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Mathematical Model to Estimate Storm Hydrographs using Only Hydrometric Information

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Abstract: This paper develops a procedure to estimate design hydrographs based on period return using only hydrometric data. Storm hydrographs are extremely important in the design of hydraulic structures (i.e., diversion systems, bridges, dikes, and in certain countries, spillways); therefore, the peak flow, base flow, total volume, peak time, base time, and shape of storm hydrographs should be adequately estimated. The solution was carried out through an optimization method for minimizing the mean squared error for maximum recorded volumes and maximum modeled volumes. This procedure was applied to the Páez in the Lengupá River (1090 km²) and the Puente Balseadero station in the Magdalena River (5592 km²) located at Colombia. Calibration was accomplished using recorded hydrographs, and verification was achieved using the maximum recorded hydrographs at each site. Results show that the mathematical procedure can be used to compute design hydrographs based on period return. The procedure is very important because helps engineers to estimate design hydrographs base on period return quickly and accurately because it does not use precipitation records.

Keyword: Hydrographs, Hydrometric Information, Return period, Runoff, Rivers.

1. INTRODUCTION

The design of hydraulic structures for diversion, bridges, longitudinal dikes, and in certain countries, spillways, involves calculating storm hydrographs associated with different return periods [1]. When estimating hydrographs, it is essential that they account for the peak instantaneous flow, total volume, base flow, peak time, base time, and shape. Different methods are used to estimate design storm hydrographs, which depend on what information is available. In watersheds with instrumentation, seasonal records are available, such as the following: limnigraphic stations record continuous hydrometric information; pluviographic stations record continuous precipitation information; pluviometric stations record daily precipitation information; and climatological stations record meteorological information.

The commonly used methods are based on hydrographic information [2], which are used when hydrographic records are reliable and sufficient, and rain-runoff methods [3]–[5], which are used when precipitation information is reliable and sufficient by using use for instance the model HEC-HMS [6]. These information can be used for analyzing complex systems, for instance to model the flow through a system of floodgates [7].

This study focuses on methods based on hydrographic information, which assumes that records from limnigraphic stations constitute detailed information that is representative of the rain-runoff dynamics and therefore, are extremely useful to a hydrologic study.

Methods based on hydrographic information that are used to estimate design hydrograph typically involve analyzing maximum value frequencies through statistical adjustments of the available information with probability distributions (i.e., Gumbel, Pearson, Log Pearson Type III, GEV, etc.). Clearly, when the available information is more extensive and of better quality, there will be less uncertainty in the estimations. Among these methods, we mention three: the safety factor method, the method of Escalante and the method of the Instituto de Ingeniería (IINGEN) in [8].

Traditionally, safety factor method is popular due to its rapid and easy application. The safety factor method requires identifying the maximum recorded storm event (in terms of maximum instantaneous flow and total runoff volume). The hydrograph record for this storm is used to obtain its corresponding dimensionless hydrograph by dividing each ordinate by the maximum instantaneous flow rate. To estimate design hydrographs, a frequency analysis must be conducted with the annual series of maximum flow rates, and the maximum flow rate for the return period of interest must be estimated. The design hydrograph is obtained by multiplying the ordinates of the dimensionless hydrograph by the calculated maximum flow rate. In this way, an estimation can be performed for a storm with the same characteristics of the maximum storm recorded, though the storm is generally larger (safety factor). One disadvantage of this method is that on occasion, the storm adopted as the maximum record can be subsequently exceeded by storms in future years. The Method of Escalante is a procedure that consists of estimating design hydrographs by joint modeling four variables (maximum instantaneous flow rate, total volume, and accumulated volumes before and after the maximum flow rate), which are obtained from maximum recorded hydrographs; this requires marginal functions such as Gumbel, GEV or double Gumbel. A numerical scheme based on successive approximations of the Newton Raphson is used [8]. IINGEN considers peak floods, the volume and shape of the recorded hydrographs. Storm hydrograph associated for return period are estimated using the alternating block method and daily flows [2].

In this work, the authors develop a mathematical model for estimating storm hydrographs for period return that can be used for practical applications and is very easy to implement. It considers peak floods, base flow, the volume and the shape of the storm hydrographs.

2. PROPOSED PROCEDURE

This section presents the procedure to estimate design hydrographs based on hydrometric information. The basic assumptions of this procedure are the following:

- The procedure is a mathematical procedure, not a physical model (relates the dynamics of precipitation-runoff).
- There is only one dimensionless representative hydrograph for the watershed and therefore, only one peak time.
- The procedure does not require precipitation records for its application.

- The procedure is based on a mass balance between the volume of runoff and the base flow volume.
- To separate the base flow from recorded hydrographs, two methods are mentioned [3]: the constant method and the linear method (See Figure 1). These methods have some limitations. These base flow separation methods will be used to generate storm hydrographs, which assumes that the base flow for different return periods can be approximated based on the statistical analysis of the series of monthly mean flow rates of the wettest month.

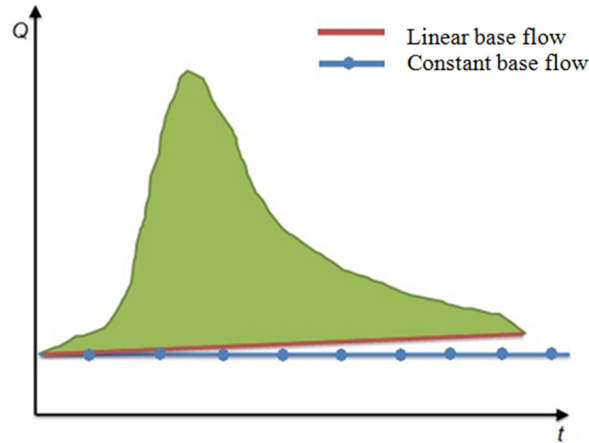


Figure 1: Separation and superposition of base flow

- The volume of the hydrographs for different return periods is determined based on the statistical analysis of the series of total volumes of the maximum annual hydrographs recorded. When there are no records of the majority of storms, the series of maximum annual volumes can be determined using the following procedure: (i) determine mean daily flow rates, (ii) determine mean daily volumes by taking into account the duration of the storms in the area, and (iii) obtain the series of maximum annual volumes.
- Because of its definition, this procedure is more applicable to small watersheds and homogeneous hydroclimatological areas.
- The procedure can be used with estimating Return Period under stationarity or Nonstationarity conditions [9].

2.1. Derivation of the procedure

The procedure is based on a balance of mass, which accounts for the variables and parameters presented in Figure 2. The figure is divided into the following parts: (a) maximum recorded hydrographs; (b) dimensionless hydrograph in terms of flow rate and time, which is determined by dividing each of the abscissas by the recorded peak time ($t_{p - \text{recorded}}$) and the ordinates by the maximum recorded flow rate (QP_i) and averaging the recorded dimensionless hydrographs; (c) hydrograph with dimensionless flow rate for the watershed; (d) specific hydrograph for a return period.

where,

- QE_i : Direct runoff flow rate of the recorded hydrograph.
- QP_i : Maximum instantaneous flow rate of the recorded hydrograph.
- QB_i : Base flow rate of the recorded hydrograph.

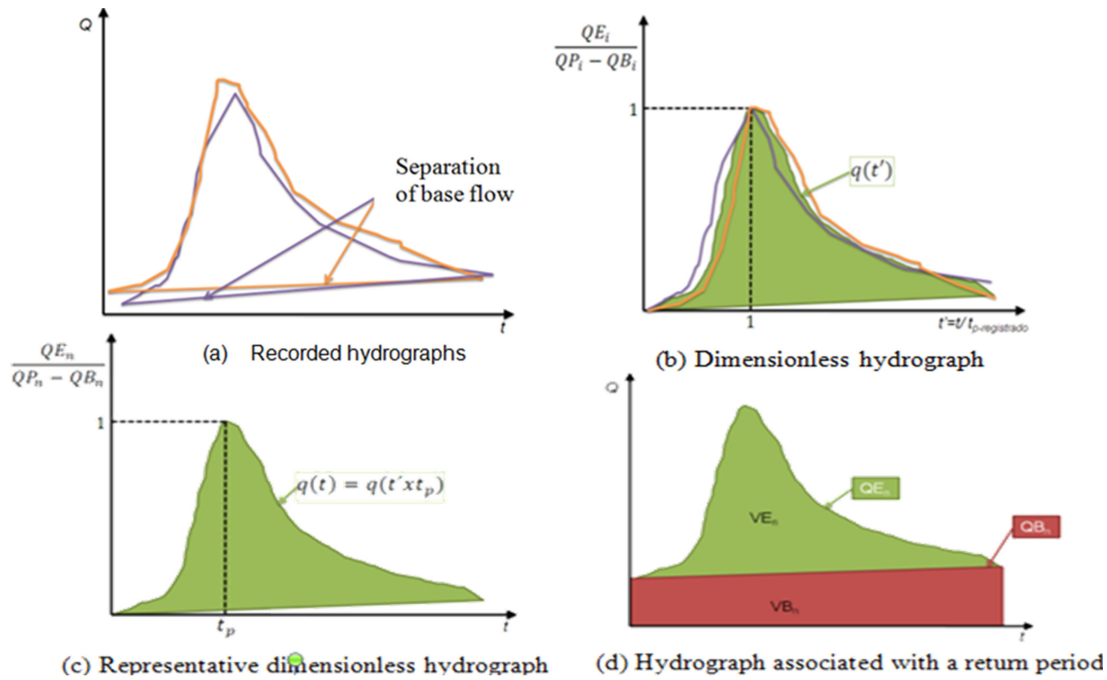


Figure 2: Variables and parameters

- i : Recorded storm hydrograph.
- t_p - recorded: Peak time of the recorded hydrograph.
- $q(t')$: Dimensionless hydrograph obtained from the recorded hydrographs, which is obtained by averaging the dimensionless recorded hydrographs in terms of flow rate and time (see Figure 2 (b)).
- QE_n : Flow rate from direct runoff for a return period.
- QP_n : Maximum instantaneous flow rate of the design hydrographs for a return period.
- QB_n : Flow rate from the base flow for a return period.
- $q(t)$: Dimensionless hydrograph in terms of flow rate of the hydrographs associated with different return periods [$q(t) = q(t' \times t_p)$].
- t_p : Peak time of the hydrographs associated with an estimated return period.
- VM_n : Total estimated volume of the storm for a return period.
- VE_n : Runoff volume for a return period.
- VB_n : Volume of base flow rate for a return period.
- n : Storm hydrograph associated with an analyzed return period.
- t : Time interval of the storm duration.

The volume of the storm associated with a return period of interest was determined as follows:

$$VM_n = VE_n + VB_n \quad (1)$$

By definition, the volume may be expressed as:

$$V = \int_{t=0}^{t=T} Q(t) dt \tag{2}$$

Then

$$VE_n = \int_{t=0}^{t=T} QE_n(t) dt = \int_{t=0}^{t=T} [QP_n - QB_n(t)]q(t) dt \tag{3}$$

$$VB_n = \int_{t=0}^{t=T} QB_n(t) dt \tag{4}$$

To determine the total volume of a storm, the following expression is used:

$$VM_n = VE_n + VB_n = \int_{t=0}^{t=T} [QP_n - QB_n(t)]q(t) dt + \int_{t=0}^{t=T} QB_n(t) dt \tag{5}$$

$$VM_n = \int_{t=0}^{t=T} QB_n(t) dt + QP_n \int_{t=0}^{t=T} q(t) dt - \int_{t=0}^{t=T} QB_n(t)q(t) dt \tag{6}$$

The previous equation can be solved using intervals:

$$VM_n = \sum_{t=0}^T QB_{n,t} \Delta t + QP_n \sum_{t=0}^T q_t \Delta t - \sum_{t=0}^T QB_{n,t} q_t \Delta t \tag{7}$$

2.2. Steps to Apply the Procedure

Application of the procedure requires the following steps.

- **Obtaining input data:** The input data required for applying the procedure are the recorded hydrographs and statistical analyses of the maximum instantaneous flow rates (QP_n), total volumes (VR_n), and base flow rate (QB_n).
- **Selection of the base flow equation:** Depending on the base flow equation selected (constant or linear) and considering equation [6], one can verify the expressions presented in Table 1.

Table 1
Base flow equations

Base flow	Base flow equation	Equation for the modeled volume
Constant	$QB_n(t) = p_n$	$VM_n = p_n T + \left(\int_{t=0}^{t=T} q(t) dt \right) (QP_n - p_n)$
Linear	$QB_n(t) = p_n \times t = p_n$	$VM_n = \frac{mT^2}{2} + p_n T + QP_n \int_{t=0}^{t=T} q(t) dt - \int_{t=0}^{t=T} QB_n(t)q(t) dt$

where,

p_n : Constant base flow rate for each return period.

m : Slope of the base flow rate

- **Calibration of the variables:** The variable subject to calibration is the peak time (t_p), and in the case of using the linear base flow equation, the base flow slope (m) is also included. To calibrate, different methods may be used, such as trial and error, the Monte Carlo method, gradient method, etc.

- **Optimization algorithm:** Estimation of the peak time and base flow equations is performed through a process of minimizing the mean squared error (MSE) for the series of maximum recorded volumes associated with different return periods (VR_n) and the series of maximum modeled volumes associated with different return periods (VM_n). The target function (TF) to be used is:

$$TF = \min(MSE) = \sqrt{\frac{1}{N} \sum_{n=1}^N (VM_n - VR_n)^2} \quad (8)$$

where

N: Number of storm hydrographs analyzed.

The optimization algorithm proposed is presented next:

– Objective

Minimize the mean squared error of the volumes modeled with respect to those recorded in the storm hydrographs domin.

– Sets

$n \in N$: Storm hydrograph associated with an analyzed return period.

$t \in T$: Time interval of the storm duration.

– Parameters

QP_n : Vector of the maximum flow rate associated with a return period.

VR_n : Vector of the maximum recorded volume of the storm associated with a return period.

p_n : Vector of constant base flow associated with a return period.

Δt : Scalar of the size of the selected interval.

$q(t')$: Dimensionless hydrograph in terms of flow rate and average time of the recorded storms.

$t_{p-\min}$: Minimum peak time recorded for the storms.

m_{\min} : Slope of the minimum base flow recorded.

m_{\max} : Slope of the maximum base flow recorded.

T: Duration of storms.

– Variables

TF: Target function.

MSE: Mean squared error.

VM_n : Volume modeled from the storm associated with a return period.

t_p : Peak time of hydrographs associated with a return period.

$q(t)$: Dimensionless hydrograph in terms of flow rate.

m : Slope of the storms associated with a return period.

- Formulation of the problem

TF: minimize (MSE)

Subject to

$$\text{MSE} = \sqrt{\frac{1}{N} \sum_{n=1}^N (\text{VM}_n - \text{VR}_n)^2}$$

$$\text{VM}_n = \int_{t=0}^{t=T} \text{QB}_n(t) dt + \text{QP}_n \int_{t=0}^{t=T} q(t) dt - \int_{t=0}^{t=T} \text{QB}_n(t) q(t) dt \quad \forall n$$

$$t_{p-\min} \leq t_p$$

$$t_p \in Z^+$$

$$m_{\min} \leq m \leq m_{\max} \text{ (Applies when considering linear base flow)}$$

$$q(t) = q(t' \times t_p) \quad \forall t$$

- **Generation of storm hydrographs:** The storm hydrographs are generated by considering the equation obtained based on Figure 2:

$$Q_n(t) = q(t)[\text{QP}_n - \text{QB}_n(t)] + \text{QB}_n(t) \tag{9}$$

where

$Q_n(t)$: Flow rate for a given time and a return period of interest.

2.3. Flow Diagram

Figure 3 shows the procedure in order to compute the design hydrographs based on period return.

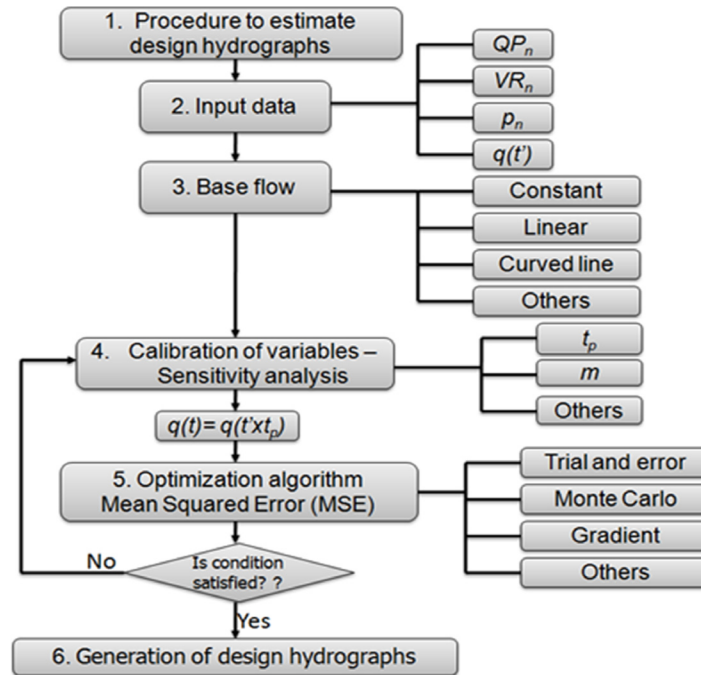


Figure 3: Flow diagram

3. APPLYING THE PROPOSED PROCEDURE

Application of the proposed procedure was achieved using hydrometric records from the stations at the Lengupá river – Páez and Magdalena river – Puente Balseadero, which have catchment areas of 1090 and 5592 km², respectively, and are located in Colombia, South America.

Lengupá River – Páez Station

The Páez Station is a limnigraphic station and has available records for a period of 28 years (1976 to 2003). The hydrometric records of the Lengupá river – Páez were used to perform the statistical adjustment with the Gumbel probability distribution (maximum likelihood method) using the Hyfran program [10] for the base flow rate, maximum instantaneous flow rate, and total volume for different return periods. Table 2 shows the obtained results.

Table 2
Lengupá River – Páez Station. Input data

Return period (years)	Base flow rate (p_n) (m^3/s)	Maximum instantaneous flow rate (QP_n) (m^3/s)	Total volume (VR_n) (hm^3)
200	221.2	1235.0	66.7
100	204.3	1136.0	61.4
50	187.3	1036.0	56.2
20	164.6	903.4	49.1
10	147.0	800.8	43.6
5	128.8	693.8	38.0

Because not all the records of the maximum annual recorded storms were acquired, the annual series of total volumes was calculated based on the mean daily maximum flow rates in 24 hours (approximate duration of hydrographs). Figure 4 shows the maximum recorded hydrographs at the Lengupá River – Páez station.

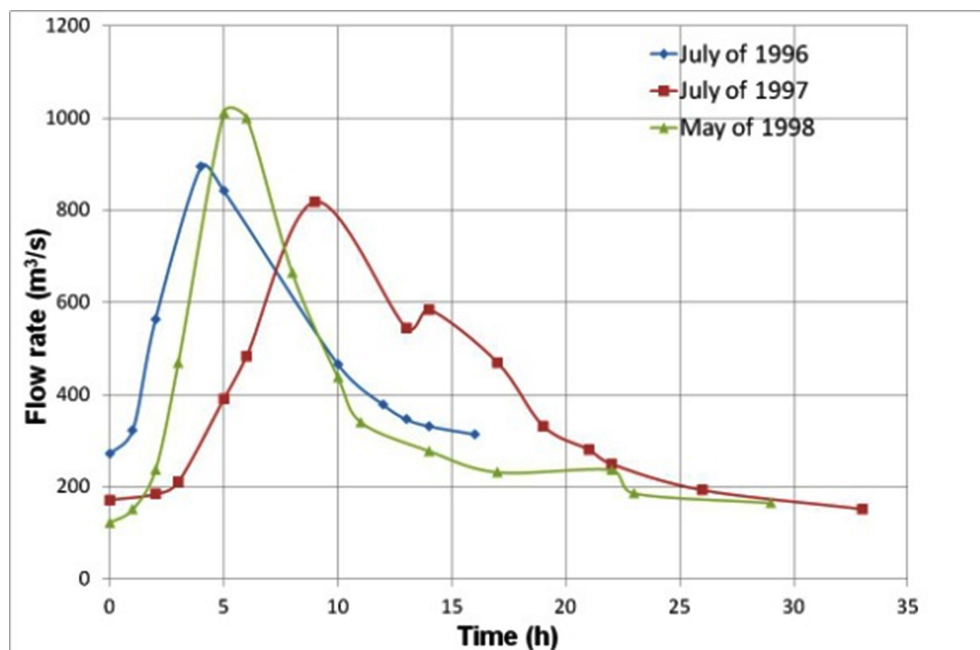


Figure 4: Lengupá River– Páez Station. Recorded hydrographs

The procedure was applied considering the storm hydrographs recorded in 1996 and 1998, and the hydrograph recorded in 1997 was used for validation. The procedure was applied considering constant and linear base flow. Tables 3 and 4 show the obtained results.

Table 3
Lengupá River – Páez Station. Calibration of variables

Base flow equation	Calibrated Variables	
	t_p (hours)	m
Constant	10	–
Linear	10	2.37

Table 4
Lengupá River – Páez Station

Return period (years)	Volume recorded (VR_n) (hm^3)	Volume modeled (VM_n) (hm^3)	
		Constant base flow	Linear base flow
200	66.7	66.3	66.6
100	61.4	61.0	61.4
50	56.2	55.7	56.1
20	49.1	48.6	49.1
10	43.6	43.1	43.7
5	38.0	37.4	38.1
	MSE	0.50	0.07

The Table 4 shows that when considering linear base flow, there is an MSE of 0.07 hm^3 , which is lower than that obtained when considering a constant base flow (0.50 hm^3). Figure 5 shows the generated storms, which considers a constant base flow. When a linear base flow was considered, the results were similar.

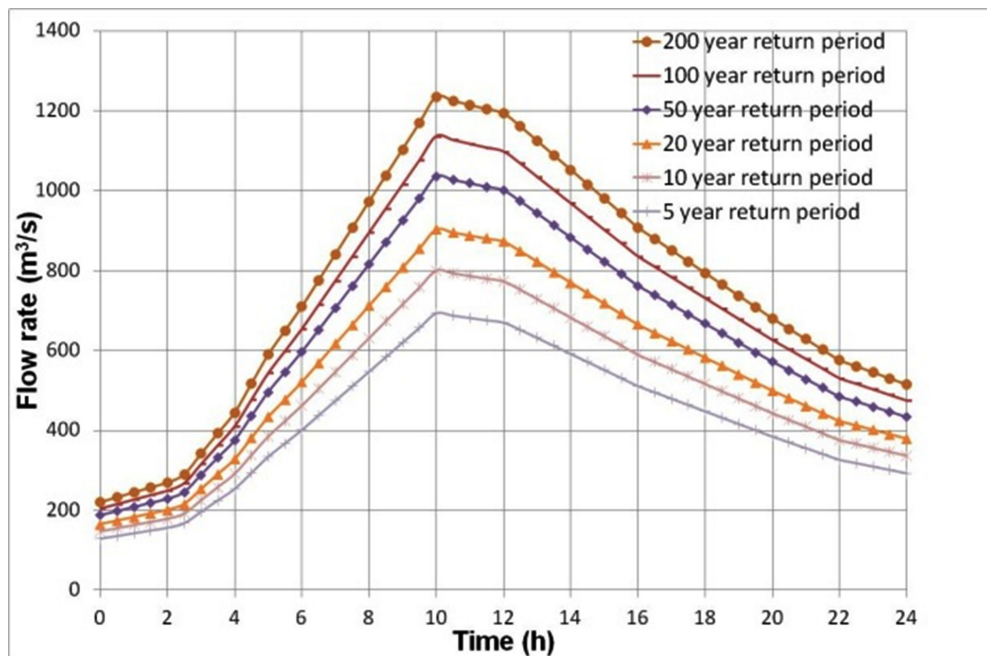


Figure 5: Lengupá River– Páez Station. Storm hydrographs considering constant base flow

Magdalena River – Puente Balseadero Station

The Magdalena River – Puente Balseadero station is a limnigraphic station and has available records spanning 31 years (1972 to 2002). Based on the hydrometric records of the Magdalena River – Puente Balseadero station, a statistical adjustment was performed using the Gumbel probability distribution (maximum likelihood method) using the program Hyfran (Chaire en Hydrologie Statistique (CHS). Hyfran, 2002) for the base flow rate, peak flow rate, and maximum volume for different return periods. Table 5 shows the obtained results. Because not all the records on maximum annual recorded storms could be obtained, the annual series of total volumes was calculated based on the maximum average daily flow rates in 48 hours (approximate duration of storms). Figure 6 shows the maximum recorded hydrographs in the Magdalena River – Puente Balseadero station.

Table 5
Magdalena River–Puente Balseadero Station. Input data

Return period (years)	Base flow rate (p_n) (m^3/s)	Maximum instantaneous flow rate (QP_n) (m^3/s)	Total volume (VR_n) (hm^3)
200	637.4	2990	292.1
100	595.3	2770	270.5
50	553	2560	248.8
20	496.7	2270	219.9
10	453.1	2050	197.6
5	407.7	1810	174.3

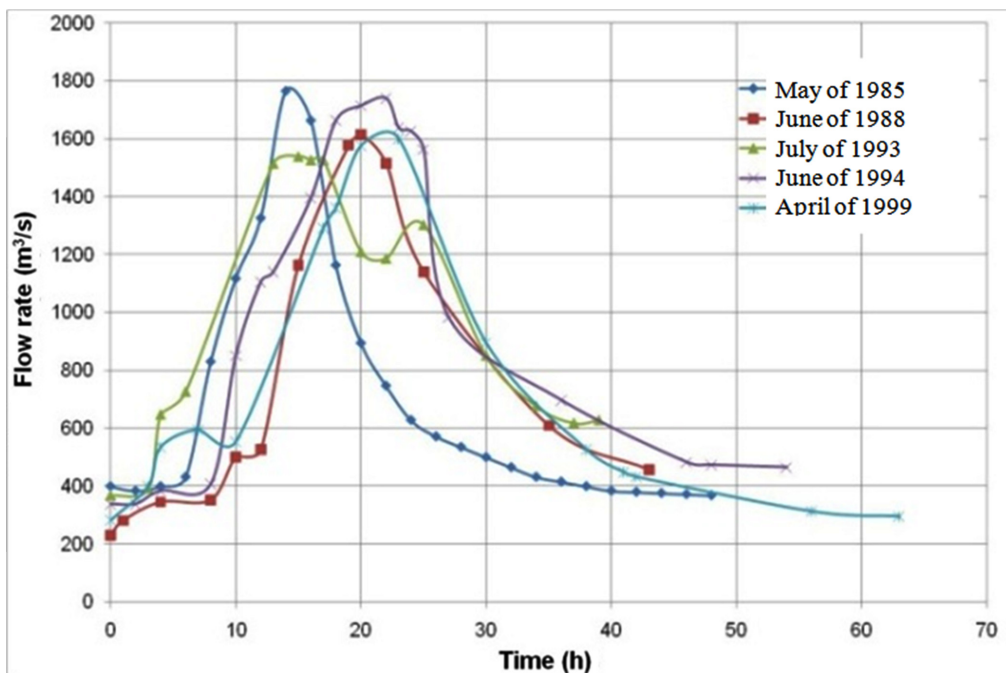


Figure 6: Magdalena River – Puente Balseadero Station. Recorded hydrographs

Application of the procedure was achieved using the recorded storm hydrographs for 1985, 1988, 1993, and 1999, and the hydrograph recorded for 1994 was used for validation. The procedure was applied considering constant and linear base flows. Tables 6 and 7 show the obtained results.

Table 6
Magdalena River – Puente Balseadero Station. Calibration of variables

Base flow equation	Calibrated variables	
	t_p (hours)	m
Constant	23	–
Linear	25	–0.65

Table 7
Magdalena River – Puente Balseadero Station. Model results

Return period (years)	Volume recorded (VR_n) (hm^3)	Volume modeled (VM_n) (hm^3)	
		Constant base flow	Linear base flow
200	292.10	291.48	289.69
100	270.50	270.49	268.74
50	248.80	250.25	248.53
20	219.90	222.51	220.83
10	197.60	201.38	199.73
5	174.30	178.54	176.92
MSE		2.63	1.88

The preceding table shows that, when flow is considered on a linear basis, an EMC of 1.88 hm^3 is observed, which is less than that obtained when flow is considered on a constant basis (2.63 hm^3). Figure 7 shows the storms that result from considering flow on a constant basis. Similar results occur when a linear flow basis is considered.

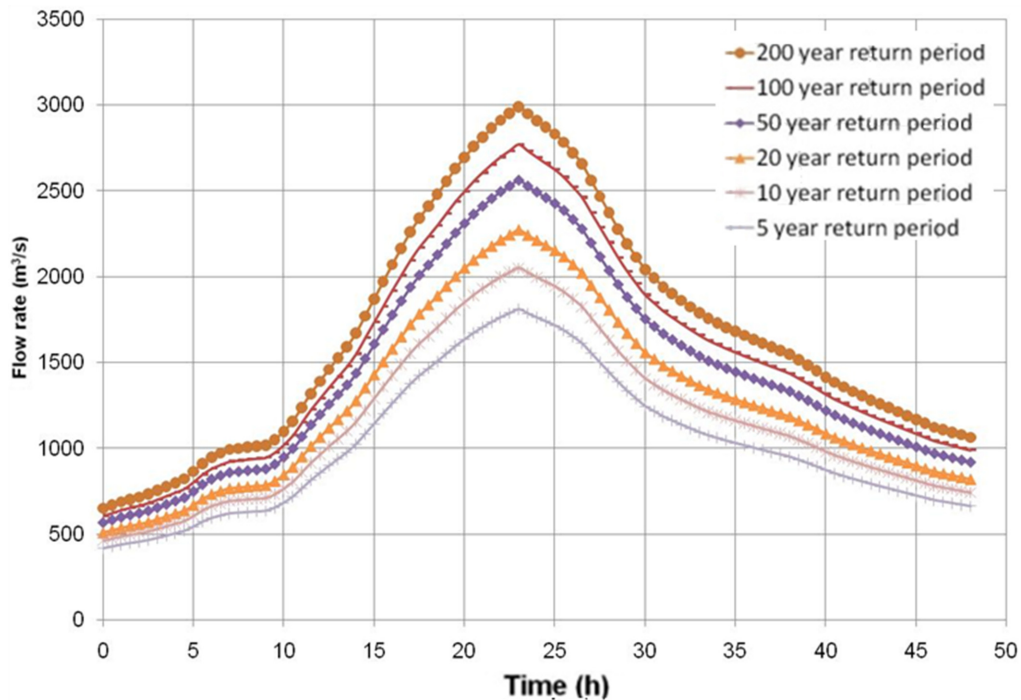


Figure 7: Magdalena River – Puente Balseadero Station. Storm hydrographs considering constant base flow

4. SENSITIVITY ANALYSIS FOR THE PROPOSED PROCEDURE

The sensitivity analysis performed on the procedure consisted of varying the selection of the dimensionless hydrograph. The following are the modeling cases that were performed:

- Lengupá River – Páez Station: A recorded hydrograph was considered for the design.
- Magdalena River – Puente Balseadero Station: Two recorded hydrographs were considered as well as three recorded hydrographs.

On the Figures 8 and 9 shows that the uncertainty is not greater when using different recorded hydrographs to estimate design hydrographs. The envelopes obtained for other return periods exhibit similar shapes as that of the 100-year storm.

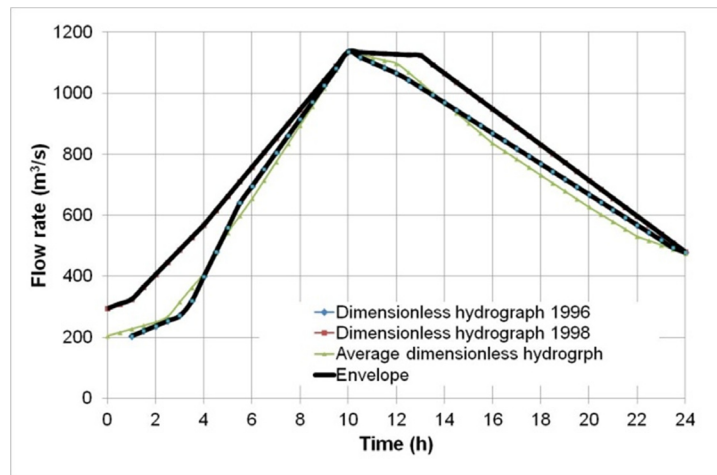


Figure 8: Lengupá River– Páez Station. Considering Constant base flow. Sensitivity analysis. 100-year storm

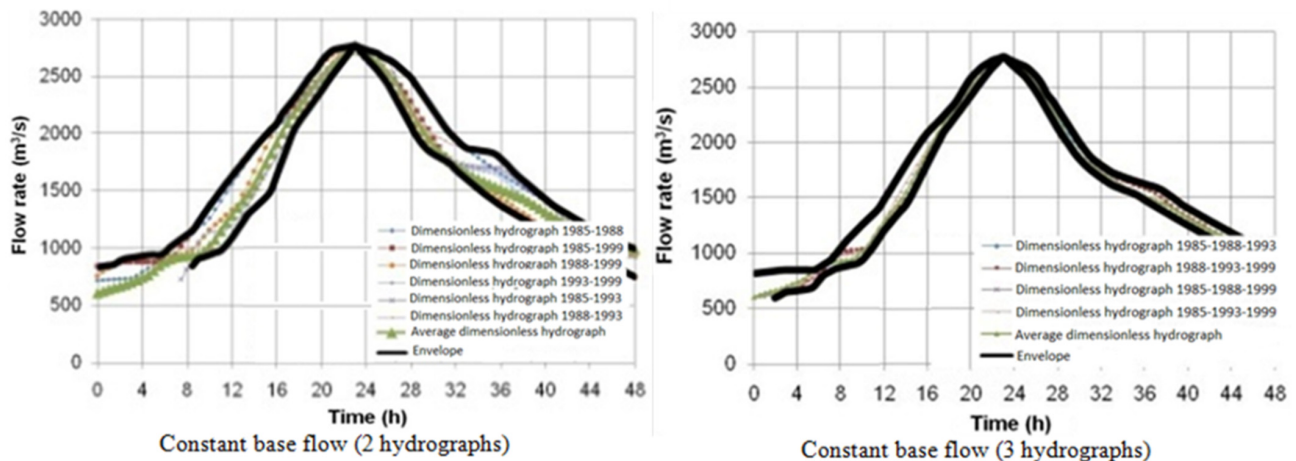


Figure 9: Magdalena River– Puente Balseadero Station. Sensitivity Analysis. 100-year storm

5. VALIDATION OF THE PROPOSED PROCEDURE

Validation of the procedure consisted of comparing the maximum recorded hydrographs in terms of the maximum instantaneous flow rate and total volume with respect to the design hydrographs generated with the proposed procedure. To that end, we used the maximum recorded storms in July of 1997 and June of 1994 for the Lengupá

River – Páez and Magdalena River – Puente Balseadero stations, respectively. Figure 10 shows the obtained results from the procedure validation when considering constant base flow.

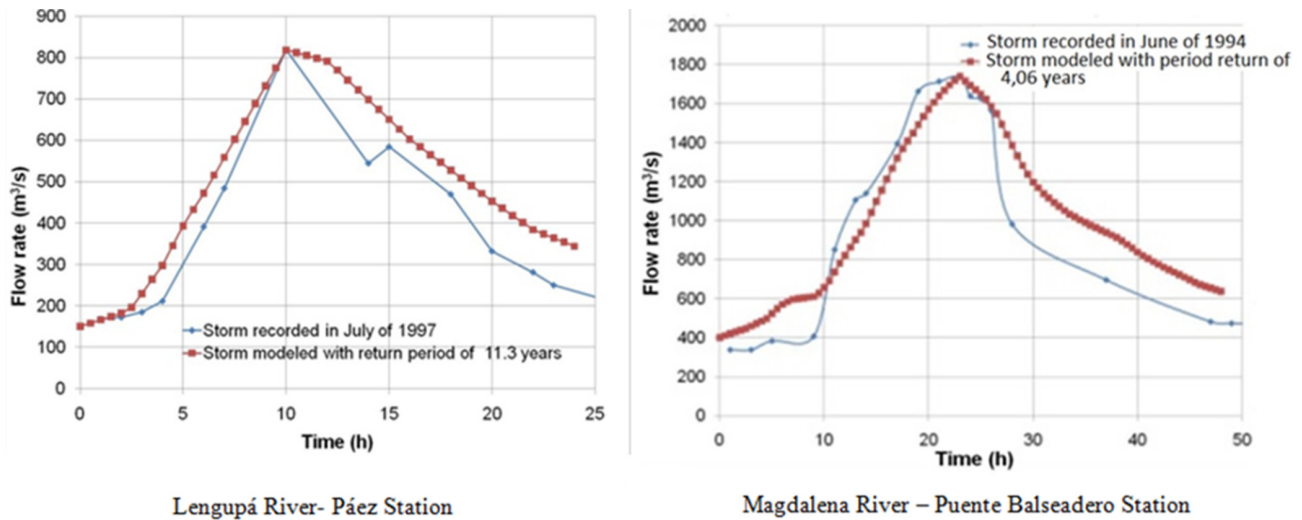


Figure 10: Validation of the proposed procedure.

The results obtained when considering a linear base flow were extremely similar (see Table 8).

Table 8
Validation of the proposed procedure

Station	Date	Return period (years)	Recorded storm		Modeled storm	
			$VR_i (hm^3)$	$t_p (h)$	$VM_n (hm^3)$	$t_p (h)$
Considering constant base flow						
Lengupá River – Páez	Jul-97	11.3	44.3	9	45.2	10
Magdalena River – Puente Balseadero	Jun-94	4.1	157.2	22	171.7	23
Considering linear base flow						
Lengupá River – Páez	Jul-97	11.3	44.3	9	44.7	10
Magdalena River – Puente Balseadero	Jun-94	4.1	157.2	22	170.1	25

Based on the previous results, the procedure to estimate design hydrographs adequately represents the maximum recorded storms because it is consistent in the estimation of the maximum instantaneous flow rate, total volume, and peak time.

6. CONCLUSIONS AND RECOMMENDATIONS

The proposed procedure seeks to minimize the mean squared error (MSE) of the maximum recorded volumes with respect to the modeled volumes, which is subject to restrictions on peak time and base flow slope. Based on the results obtained in the Lengupá River – Páez (catchment area: of 1090 km²) and Magdalena River – Puente Balseadero (catchment area: of 5592 km²) stations, we can conclude that the solutions are good because the procedure is able to adequately calibrate the maximum instantaneous flow rate, maximum volume, and base flow rate associated with a return period. When considering linear base flow, better results are obtained than that with constant base flow. When selecting the dimensionless hydrograph, it is recommended to use the highest number of maximum recorded storms because better results were produced. However, using a lower number of recorded hydrographs also yielded good results. The maximum recorded storms for the Lengupá river – Páez

and Magdalena river – Puente Balseadero stations were used for validation, and it was found that the proposed procedure adjusted them appropriately in terms of the maximum instantaneous flow rate, maximum volume, base flow rate, hydrograph shape, and peak time. When applying the proposed procedure, there was less uncertainty in the results for watersheds with smaller areas (Lengupá River – Páez, catchment area of 1090 km²) compared with watersheds with larger areas (Magdalena River – Puente Balseadero, catchment area of 5592 km²) because better estimations of the maximum recorded volumes were obtained.

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