

Neuro Fuzzy Method for Efficient Torque Response of Induction Motor Drive

J.N. Chandra Sekhar* and G.V. Marutheswar*

Abstract: This article proposes a modified Neuro Fuzzy (NF) approach dependent Direct Torque Controller (DTC) model for a non-linear system like Induction Machine Drive. A plain tuning approach has been considered which considerably optimizes the deviation between actual and required acceleration of the drive by modifying the parameters of the subsequent layers of NF Controller. For a traditional Voltage Source Inverter (VSI) fed DTC model the fixed magnitude limits of flux and torque hysteresis controllers exhibit high ripple content. In order to reduce such abnormalities regarding torque variations the hysteresis band limits are subjected to the online tuning algorithm. Further the proposed controller is effectively implemented for three-level and five-level VSI using MATLAB/SIMULINK tool and simulation is performed at various operating conditions. Finally the results display the reduced torque ripple when compared with classical PID controller.

Index Term: Induction Motor Drive, Direct Torque Control, Voltage Source Inverter, Neuro Fuzzy Controller, Torque Ripple.

1. INTRODUCTION

Most of today's industrial requirements demand rotating electrical motors which can operate under variable speed applications and can handle dynamic load changes. Induction motors (IMs) are mostly preferred in industry because of their rugged design feature, good robustness, and ease of maintenance [1]. At the late 1980s [2-3] to the present, direct torque control (DTC) has gathered the interests of ongoing research domain. Superior transient response, negligible sensitivity with regard to the machine parameters deviations, good-ease in control algorithm are some of its notable features. Conversely, with notable torque ripple and variable switching operation in the conventional DTC encumber its performance in variable speed drives. Further such operational behavior of DTC is mainly because of hysteresis level arrangement with respect to torque and flux. Various DTC control algorithms [3] are presented which address the concept constant switching mode. Conventional controllers like PI, PID generally are function of the system mathematical models [7]. Also these methods are inefficient when prone to dynamic speed deviations, load variations. In the current research scenario considerable developments on varied intelligent control methods are available like fuzzy expert control system, neural networks, artificial intelligence method which is dedicated to solve the discussed non-linear problems [8]. Of these, the fuzzy expert method may be applied to the systems that exhibit vagueness or uncertainty [9]. Then artificial neural network (ANN) method which is regarded as information processing method which impersonates the process found in biological neurons [10]. However, either fuzzy expert method or ANN carries its own demerits. In a simple fuzzy expert is utilized in the electrical motor drive system then the speed parameter is considered with tapered variations and needs good deal of tuning if high performance is to be achieved [11]. And in an ANN system, the task lies in handling the distinct training data algorithms applicable for all the operating features [12]. Since the early decade of 2010, the research community significantly focused on the application of NF Controller for non-linear electrical drive models [11]–[14]. In such systems, the traditional NF Controllers used in previous papers have an outsized number of membership functions and fuzzy rules which enhance the computational burden. In the current work, a NF Controller makes use of only three membership functions for each identified input and subsequently the output is independent of membership function. Flux variations are optimized

* S.V.U. College of Engineering, Tirupati. Email: chandu.jinka@gmail.com and marutheswargv@gmail.com

using NF Controller in [23] but the transient response of speed is not well addressed. A three-level torque hysteresis controller with preset band limits is employed for the traditional DTC model [24]. In this system, the high torque ripple gradient is observed which cross the bandwidth periphery of the hysteresis controller. Analogous to this, a Neuro-Fuzzy (NF) Controller for the IM motor drive, which has the advantages of both FLC and ANN, is presented in this paper. Further, an impartial Takagi–Sugeno–Kang (TSK)-type NF Controller is proposed to enhance the transient speed response of the considered system. The simulation studies are observed by employing the 5-level and 3-level inverter using space vector modulation (SVM)-DTC and compared with traditional PID and NF Controller.

2. INDUCTION MOTOR MODEL FOR DTC

Modern industrial applications widely rely on variable speed motors preferably Induction motor. With regard to the operational features of induction motor it is considered to be a non-linear system with composite machine parameters and this aspect of the motor attracted the interest of research community. The input machine variables and the output machine variables of the motor needs to be tuned with the preset control signals [15,16]. Conventionally, Vector control has allowed good transient feature by decoupling torque and flux variables [17]. Further this strategy involves some difficulties like fixed reference frame, diverted flux orientation, co-ordinate transformation and high sensitivity to the machine parameters [18, 19]. Consequently, the direct torque control (DTC) approach has emerged with significant acceptance in the field of high performance motor drive applications [20, 22]. But the traditional DTC method suffers with notable demerits such as variable switching operation of the voltage source inverter module, good amount of torque ripple under dynamic load changes [18,22]. Such operational defects may be solved using PID controllers in conjunction with space vector modulation (SVM) to some extent.

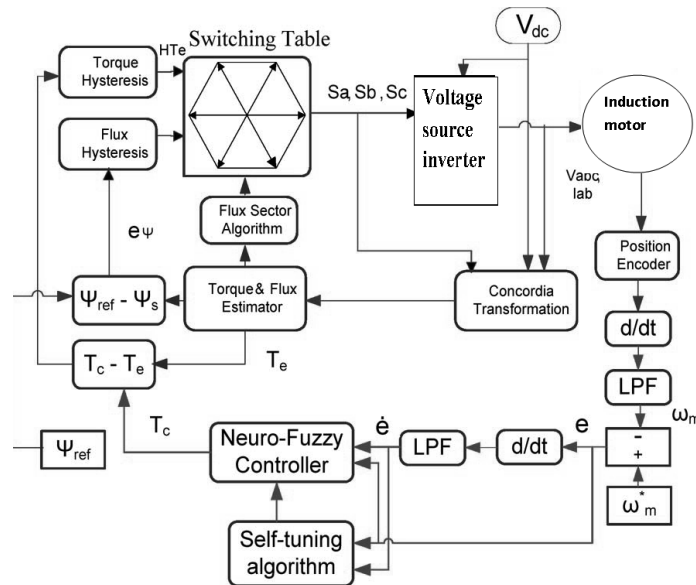


Figure 1: NF Controller based direct torque control

The dynamic model of an IM with respect to the stationary frame of reference is as follows:

$$\begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s + pL_s & 0 & pL_m & 0 \\ 0 & R_s + pL_s & 0 & pL_m \\ pL_m & Pw_m L_m & R_r + pL_r & Pw_m L_r \\ -Pw_m L_m & pL_m & -Pw_m L_r & R_r + pL_r \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} + \begin{bmatrix} \dot{i}_{sd} \\ \dot{i}_{sq} \\ \dot{i}_{rd} \\ \dot{i}_{rq} \end{bmatrix} \quad (1)$$

$$V_{sr} = \frac{V_{dc}}{3} (2S_r - S_y - S_b) \tag{2.1}$$

$$V_{sy} = \frac{V_{dc}}{3} (2S_r - S_y - S_b) \tag{2.2}$$

$$V_{sb} = \frac{V_{dc}}{3} (2S_r - S_y - S_b) \tag{2.3}$$

The stator voltage vector resolved for *d*-axis and *q*-axis references using the co-ordinate transformation is simplified as

$$\begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix} = \begin{bmatrix} 1 & 1/2 & 1/2 \\ 0 & \sqrt{3}/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} V_{sr} \\ V_{sy} \\ V_{sb} \end{bmatrix} \tag{3}$$

The motor developed electromagnetic torque can be represented by

$$T_e = \frac{3}{2} P [\psi_{sd} I_{sq} - \psi_{sq} I_{sd}] \tag{4}$$

The orientation angle for stator flux, and the resolved components are as

$$\theta_s = \tan^{-1} \left(\frac{\psi_{sq}}{\psi_{sd}} \right) \tag{5.1}$$

$$\psi_s = \sqrt{\psi_{sd}^2 + \psi_{sq}^2} \tag{5.2}$$

$$\psi_{sd} = \int (V_{sd} - R_s I_{sd}) dt \tag{6.1}$$

$$\psi_{sq} = \int (V_{sq} - R_s I_{sq}) dt \tag{6.2}$$

3. PROPOSED CONTROLLER MODEL

A. Controller Description

The Neuro-adaptive method functions similar to that of the neural networks. Basically Neuro-adaptive learning methods present a simple way for representation of fuzzy system with a defined control algorithm to learn and train the required data set. The deviation in speed be “E” and change in deviation in speed be “ΔE” which are considered as the inputs, and T_c is the response of the NF Controller. Every linguistic parameter comprises of three predefined membership functions (mfs). The Gaussian membership function is mapped to mf-ZE (zero), and the sigmoid membership function is mapped to both mf-PS (positive small) and mf-NE (negative). Thus such representation provides naturally nine rules with regard to the proposed controller.

The representation of the proposed NF Controller can be viewed in Figure 2. Also the description of each layer of the controller model can be assessed as follows.

1. *Layer-I*: In this layer, the regulated speed deviation and the rate of change of actual speed are evaluated. The response of the Layer-I, which acts as input to the next Layer-II, is as

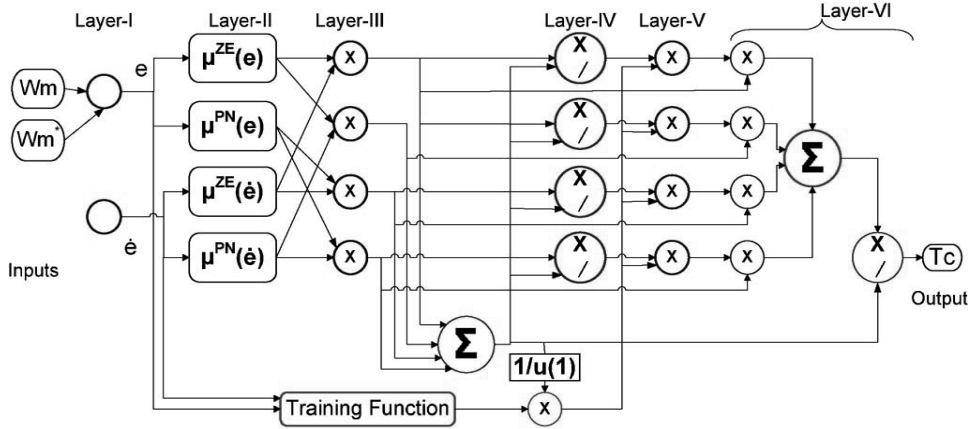


Figure 2. Structure of the proposed NF Controller

$$\delta = X_1^I = \frac{w_m^* - w_m}{w_m^*} \tag{8.1}$$

$$\delta^1 = X_2^I = \frac{w_m[n] - w_m[n-1]}{T_s} \tag{8.2}$$

2. *Layer-II*: This layer comprises the fuzzy features and each crisp input variable is fuzzified by using the respective perused membership functions. It can be observed that the geometric progression of $\mu^{ZE}(\delta)$ and $\mu^{ZE}(\delta^1)$ are symmetric about the origin. And, the $\mu^{PS}(\delta)$ and $\mu^{NE}(\delta)$ act as mirror images of each other. Describing equations at each node of the membership functions in Layer-II are simplified as

$$X_1^{II} = \mu^{PN}(\delta) \left\{ \begin{array}{ll} 0.32|\delta| & 0 \leq |\delta| \leq 0.26 \\ 1.72|\delta| - 0.364 & 0.26 < |\delta| \leq 0.75 \\ 0.3|\delta| + 0.7 & 0.75 < |\delta| \leq 1 \\ 1 & |\delta| > 1 \end{array} \right\} \tag{9.1}$$

$$X_2^{II} = \mu^{ZE}(\delta) \left\{ \begin{array}{ll} 1 - \frac{27}{26}|\delta| & 0 \leq |\delta| \leq 0.26 \\ 1.12 - 1.49|\delta| & 0.26 < |\delta| \leq 0.75 \\ 0.25(1 - |\delta|) & 0.75 < |\delta| \leq 1 \\ 0 & |\delta| > 1 \end{array} \right\} \tag{9.2}$$

$$X_3^{II} = \mu^{PN}(\delta^1) \left\{ \begin{array}{ll} 3.07 \times 10^{-3}|\delta| & 0 \leq |\delta| \leq 26 \\ 0.017|\delta| - 0.366 & 26 < |\delta| \leq 76 \\ 1.37 \times 10^{-3}|\delta| - 1043 & 76 < |\delta| \leq 120 \\ 1 & |\delta| > 120 \end{array} \right\} \tag{9.3}$$

$$X_4^{II} = \mu^{ZE}(\delta^1) \left\{ \begin{array}{ll} 1 - 0.005|\delta| & 0 \leq |\delta| < 10 \\ 0.81 - 0.014|\delta| & 10 < |\delta| \leq 70 \\ 0.315 - 0.003|\delta| & 70 < |\delta| \leq 105 \\ 0 & |\delta| > 105 \end{array} \right\} \tag{9.4}$$

3. *Layer-III*: In this layer the initiation of evaluating fuzzy rules is done by any suitable method like maximum, sum, or length. For the proposed NF Controller, the product operation is selected to implement the “AND” logic. The resultant firing intensity of respective rule is derived from the response of each node of Layer-III. The node describing equation in Layer-III can be represented as

$$X_i^{III} = W_i = \prod_{j=1,2}^{k=34} (X_j^{II} \times X_k^{II}) \tag{10}$$

4. *Layer-IV*: This is the layer where the normalized firing strength of corresponding rule is evaluated and also the last stage of evaluating the fuzzy rule with the following node equation

$$X_i^{IV} = W_i' = \frac{W_i}{\sum W_i} \tag{11}$$

5. *Layer-V*: In this layer the crisp magnitude of the variable is taken into account in place of the fuzzified values. This is the prediction of Sugeno-type NF Controller over Mamdani type which considers fuzzified values of the variable and then defuzzifies at the last stage [1]. The response of every node in Layer-V can be shown as

$$X_i^V = Z_i^{IV} \times X_i^{IV} \tag{12}$$

6. *Layer-VI*: This layer forms the end step for the determination of response of the controller which is given as

$$X^{VI} = \frac{\sum (X_i^{III} \times X_i^V)}{\sum X_i^{III}} \tag{13}$$

From the above equations the preset torque variable can be represented as

$$T_c = X^{VI} = \frac{\sum (X_i^{III} \times Z_i^{IV} \times X_i^{IV})}{\sum X_i^{III}} \tag{14}$$

B. Tuning Algorithm for the Controller

Because of the frequent varying operating conditions of the drive system, the ultimate response of the NF Controller is hard to achieve for which it require a self tuning mechanism. A corroboration signal “r” can be defined which is the difference between the preset acceleration “y” and the actual acceleration ($d\omega_m/dt$) of the motor. This corroboration signal is employed to produce the control factor to obtain the required speed response. The corresponding mathematical model is given as follows:

$$y = \begin{cases} 0.99|\delta| \times K_1 \times \text{sign}(\delta) & 0 \leq |\delta| \leq 0.02 \\ (2.854|\delta| - 0.0373) \times K_1 \times \text{sign}(\delta) & 0.02 < |\delta| \leq 0.04 \\ (3.3|\delta| - 0.055) \times K_1 \times \text{sign}(\delta) & 0.04 < |\delta| \leq 0.32 \\ K_1 \times \text{sign}(\delta) & |\delta| > 0.32 \end{cases} \tag{15}$$

The error term between the preset and the actual acceleration of the motor for the considered controller may be simplified as

$$E = \frac{1}{2} r^2 = \frac{1}{2} \left(y - \frac{dw_m}{dt} \right)^2 = \frac{1}{2} (y - X_2^I)^2 \tag{16}$$

The subsequent parameter Z_i^{IV} in the Layer-V is rationalized by the controller in every sampling instant to optimize acceleration error term to zero which can be followed as below

$$X_2^I = K_2 \times T_c = K_2 \times X^{VI} \tag{17}$$

By using all the above discussions the updation expression can be expressed as

$$Z_i^{IV}(n) = Z_i^{IV}(n-1) + \eta \left(y - \frac{dw_m}{dt} \right) \frac{X_i^{III}}{\sum X_j^{III}} \tag{18}$$

C. Hysteresis Controller for Motor Torque

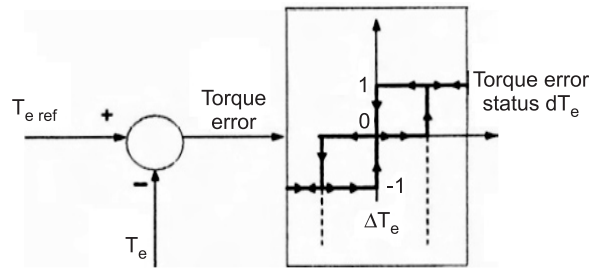


Figure 3: 3-level hysteresis controller for motor torque

In Figure 4, the hysteresis controller with three level torque magnitude is utilized for the current work. As mentioned that torque has three levels which if represented in digits can have as -1, 0, 1 for the traditional DTC model [23]. Also $T_\delta[n]$ and $T_\delta[n-1]$ are considered as present and past samples of the output digits generated by the torque hysteresis controller.

For example, the output of the torque hysteresis controller can be 1 if either $\delta_T[n] > B_{TU}$ or $\delta_T[n] > K_0$ AND $T_\delta[n-1] = 1$.

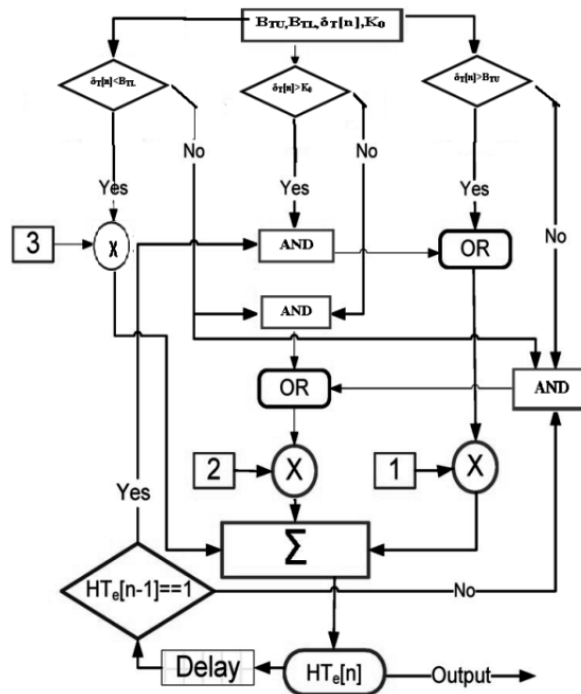


Figure 4. Flowchart for 3-level hysteresis controller for motor torque

The term torque error δ_T is the difference between the preset value and actual values of the system. The significant feature behind the idea is to make use of the zero voltage vectors for the minimization of the torque and to evade the application of the negative voltage vector to the achievable limit. Further the reduction in the negative gradient of the torque gets reduced and finally ripples in the torque response can be controlled. With the variable bandwidth of hysteresis controller optimization in the torque ripple is achieved. The term dT_e is the slope of the torque ripple and is represented by

$$dT_\delta = (T_\delta[n] - T_\delta[n - 1]) \quad (19)$$

the allowable band limits of the 3-level torque hysteresis controller are given by

$$B_{TU}^n = B_{TU} - K_U \times \Delta B_T \quad (20)$$

$$B_{TL}^n = B_{TL} - K_L \times \Delta B_T \quad (21)$$

Due to changes in the load and the speed of the motor drive, the gradient of positive torque ripple is inversely varied and the converse is the case for the negative torque gradient.

4. SIMULATION RESULTS

The performance of the proposed NF Controller utilizing DTC scheme for IM drive has been studied for both three-level and five-level inverter configurations for various operating modes. The simulated response is observed with a conventional DTC model where a PID controller with constant torque and flux hysteresis level controllers are employed. The obtained simulation results exhibit considerable variations in the developed electromagnetic torque subjected to the following variable load cases:

Case 1. Step Speed change 120 rad/sec to 160 rad/sec at half the rated load applied at 0.3 sec.

Case 2. Step Torque change 10 N-m to 17 N-m at rated speed applied at 0.3 sec.

Case 3. At rated full load value i.e. 20 N-m and at rated speed applied at 0.3 sec.

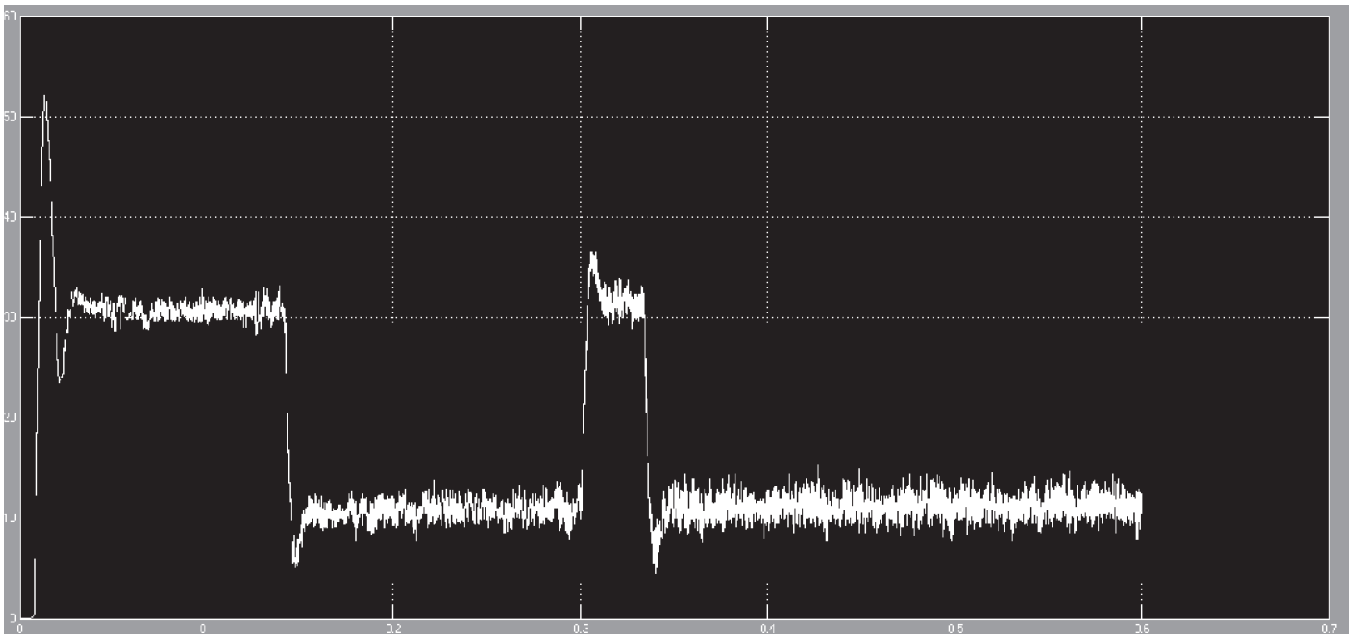
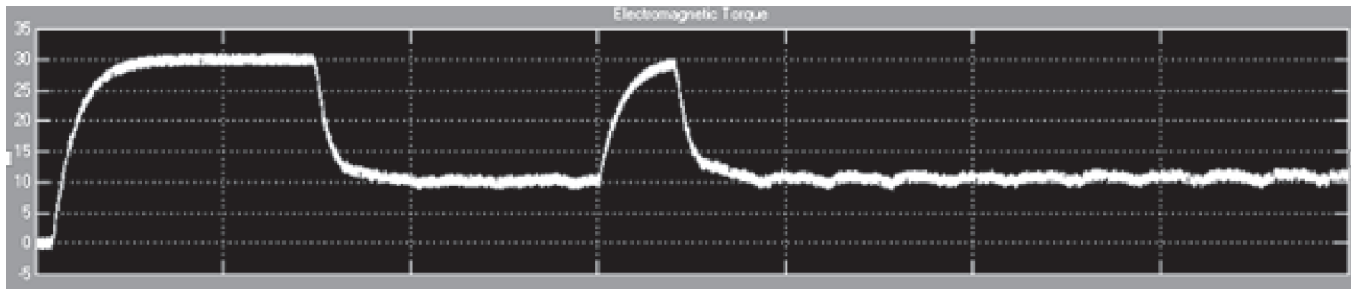


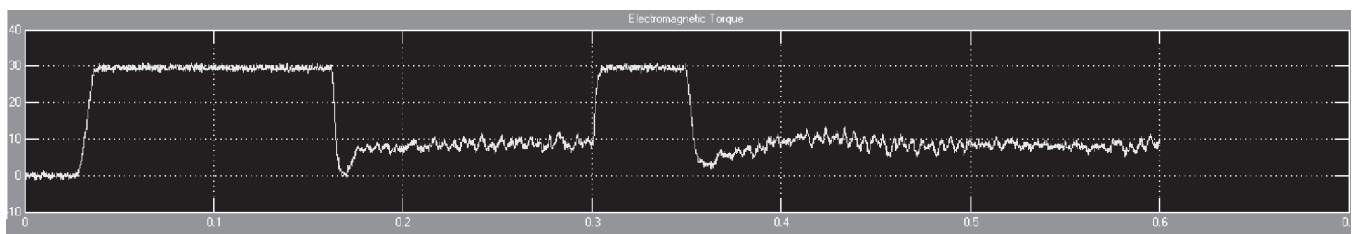
Figure 5: Torque response of the system for Case 1: PID controller

For simulation studies a step change in reference speed from 120 to 160 rad/sec when the motor is operating at 50% rated load (10 N-m) is applied at $t = 0.3$ sec. Figure 5 depicts the torque response using the conventional PID based DTC models. The proposed DTC scheme is superior to the conventional

method by exhibiting reduced torque ripple which is depicted in Figure 6 (three-level VSI) and Figure 7 (five-level).

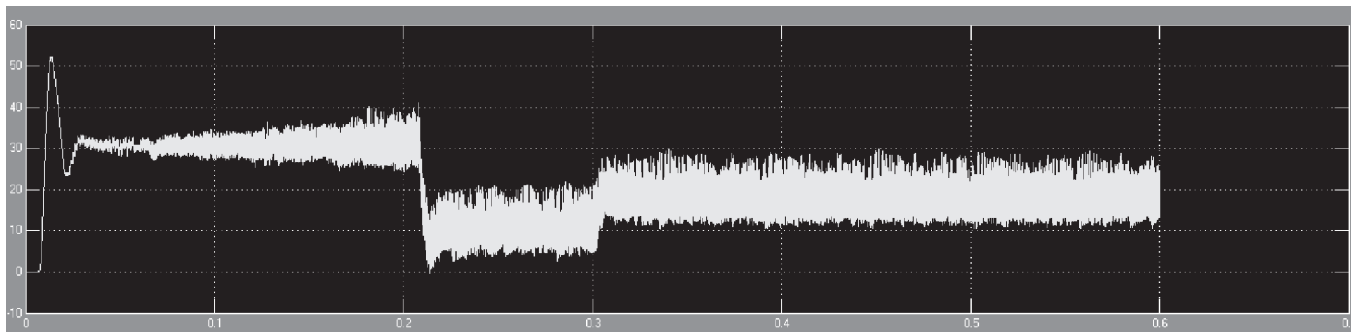


**Figure 6: Torque response of the system for Case 1:
Proposed NF Controller for three-level VSI**

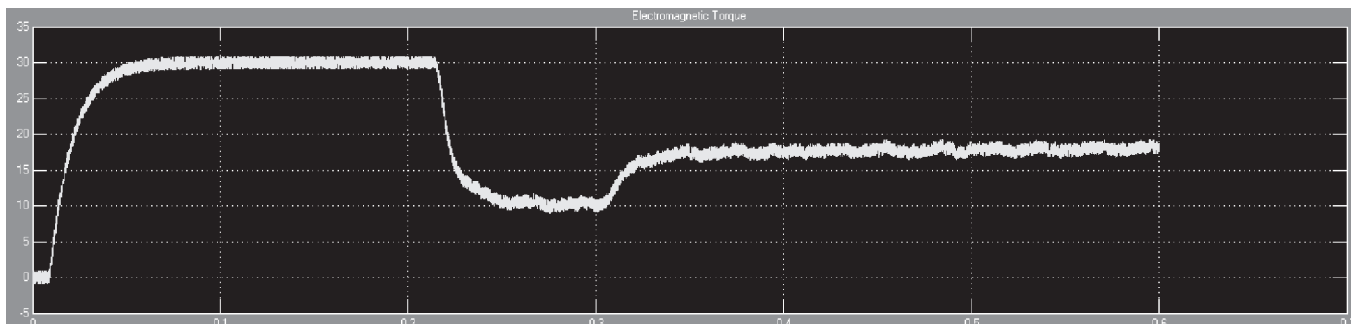


**Figure 7: Torque response of the system for Case 1:
Proposed NF Controller for five-level VSI**

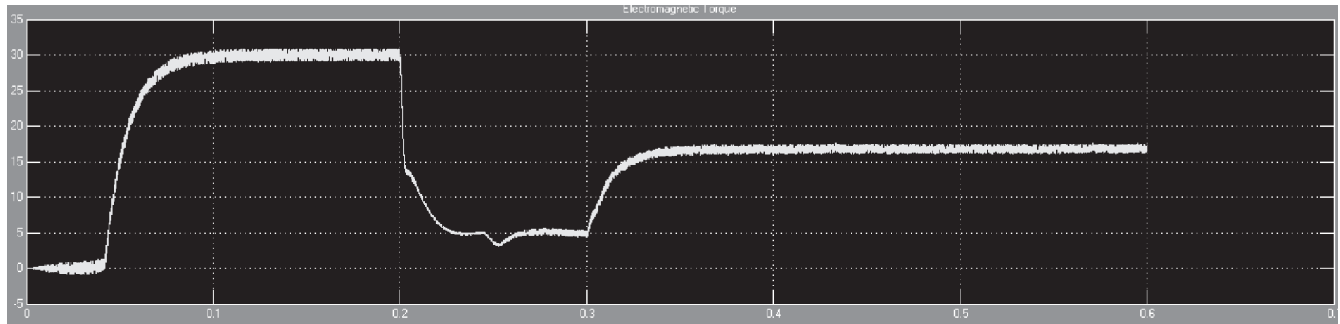
The next operating test case is carried for a step change in load torque from 10 to 17 N-m at $t = 0.3$ sec when the motor is operating at the rated speed of 180 rad/sec. It can be observed from Figure 8, Figure 9 and Figure 10 that the torque ripple has been notably reduced in the proposed scheme in dynamic operating states.



**Figure 8: Torque response of the system for Case 2:
PID controller**

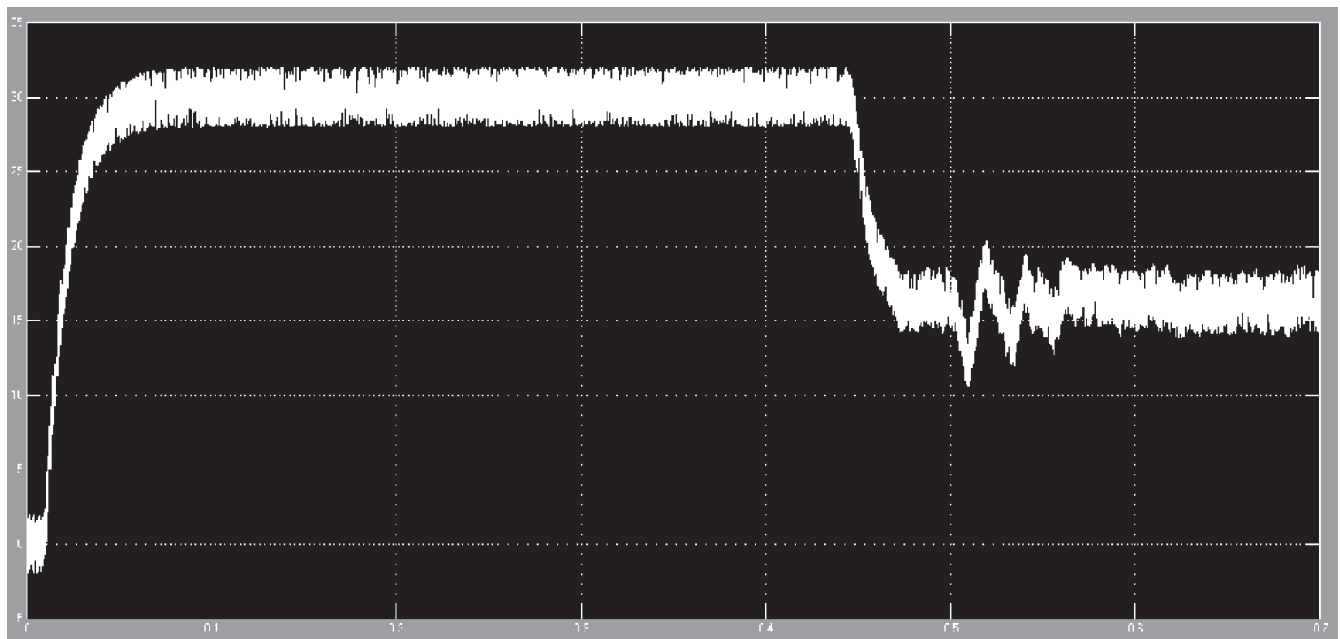


**Figure 9: Torque response of the system for Case 2:
Proposed NF Controller for three level VSI**

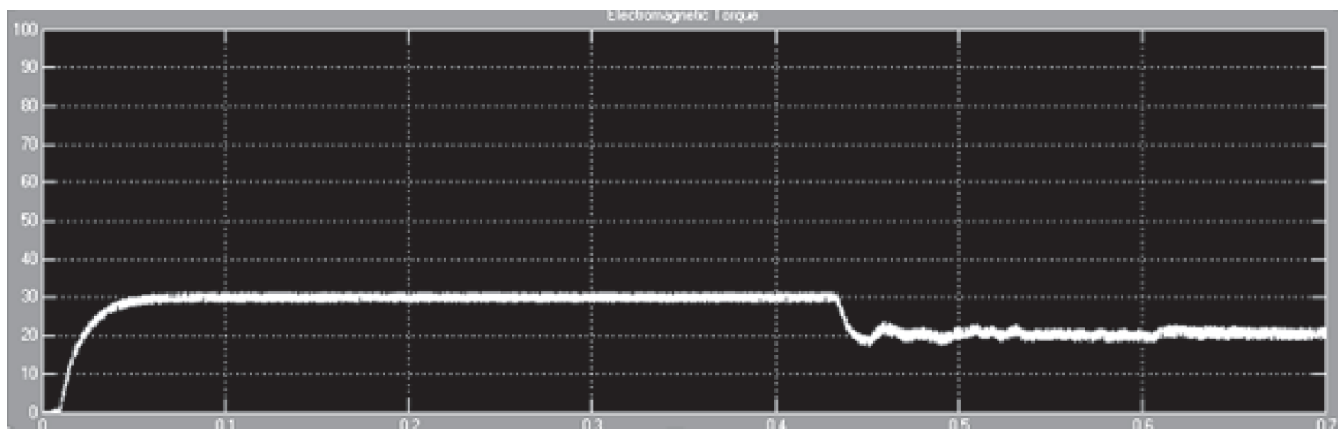


**Figure 10: Torque response of the system for Case 2:
Proposed NF Controller for five-level VSI**

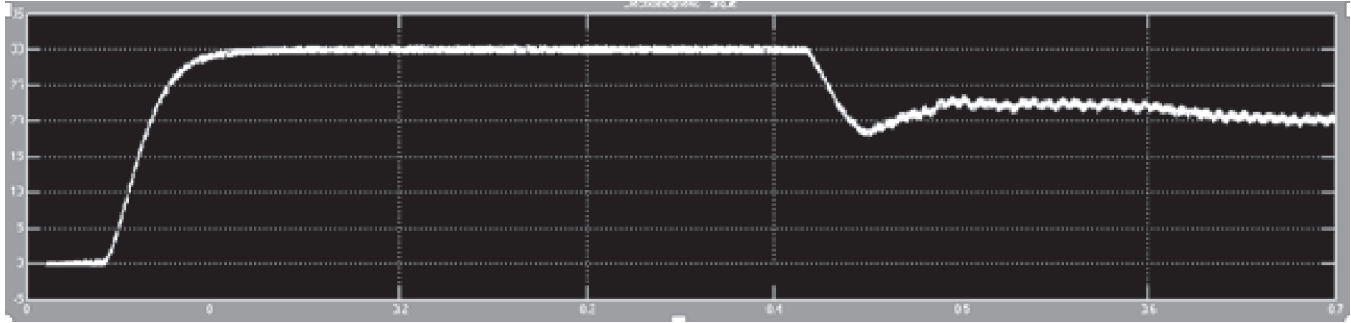
The next operating test case is carried at the rated full load value i.e. 20 N-m and at rated speed of 180 rad/sec. It can be seen from Figure 11, Figure 12 and Figure 13 that the torque ripple has been notably reduced in the proposed scheme in both transient and steady states when compared to the conventional PID based DTC model.



**Figure 11: Torque response of the system for Case 3:
PID controller**



**Figure 12: Torque response of the system for Case 3:
Proposed NF Controller for three-level VSI**



**Figure 13: Torque response of the system for Case 3:
Proposed NF Controller for five-level VSI**

**Table 1
Comparative Study of Various Controllers**

Controller \ Parameter	% Torque Ripple Case : 1	% Torque Ripple Case : 2	% Torque Ripple Case : 3
PID (3-level)	38%	28%	29%
PID (5-level)	28.5%	19.3%	20.6%
NF method (3-level)	26.1%	17.64%	18.18%
NF method (5-level)	23.25%	7.8%	6.3%

5. CONCLUSION

A modified Neuro Fuzzy (NF) Controller based Direct Torque Control method utilizing an erratic torque hysteresis level comparator is considered in this work for two configurations of Voltage Source Inverter. The proposed controller displays better improvement in the dynamic performance of the Induction Motor Drive. The proposed NF Controller is tuned to support the minimum computation effort which makes it suitable for real time implementation. To the end the performance comparison of the proposed three-level and five-level NF Controller with conventional PID utilizing DTC-based IM drive has also been presented in Table 1. Finally the tabular observations reveal the better transient operational ability of the proposed controller for which minimum torque ripples have been obtained.

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