]$

Now problem (1.1) is equivalent to the integral inclusion as

$$u(t,\omega)\in\phi(0,\omega)-f(0,\omega,\phi)+f(t,\omega,u_t)+\int_0^t G(t,\omega,u_s)ds+\int_0^t H(t,\omega,u_s)ds+\int_0^t T(t,\omega,u_s)ds,$$

if $t\in I$. Satisfying

$$u(t,\omega)=\phi(t,\omega),\ \text{if } t\in I_0.$$

Define three multi- valued operators $A,B,C,D:[\alpha,\beta]\to\rho_p(X)$ by

$$A(\omega)u(t,\omega)=-f(0,\omega,\phi)+f(t,\omega,u_t),if\ \ t\in I,$$
$$=0,if\quad t\in I_0.$$

$$B(\omega)u(t,\omega)=\phi(0,\omega)+\int_0^t G(s,\omega,u_s)ds,\ \ if\ \ t\in I,$$
$$B(\omega)u(t,\omega)=\phi(t,\omega),\ if\quad t\in I_0$$

$$C(\omega)u(t,\omega)=\int_0^t H(s,\omega,u_s)ds,if\ \ t\in I,$$
$$C(\omega)u(t,\omega)=0,if\ \ t\in I_0.$$

and

$$D(\omega)u(t,\omega)=\int_0^t T(s,\omega,u_s)ds,if\ \ t\in I,$$
$$D(\omega)u(t,\omega)=0,if\ \ t\in I_0.$$

each $\omega\in\Omega$, Clearly, the multi-valued random operators *A,B* and *C* are well defined in view of hypotheses (C_2) and (A_2) and map $[\alpha,\beta]$ into X . Now the problem (1.1) is transformed into an operator inclusion as

$$u(t,\omega)\in A(\omega)u(t,\omega)+B(\omega)u(t,\omega)+C(\omega)u(t,\omega)+D(\omega)u(t,\omega),\ \ \omega\in\Omega,t\in J.$$

We shall show that *A,B ,C and D* satisfy all the conditions of Corollary 3.1 on $[\alpha,\beta]$.

Step I: Firstly, we show that *A* is monotone increasing and *B* and *C* are right monotone increasing on $[\alpha,\beta]$.Let $u,v\in[\alpha,\beta]$ be such that $x\le y.$ Then,

$$\begin{aligned} A(\omega)u(t,\omega) &= -f(0,\omega,\phi)+f(t,\omega,u_t), if \ \ t\in I, \\ &= 0, if \quad t\in I_0. \\ &\le -f(0,\omega,\phi)+f(t,\omega,v_t), if \ \ t\in I, \omega\in\Omega \\ &\le 0, if \quad t\in I_0. \\ &= Av(t,\omega) \end{aligned}$$

For all $\omega\in\Omega$, $t\in J$. Hence, $Au\le Av,$ and so, the operator *A* is monotone increasing on $[\alpha,\beta]$. Since (A_7) and (A_{10}), we have that $S^1_G(u)\overset{i}{\le}S^1_G(v)$ and $S^1_H(u)\overset{i}{\le}S^1_H(v)$. As a result, we obtain $Bu\overset{i}{\le}Bv$ and $Cu\overset{i}{\le}Cv.$ Thus *B* and *C* are right monotone increasing on $[\alpha,\beta]$. By (A_{11}), $\alpha\le A\alpha+B\alpha+C\alpha+D\alpha$ and $A\beta+B\beta+C\beta+D\beta\le\beta$.

Step II: Next, we show that *A* is a contraction operator on $[\alpha,\beta]$. Let $u,v\in[\alpha,\beta]$ be arbitray. Then by hypothesis (A_8),

$$\| Au-Av\|\le\sup_{t\in J}|f(t,\omega,u_t)-f(t,\omega,v_t)|\le\sup_{t\in J}l(t,\omega)\|u_t-v_t\|_C\le\|l\|\|u-v\|.$$

This shows that *A* is contraction on $[\alpha,\beta]$ with the contraction constant $\|l\|<1$.

Step III: Secondly, we prove that the multi-valued operator *B* satisfy Theorem 2.2. It can be proved as in the Step I That B is a right monotone increasing mapping on [a,b]. We only prove that it is completely continuous on $[\alpha,\beta]$. First we prove that B maps bounded sets into bounded sets . If *S* is a bounded set in *X*, then there exists $r>0$ such that $\|u\|\le r$ for all $u\in S$. Now for each $x\in Bu,$ there exists a $w\in S_{\frac{1}{G}}(u)$ such that

$$\begin{aligned} x(t,\omega) &= \phi(0,\omega)+\int_0^t w(s,\omega)ds, if \ t\in I. \\ &= \phi(t,\omega), if \ \ t\in I_0. \end{aligned}$$

Then, for each $t\in J$,

$$|x(t,\omega)|\le\|\phi\|_C+\int_0^t|w(s,\omega)|\,ds\le\|\phi\|_C+\int_0^t h_r(s,\omega)ds\le\|\phi\|_C+\|h_r\|_{L^1}.$$

This further implies that $\| x \| \leq \| \phi \|_C + \| h_r \|_{L^1}$ for all $x \in Bu \subset \cup B(S)$. Hence, $\bigcup B(S)$ is bounded.

Next we prove that *B* maps from bounded sets into equicontinuous sets. Let *S* be, as above, a bounded set and $x \in Bu$ for some $u \in S$. Then there exists $w \in S_G^1(u)$ ast

$$x(t,\omega) = \phi(0,\omega) + \int_0^t w(s,\omega)ds, if\ t \in I.$$
$$= \phi(t,\omega), if\ \ t \in I_0.$$

Then for any $t_1, t_2 \in I$ with $t_1 \leq t_2$, we have

$$| x(t_1,\omega) - x(t_2,\omega) | \leq | \int_0^{t_1} w(s,\omega)ds - \int_0^{t_2} w(s,\omega)ds | = \int_{t_1}^{t_2} \left| w(s,\omega) \right| ds \leq \int_{t_1}^{t_2} h_r(s,\omega)ds.$$

If $t_1, t_2 \in I_0$, then $\left| x(t_1) - x(t_2) \right| = | \phi(t_1) - \phi(t_2) |$. For the case when $t_1 \leq 0 \leq t_2$. we have that $| x(t_1,\omega) - x(t_2,\omega) | \leq | \phi(t_1,\omega) - \phi(0,\omega) | + \int_0^{t_2} | w(s,\omega) | ds \leq | \phi(t_1,\omega) - \phi(0,\omega) | + \int_0^{t_2} h_r(s,\omega)ds.$

Hence, in all three cases, we have

$$| x(t_1,\omega) - x(t_2,\omega) | \to 0 \text{ as } t_1 \to t_2.$$

As a result, $\bigcup B(Q)$ is an equicontinuous set in *X*. From Arzela Ascoli theorem ,that the multi B is totally bounded on *X*. Consequently, $B : \Omega \times [\alpha, \beta] \to \rho_{cp}(X)$ is a compact multi-valued random operator.

Step IV: We prove that *B* has a closed graph in *X*. Let $\{u_n\} \subset X$ be a sequence such that $u_n \to u_*$ and let $\{v_n\}$ be a sequence defined by $v_n \in Bu_n$ for each $n \in N$ such that $v_n \to v_*$ .We will show that $v_* \in Bu_*$. Since $v_n \in Bu_n$, there exists a $w_n \in S_{\frac{1}{G}}(u_n)$ such that

$$v_n(t,\omega) = \phi(0,\omega) + \int_0^t w_n(s,\omega)ds,\ if \quad t \in I,$$
$$= \phi(t,\omega),\ if \quad\ t \in I.$$

Suppose the linear and continuous operator $K : L^1(I,R) \to C(I,R)$ defined as

$$Kw(t,\omega) = \int_0^t w(s,\omega)ds.$$

Now, when $n \to \infty$, we obtain

$$|v_n(t,\omega) - \phi(0,\omega) - (v_*(t,\omega) - \phi(0,\omega))| \le |v_n(t,\omega) - v_*(t,\omega)| \le \| v_n - v_* \| \to 0.$$

Therefore, from Lemma 4.2 it follows that (KoS_G^1) is a closed graph operator and from the definition of K one has

$$v_n(t,\omega) - \phi(0,\omega) \in (K0S_F^1(u_n)).$$

As $u_n \to u_*$ and $v_n \to v_*$, there is a $w \in S_G^1(u_*)$ such that

$$v_*(t,\omega) = \phi(0,\omega) + \int_0^t w_*(s,\omega)ds, \; if \quad t \in I,$$
$$= \phi(t,\omega), \; if \quad t \in I_0.$$

Hence, B is an upper semi- continuous multi-valued operator on $[\alpha, \beta]$.

Step V: Then, we prove that the multi-valued operator *C* satisfy Theorem 3.2. First, we show that *C* has compact values on $[\alpha, \beta]$. Observe first that the operator *C* is equivalent to

$$Cu(t,\omega) = (LoS_H^1)(u)(t,\omega), if \quad t \in I,$$
$$= 0, if \quad t \in I_{0,}.$$

Where $L : L^1(I,R) \to X$ is the continuous operator defined by

$$Lw(t,\omega) = \int_0^t w(s,\omega)ds, \; if \quad t \in I.$$

To show *C* has compact values, it then suffices to prove that the composition operator LoS_H^1 has compact values on $[\alpha, \beta]$. Let $u \in [\alpha, \beta]$ be arbitrary and let $\{w_n\}$ be a sequence in $S_H^1(x)$ Then, by definition of S_H^1, $w_n(t,\omega) \in H(t,\omega,u_t)$ a.e. for $t \in I$. Since $H(t,\omega,u_t)$ is compact, there is a convergent subsequence of $w_n(t,\omega)$ that converges in measure to some $w(t,\omega)$, where $w(t,\omega) \in H(t,\omega,u_t)$ a.e. for $t \in I$. Since continuity that $Lw_n(t,\omega) \to Lw(t,\omega)$ pointwise on *I* as $n \to \infty$. In order to show that the convergence is uniform, we first show that $\{Lw_n\}$ is an equi- continuous sequence. Let $t, \tau \in I$; then

$$| Lw_n(t,\omega) - Lw_n(\tau,\omega) | \le \left| \int_0^t w_n(s,\omega)ds - \int_0^\tau w_n(s,\omega)ds \right| \le \left| \int_\tau^t | w_n(s,\omega) | ds \right|.$$

Now, $w_n \in L^1(I,R)$, so the right hand side, tends to 0 as $t \to \tau$. Therefore, $\{Lw_n\}$ is equi-continuous. By Ascoli theorem, it has uniformly convergent subsequence. We then have $Lw_{n_j} \to Lw \in (LoS_H^1)(u)$ as $j \to \infty$, and so $(LoS_H^1)(u)$ is compact. Therefore, C is a compact –valued multi-valued operator on $[\alpha,\beta]$.

Let $\{v_n\}$ be a sequence in $\cup C([\alpha,\beta])$ defined by $v_n \in Cu_n, n \in N,$ where $\{u_n\}$ is a monotone increasing sequence in $[\alpha,\beta]$. Then there is a sequence $w_n \in S_H^1(u_n)$ such that

$$\begin{aligned} v_n(t,\omega) &= \int_0^t w_n(s,\omega)ds, if \quad t \in I, \\ &= 0, \; if \quad t \in I_{0.} \end{aligned}$$

We show that $\{v_n\}$ has a cluster point. , we have

$$| v_n(t,\omega) | \le \int_0^t | w(s,\omega) | ds \le \int_0^t h_r(s,\omega)ds \le \| h_r \|_{L^1}$$

For all $\omega \in \Omega$, $t \in J$. This implies that $\| v_n \| \le \| h_r \|_{L^1}$ and so, $\{v_n\}$ is uniformly bounded. Next we show that $\{v_n\}$ equicontinuous. For any $t_1, t_2 \in I$ with $t_1 \le t_2$, that

$$| v_n(t_1,\omega) - v_n(t_2,\omega) | \le \left| \int_0^{t_1} w_n(s,\omega)ds - \int_0^{t_2} w_n(s,\omega)ds \right| \le \int_{t_1}^{t_3} h_r(s,\omega)ds.$$

If $t_1, t_2 \in I_0$ then $| v_n(t_1,\omega) - v_n(t_2,\omega) | = 0.$ The case that, where $t_1 \le 0 \le t_2$,we have

$$| v_n(t_{1,\omega}) - v_n(t_2,\omega) | \le \left| \int_0^{t_2} w_n(s,\omega)ds \right| \le | \rho(t_2,\omega) - \rho(0,\omega) |,$$

Where $\rho(t,\omega) = \int_0^t h_r(s,\omega)ds.$ Hence, in all three cases, we have

$$| x(t_1,\omega) - x(t_2,\omega) | \to 0 \text{ as } t_1 \to t_2.$$

As a result $\{v_n\}$ is an equicontinuous set in *X*. By Arzela-Ascoli theorem that the sequence $\{v_n\}$ has a cluster point. Thus,Corollary 3.1 are satisfied and therefore the operator inclusions $u(\omega) \in A(\omega)u(\omega) + B(\omega)u(\omega) + C(\omega)u(\omega) + D(\omega)u(\omega)$ has a random solution in [*a*,*b*]. This implies that the problem (1.1) has a random solution in $[\alpha, \beta]$ on *J*.

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S. B. Patil
Department of Mathematics, Sevadas Jr.College,
Vasantnagar, Kotgyal Tq.: Mukhed,
Dist.: Nanded(MS) India
E-mail-sbpatil333@rediffmail.com

D.S. Palimkar
Department of Mathematics,
Vasantrao Naik College,
Nanded [MS] India,
Email: dspalimkar@rediffmail.com