

# CREDIT SCORING OF BANK RETAIL CLIENTS USING EMOTIONAL LEARNING-FUZZY INFERENCE SYSTEM

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**Abstract:** *This study presents an Emotional Learning-Fuzzy Inference System (ELFIS) to assess credit risk of bank retail clients (private individuals and SMEs). The main advantage of scoring models is to allow the banks to implement automated decision systems to manage their retail clients. The result of credit scoring is validated with the use of Adaptive Network Based Fuzzy Inference Systems (ANFIS). In the first step, this system designs its input-output structure. Credit risk is valued by the probability of delay the due payments. The data are gathered for 108 retail SME clients of a bank in the year 2013. To have the best performance of the models and to avoid data scale, data are preprocessed with standard methods. Then statistical methods are used to analyze data and model validation. The approach is capable of modeling credit scoring with appropriate cognitive and emotional signals. This is the first study that uses ELFIS approach for credit risk of bank retail clients.*

**Keywords:** *Emotional Learning Fuzzy Inference System (ELFIS); Credit Risk; Adaptive Neuro-Fuzzy Inference System (ANFIS); Bank Retail Clients*

## 1. INTRODUCTION

Banks, as a large part of financial systems, have encountered different challenges among them credit risk assessment of their retail clients is a vital challenge. Given that some customers after receiving credit sources are not able to repay their debts. Thus credit risk assessment of clients is an important task in managing financial resources.

Although sometimes banks continue to allocate credit to their customers without measuring, but given the importance of the need for complete knowledge of the sources of risk, accessing advanced methods of determining credit risk is inevitable. Therefore, a change in the method of credit risk scoring could help improved implementation of risk management.

In the literature, there are several methods to measure credit risk of different clients. Traditional methods including linear discriminant analysis,

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logistic regression, k nearest neighbor, kernel density estimation, and decision trees (Sabato, 2010) as well as soft computing methods are well developed to model credit risk in different modeling environments (Lee and Chen, 2005; Blanco et al, 2013; Desai et al., 1996). Normally credit risk of retail clients reflects the capability and history of retail client to repay their obligations (Back, 2005). Because the history of repayments is a reliable and the only source of credit risk, hence these kinds of data usually are used to develop and train scoring tools (Karan et al., 2008; Karan et al., 2013). Of course, along with historical data related to the repayment obligations, other non-financial measures can be found in the literature that have been used to predict credit risk (Beaver, 1967; Altman and Sabato, 2007).

Akkoç (2012) proposed a three stage hybrid Adaptive Neuro Fuzzy Inference System credit scoring model. The proposed model's performance was compared with conventional and commonly utilized models with the credit card data of an international bank operating in Turkey. Results demonstrated improved performance than the Linear Discriminant Analysis, Logistic Regression Analysis, and Artificial Neural Network (ANN) approaches, in terms of average correct classification rate and estimated misclassification cost.

Tsai and Wu (2008) investigated the performance of a single classifier as the baseline classifier to compare with multiple classifiers and diversified multiple classifiers by using neural networks based on three datasets. They suggested that it is better to consider these three classifier architectures to make the optimal financial decision in Bankruptcy prediction and credit scoring.

West (2000) investigated the credit scoring accuracy of five neural network models: multilayer perceptron, mixture-of-experts, radial basis function, learning vector quantization, and fuzzy adaptive resonance. Their results suggested that among these models, the mixture-of-experts and radial basis function neural network models are proffered for credit scoring applications.

In brief, assessing credit risk of bank clients is a complex task because many different economic, financial, psychological and sociological variables may play role in credit risk (Dimitras et al., 1996; Mannasoo and Mayes, 2009). The main purpose of this study is to develop improved soft computing method, based on the use of emotional learning fuzzy inference systems (ELFIS), for risk scoring of bank retail clients. The proposed ELFIS of this study is relevant for purposeful credit risk assessment. In the other words,

ELFIS as the scoring model can be trained in a way that the accuracy of the credit scoring for clients of high risk is improved. This is achieved by introducing an appropriate emotional signal which is more sensible to the credit scoring errors for clients of high risk. This is the first study that proposes ELFIS for improved credit scoring of bank retail clients.

Recent studies in the use of emotional learning algorithms have made significant advancement in the emotional learning-based models. Q-learning (Watkins and Dayan, 1992) and its new version, backward Q-learning (Wang et al. 2013) are among the simplest policies. Other learning algorithms include the models by Lotfi and Akbarzadeh (2013) and Lotfi and Akbarzadeh (2014). Abdi et al. 2012) proposed an emotional temporal difference learning algorithm to enhance neurofuzzy models. The key point about the application of emotional learning in all these algorithms is the capability created by emotional learning to purposeful model training (Ebrahimipour et al., 2013) Purposeful model training means to train forecasting model in a special way to improve its performance where it is desirable. Peak points, changing points, minimum level, and maximum level are some examples of desirable areas. It is noted that in this study the desirable area is that group of bank retail clients which are more potential to be a risky client. This study use ELFIS to improve neuro-fuzzy models for credit risk scoring of high risk clients.

The remainder of the paper is organized as follows. In section 2, a complete discussion of ELFIS is presented. Section 3 discusses a case study of retail credit scoring in an Iranian bank. The result of ELFIS for credit scoring in the case study is presented in section 4. The paper ends in section 5 with main findings and conclusions.

## **2. THEORY AND METHODOLOGY**

It is not easy to model uncertain behavior of high risk retail clients by conventional methods due to various changes in behavior, uncertainty, and lack of data. Therefore, exploiting the special features of emotional learning methods can be an efficient strategy to produce more accurate credit behavior. In this study, a new approach, namely Emotional Learning Fuzzy Inference System (ELFIS) is introduced and implemented for improvement of credit risk scoring. Also, to assess the acceptability and superiority of the ELFIS approach, another soft computing model namely Adaptive Neuro-Fuzzy Inference System (ANFIS) is employed and its results is compared with the results of ELFIS. The theory and methodological discussion of these techniques are presented in the following sub sections.

## 2.1 Emotional Learning Fuzzy Inference System

The Emotional learning method is a psychologically motivated approach that is employed to reduce the complexity of computations in forecasting problems. Using emotional cue in a forecasting model leads to lower the forecasting error in some regions. "It is notable that the emotional learning based fuzzy inference system (ELFIS) has the advantages of simplicity and low computational complexity in comparison with other multi-objective optimization methods. The emotional signal can be produced by any combination of objectives or goals which improve estimation or prediction" (Lucos et al., 2003).

In this learning method, a loss function will be defined as a function of emotional signal and the training algorithm will be simply developed to minimize this loss function. Therefore, the predictor model will be trained to provide the desired performance in a cognitive manner. The emotional learning algorithm has been used to enhance the performance of an adopted network trained by ANFIS predictor. The Takagi-Sugeno fuzzy (Takagi and Sugeno, 1985) inference system is constructed by fuzzy rules of the following type:

$$\begin{aligned} \text{Rule}_i : & \text{If } u_1 = A_{i1} \quad \text{And} \quad \dots \quad \text{And} \quad u_p = A_{ip} \\ & \text{Then } \hat{y} = f_i(u_1, u_2, \dots, u_p) \end{aligned} \quad (1)$$

Where  $i = 1 \dots M$  and  $M$  is the number of fuzzy rules.  $u_1, \dots, u_p$  are the inputs of network, each  $A_{ij}$  denotes the fuzzy set for input  $u_j$  in rule  $i$  and  $f_i(\cdot)$  is a crisp function which is defined as a linear combination of inputs in most applications such as:

$$\hat{y} = \omega_{i0} + \omega_{i1}u_1 + \omega_{i2}u_2 + \dots + \omega_{ip}u_p \quad (2)$$

Matrix form  $\hat{y} = a^T(\underline{u})W$

Thus the output of this model can be calculated by

$$\hat{y} = \frac{\sum_{i=1}^M f_i(\underline{u})\mu_i(\underline{u})}{\sum_{i=1}^M \mu_i(\underline{u})}; \quad \mu_i(\underline{u}) = \prod_{j=1}^p \mu_{ij}(u_j) \quad (3)$$

Where  $\mu_{ij}(u_j)$  is the membership function of  $j$ th input in the  $i$ th rule and  $\mu_i(\underline{u})$  is the degree of validity of the  $i$ th rule.

A loss function is defined on the base of emotional signal. A simple form is:

$$J = \frac{1}{2} K \sum_{i=1}^N es(i)^2 \quad (4)$$

Where  $es(i)$  is the of emotional signal to the  $i$ th sample of training data, and  $K$  is a weighting matrix, which can be simply replaced by unity. Learning is adjusting the weights of model by means of a nonlinear optimization method, e.g. the steepest descent or conjugate gradient. With steepest descent, the weights are adjusted by the following variations:

$$\Delta\omega = -\eta \frac{\partial J}{\partial \omega} \quad (5)$$

Where  $\eta$  is the learning rate of the corresponding neurofuzzy controller and the right hand side can be calculated by chain rule:

$$\frac{\partial J}{\partial \omega} = \frac{\partial J}{\partial es} \cdot \frac{\partial es}{\partial y} \cdot \frac{\partial y}{\partial \omega} \quad (6)$$

According to (1):  $\frac{\partial J}{\partial es} = K \cdot es$

And  $\frac{\partial y}{\partial \omega}$  is accessible from (3) where  $f_i(\cdot)$  is a linear function of the weights. Calculating the remaining part,  $\frac{\partial es}{\partial y}$ , is not straightforward in most cases. This is the price to be paid for the freedom to choose any desired emotional cue as well as not having to impose presuppose any predefined model. However, it can be approximated via simplifying assumptions. If, for example error is defined by

$$e = \hat{y} - y \quad (7)$$

where  $\hat{y}$  is the output to be estimated, then

$$\frac{\partial es}{\partial y} = -\frac{\partial es}{\partial e} \quad (8)$$

can be replaced by its sign (-1) in (6). The algorithm is after all, supposed to be satisfying rather than optimizing. Finally the weights will be updated by the following formula:

$$\Delta\omega = -K.\eta.es.\frac{\partial y}{\partial\omega} = -K.\eta.es.\frac{\sum_{i=1}^M u_i\mu_i(\underline{u})}{\sum_{i=1}^M \mu_i(\underline{u})} \quad (9)$$

The emotional cue is computed by a linguistic fuzzy inference system (fis) with error and rate of error change as inputs and the last targeted output.

### 2.2 Adaptive Network-Based Fuzzy Inference System

Fuzzy Logic Controllers (FLC) has played an important role in the design and enhancement of a vast number of applications. The proper selection of the number, the type and the parameter of the fuzzy membership functions and rules is crucial for achieving the desired performance and in most situations, it is difficult. Yet, it has been done in many applications through trial and error. This fact highlights the significance of tuning fuzzy system.

Adaptive Neuro-Fuzzy Inference Systems are fuzzy Sugeno models put in the framework of adaptive systems to facilitate learning and adaptation. Such framework makes FLC more systematic and less relying on expert knowledge. To present the ANFIS architecture, let us consider two-fuzzy rules based on a first order Sugeno model:

<p><i>Rule 1: if (x is A<sub>1</sub>) and (y is B<sub>1</sub>) then (f<sub>1</sub> = p<sub>1</sub>x + q<sub>1</sub>y + r<sub>1</sub>)</i>  <i>Rule 2: if (x is A<sub>2</sub>) and (y is B<sub>2</sub>) then (f<sub>2</sub> = p<sub>2</sub>x + q<sub>2</sub>y + r<sub>2</sub>)</i></p>	(10)
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One possible ANFIS architecture to implement these two rules is shown in Figure 1. In the following presentation  $O_{Li}$  denotes the output of node  $i$  in a layer  $L$ .

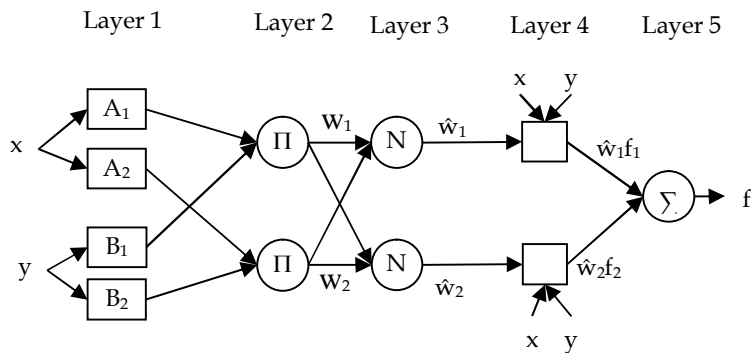


Figure 1: Construct of ANFIS

*Layer 1:* All the nodes in this layer are adaptive nodes,  $i$  is the degree of the membership of the input to the fuzzy membership function (MF) represented by the node:

$$\begin{aligned} O_{1,i} &= \mu_{A_i}(x) & i &= 1,2 \\ O_{1,i} &= \mu_{B_{i-2}}(y) & i &= 3,4 \end{aligned} \quad (10)$$

$A_i$  and  $B_i$  can be any appropriate fuzzy sets in parameter form.

*Layer 2:* The nodes in this layer are fixed (not adaptive). These are labeled  $M$  to indicate that they play the role of a simple multiplier. The outputs of these nodes are given by:

$$O_{2,i} = w_i = \mu_{A_i}(x)\mu_{B_i}(y) \quad i=1,2 \quad (11)$$

The output of each node in this layer represents the firing strength of the rule.

*Layer 3:* Nodes in this layer are also fixed nodes. These are labeled  $N$  to indicate that these perform a normalization of the firing strength from previous layer. The output of each node in this layer is given by:

$$O_{3,i} = \bar{w}_i = \frac{w_i}{w_1 + w_2} \quad i = 1,2 \quad (12)$$

*Layer 4:* All the nodes in this layer are adaptive nodes. The output of each node is simply the product of the normalized firing strength and a first order polynomial:

$$O_{4,i} = \bar{w}_i f_i = \bar{w}_i (p_i x + q_i y + r_i) \quad i = 1,2 \quad (13)$$

Where  $p_i$ ,  $q_i$  and  $r_i$  are design parameters (consequent parameter since they deal with the then-part of the fuzzy rule).

*Layer 5:* This layer has only one node labeled  $S$  to indicate that it performs the function of a simple summer. The output of this single node is given by:

$$O_{5,i} = f = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \quad i = 1,2 \quad (14)$$

The ANFIS architecture is not unique. Some layers can be combined and still produce the same output. In this ANFIS architecture, there are two adaptive layers (1, 4). Layer 1 has three modifiable parameters ( $a_i$ ,  $b_i$  and  $c_i$ ) pertaining to the input MFs. These parameters are called premise parameters. Layer 4 has also three modifiable parameters ( $p_i$ ,  $q_i$  and  $r_i$ ) pertaining to the first order polynomial. These parameters are called consequent parameters (Jang, 1997).

### 3. CASE STUDY

In this section, a case study is considered to implement credit scoring with the use of proposed ELFIS. The confidential data related to the loan repayments of 108 retail client in an Iranian bank are collected in the year of 2013. Some of these retail clients have delayed repayment of their due loans and considered to be a high risk client. Based on the interview with bank experts, among 108 clients, 51 retail clients are considered to be high risk clients. A risky client has assigned a score indicating the probability of having delay in repayment of debts. The purpose of credit scoring model is to forecast this score for each client based on some explanatory variables.

The designed ELFIS and ANFIS models use standard indicators of *Sel* and *Transaction frequency (TR Frequency)* as their inputs and *delay probability* as their output. The selected output variable (DelProb) is calculated in Equation 15.

$$\begin{aligned} \text{DelProb} &= \frac{\text{Number of delayed Payments}}{\text{Total Number of Payments}} \\ &= \frac{\text{Number of delayed Payments}}{\left( \frac{365}{\text{Mean Time between Paymenets}} \right)} \end{aligned} \quad (15)$$

The row data are presented in the Appendix. The calculated delay probabilities for the bank clients in the case study are depicted in Figure 1.



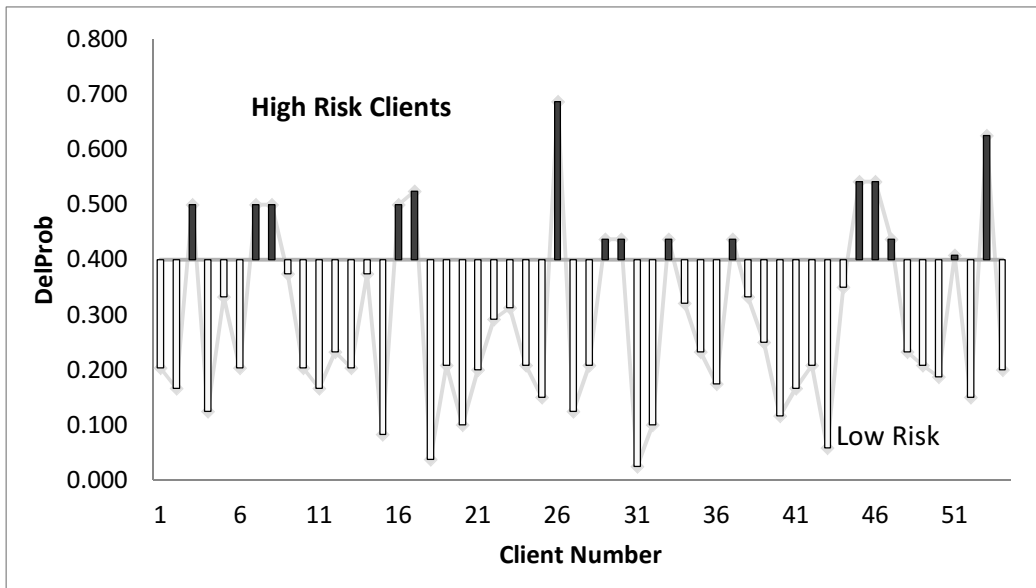


Figure 2. Delay probabilities for the bank clients

### 3.3 ANFIS

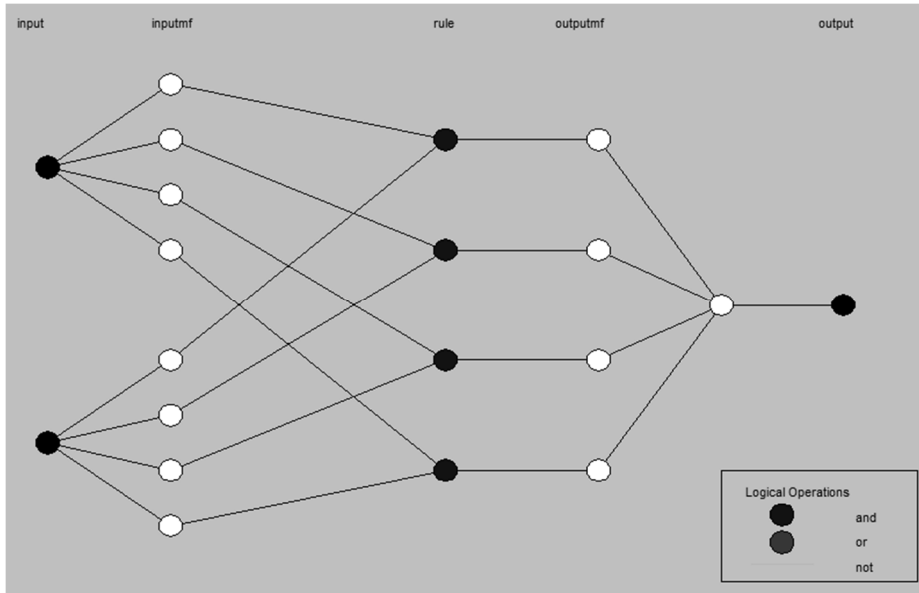
The first step to develop ELFIS is to develop an initial fuzzy inference system. To have a fair and significant judgment concerning the performance of ELFIS for credit scoring, ANFIS is implemented to create the initial FIS for ELFIS. We use MATLAB software to create an ANFIS predictor. The ANFIS procedure and learning is implemented using the standard fuzzy toolbox in MATLAB software. Table 1 presents the properties of ANFIS predictor.

Table 1.  
Properties of ANFIS

Training	Input Layer		Output Layer	Fuzzy Logic Info.	
Training Method	Number of MF's*	MF Type	MF Type	Defuzzification Method	Type
Hybrid	4,4	Bell-shaped	Linear	Weighted Average	Sugeno

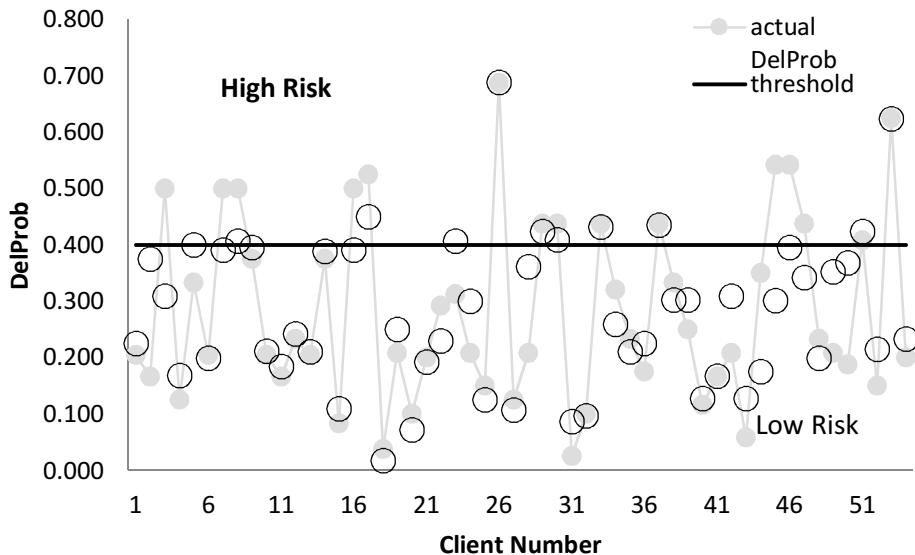
To generate an initial FIS for ANFIS, the genfis2 fuzzy toolbox of MATLAB is used. This FIS generator has generated 4 fuzzy clusters and accordingly 4 fuzzy rules for the initial FIS. Then ANFIS is trained using the actual data in the Appendix. The structure of ANFIS model is shown in Figure 3.

Figure 3: ANFIS structure



The results of ANFIS for the predicted credit scoring compared to the actual delay probabilities are shown in Figure 4. As seen, ANFIS predictor has a satisfactory results for low risk clients however its results for high risk clients is not very well and has a relatively big error.

Figure 4: The results of ANFIS for credit scoring



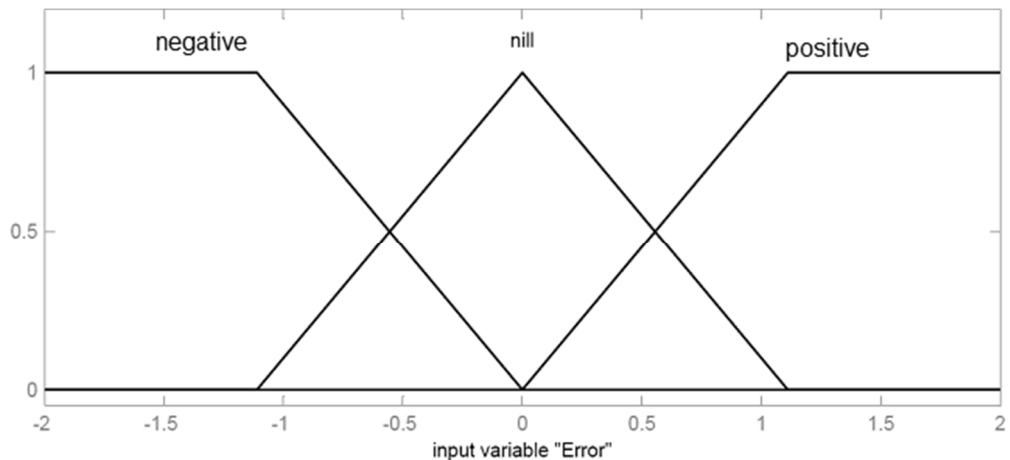
### 3.2 ELFIS

After having an initial FIS for ELFIS training, The second step to implement ELFIS is to generate an emotional cue. Emotional cue is an FIS that generates emotional signal from a set of predefined inputs. The key to determine inputs for emotional cue is to determine a set of variables that explain the purpose of emotional learning. Here, in credit scoring, the purpose of emotional learning is to improve the performance of credit scoring model for high risk clients. Therefore, the emotional signal is directly related to the magnitude of Delay probability and one of the inputs for emotional cue is *DelProb*. Another input for the emotional cue is the error of FIS. The error for each observation is calculated as the difference between the actual delay probability (actual *DelProb*) and the predicted delay probability from ELFIS (ELFIS *DelProb*) as shown in Equation 16.

Figure 5: Fuzzy membership functions for "Error"

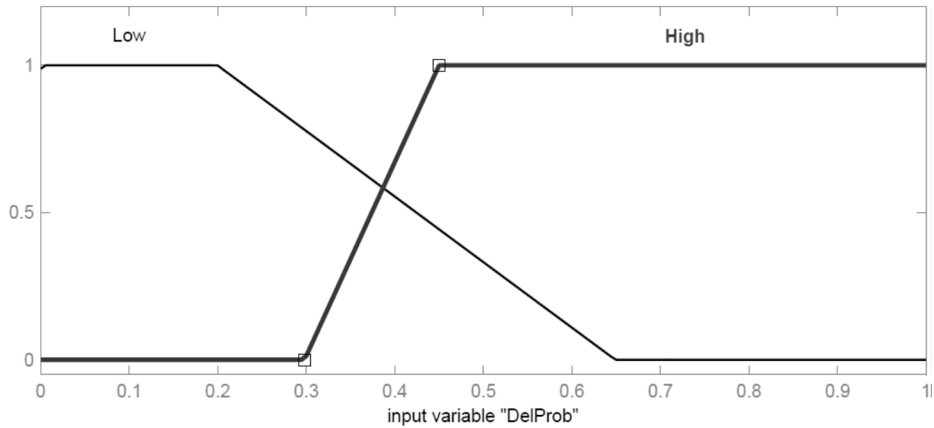
$$\text{Error} = \text{actualDelProb} - \text{ELFISDelProb} \quad (16)$$

Three fuzzy membership functions of type trapezoid are considered for the error values (Figure 5)



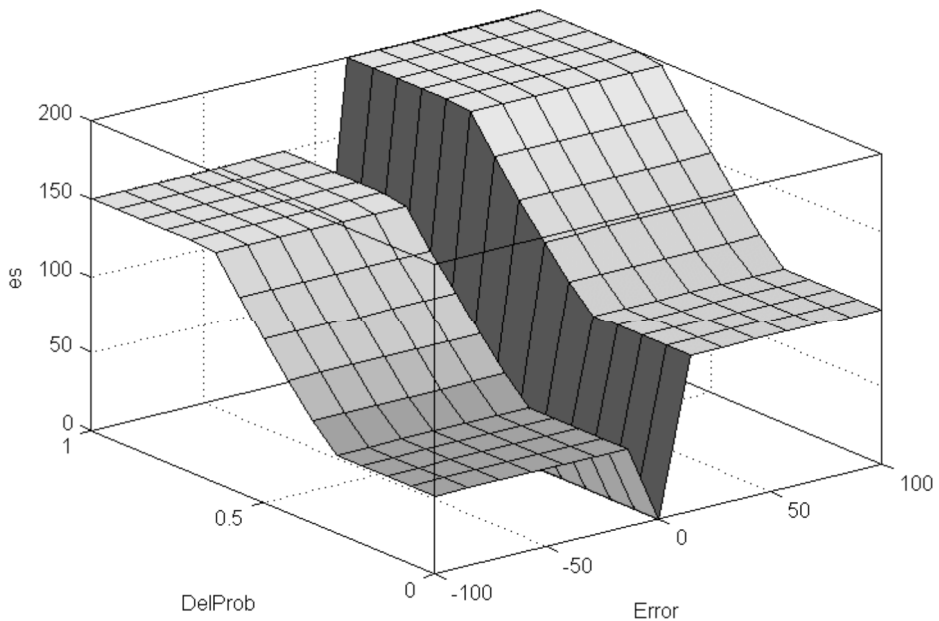
Two fuzzy membership functions of type trapezoid are considered for the delay probability values (Figure 6). These membership functions represent the Low and High delay probabilities for low-risk and high-risk clients.

Figure 6: Fuzzy membership functions for "DelProb"



By defining appropriate membership functions for each of the inputs and 6 linguistic fuzzy rules, the desired behavior of emotional critic is depicted in Figure 7. It is noted that during ELFIS training, the emotional signal generated by this emotional cue is minimized.

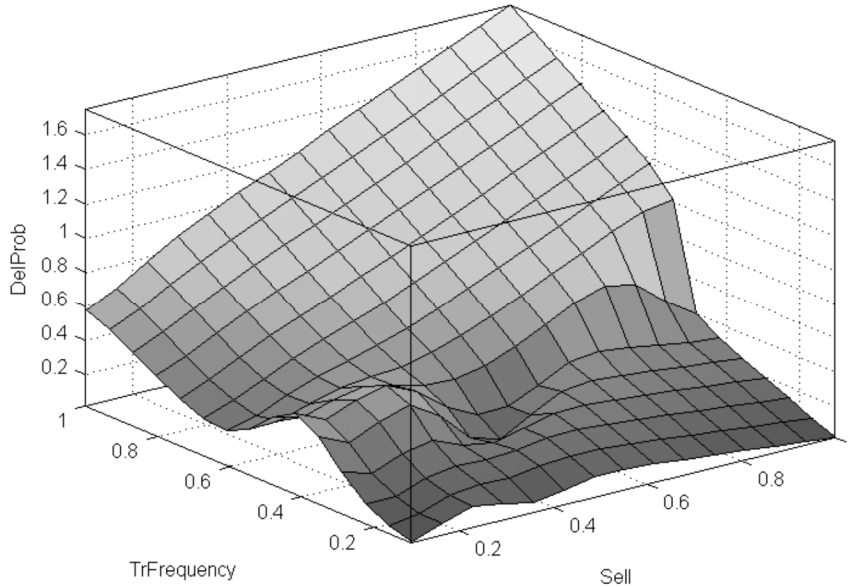
Figure 7: The emotional cue for ELFIS credit scoring



The ELFIS training exactly compatible with the procedure presented in section 2.1 has been implemented in the MATLAB environment and the results of this training is the surface generated in Figure 8. This surface

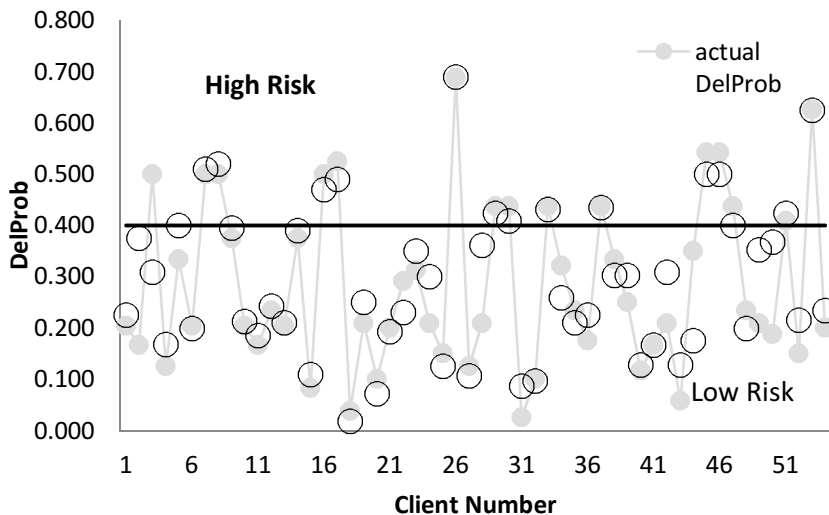
relates the input variables Sell and TrFrequency to the delay probability. It is noted that in this surface input data are normalized between 0 and 1.

Figure 8: The surface generated by ELFIS for credit scoring



The results of ELFIS for the predicted credit scoring compared to the actual delay probabilities are shown in Figure 9. As seen, ELFIS predictor has satisfactory results for both low risk clients and high-risk clients.

Figure 9: ELFIS predictor figures for each scenario



For the purpose of comparisons of the scoring performances for both ANFIS and ELFIS methods, the mean absolute percentage error (MAPE), given by Equation 18, is used as the index of scoring accuracy.

$$MAPE = \frac{1}{N} \sum_{i=1}^N \left| \frac{y_i - \hat{y}_i}{y_i} \right| \times 100\% \quad (17)$$

Where  $y_i$  and  $\hat{y}_i$  represent the actual and estimated delay probabilities of the  $i$ th client, respectively, and  $N$  is the number of clients in the sample, here  $N = 54$ .

Table 2 presents a comparison between the MAPE results of ANFIS and ELFIS for credit scoring. As seen the results of credit scoring has improved 61% in terms of MAPE error. This comparison reveals the superiority of ELFIS over ANFIS for credit scoring of retail bank clients. This also proves the realized improvements of credit scoring accuracy by the use of emotional learning and purposeful training,

**Table 2.**  
Comparison of ANFIS and ELFIS error for credit scoring

Method	ANFIS	ELFIS
MAPE	29%	12%

## 5. CONCLUSION

This study introduced an approach based on ELFIS, for improved credit scoring of bank clients. It uses a comparison mechanism to show the suitability of the emotional learning for credit scoring in real bank environments. The proposed ELFIS model used standard inputs and output. ANFIS is used to generate initial FIS for ELFIS. MAPE index is used to quantitative comparison. Applicability and superiority of the ELFIS model is shown through applying the ANFIS and ELFIS models on actual client's history data in a confidential Iranian bank in 2013. The approach is capable of accurate credit scoring for the clients with a high rate of delayed due payments. ELFIS shows better performance when compared with ANFIS. This is the first study that uses ELFIS approach for credit scoring in banking industry. Furthermore, the ELFIS of this study has interesting features of emotional learning, purposeful training, simplicity, needing not a big number of data.

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**Appendix 1**  
**Original data for annual NG demand**

ID	Sell	Transaction frequency	actual DelProb
1	500000	7	0.204
2	420000	10	0.167
3	650000	10	0.500
4	500000	5	0.125
5	475000	10	0.333
6	400000	7	0.204
7	450000	10	0.500
8	1000000	15	0.500
9	550000	10	0.375
10	700000	7	0.204
11	300000	8	0.167
12	850000	8	0.233
13	650000	7	0.204
14	750000	15	0.375
15	300000	5	0.083
16	450000	10	0.500
17	400000	14	0.525
18	2000000	3	0.038
19	700000	10	0.208
20	650000	3	0.100
21	600000	6	0.200
22	800000	7	0.292
23	1000000	15	0.313
24	300000	10	0.208
25	1200000	3	0.150
26	1500000	15	0.688
27	1800000	5	0.125

Contd...

28	600000	10	0.208
29	450000	15	0.438
30	670000	15	0.438
31	1500000	3	0.025
32	550000	3	0.100
33	575000	15	0.438
34	1000000	7	0.321
35	670000	7	0.233
36	500000	7	0.175
37	525000	15	0.438
38	1500000	10	0.333
39	1500000	10	0.250
40	250000	7	0.117
41	800000	10	0.167
42	650000	10	0.208
43	250000	7	0.058
44	350000	7	0.350
45	170000	10	0.542
46	548246	10	0.542
47	220000	15	0.438
48	400000	7	0.233
49	120000	10	0.208
50	320000	15	0.188
51	350000	14	0.408
52	800000	6	0.150
53	200000	30	0.625
54	1000000	6	0.200

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