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CERTAIN FRACTIONAL KINETIC EQUATIONS INVOLVING MULTI-VARIABLE MITTAG-LEFFLER

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Abstract: The aim of the present paper is to develop a generalized fractional kinetic equation involving generalized multi-variable Mittag-Leffler function. Using the Laplace transform, the solutions of the fractional kinetic equation are established in terms on general Mittag-Leffler function. The results obtained here are general in nature to yield a large number known and (presumably) new results as their special cases.

Keywords: Pochhemmer symbol, Fractional Calculus, Multi-variable Mittag-Leffler function, Laplace Transform, Fractional Derivative, Fractional Integration. 2010 MSC: 26A33, 33C45, 33C60, 33C70

1. INTRODUCTION AND PRELIMINARIES

Many important functions in applied sciences (which are popularly known as Mittag-Leffler functions) are defined via infinite summation (or infinite products). During the last one and a half decades, several interesting and useful extensions of many of the familiar Mittag-Leffler functions have been considered by several authors (see, for example, see the recent work [12],[13]). The above-mentioned works have largely motivated our present study. Mittag-Leffler [8] introduced the function $E_\alpha(z)$, defined

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)} \tag{1.1}$$

Where z is a complex variable and $\Gamma(.)$ is a Gamma function, $a \geq 0$. The Mittag-Leffler function is a direct generalization of the exponential function to which it reduces for $\alpha = 0$. For $0 \prec \alpha \prec 1$; it interpolates between the pure exponential and a hypergeometric function $\frac{1}{1-z}$. Its importance is realized during the last two decades due to its involvement in the problems of physics, chemistry, biology, engineering and applied sciences. Mittag-Leffler function naturally occurs as the solution of fractional order differential equation or fractional order integral equations. The generalization $E_\alpha(z)$ of was studied by Wiman [11] and he defined the function as

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$$E_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+\beta)},\ (\alpha,\beta\in\mathbb{C};\Re(\alpha)>0,\Re(\beta)>0) \tag{1.2}$$

Which is known as Wimans function or generalized Mittag- Leffer function as $E_{\alpha,1}(z) = E_{\alpha}(z)$.The main properties of these functions are given in [14] and a more comprehensive and a detailed account of Mittag-Leffer functions are presented by Dzherbashyan [15]. Prabhakar [9] introduced the function $E^{\gamma}_{\alpha,\beta}(z)$ in the form of

$$E^{\gamma}_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{\gamma_n}{\Gamma(\alpha n+\beta)}\frac{z^n}{n!}\ (\alpha,\beta,\gamma\in\mathbb{C};\Re(\alpha)>0,\Re(\beta)>0,\Re(\gamma)>0) \tag{1.3}$$

and $(\lambda)_n$ denotes the familiar Pochhammer symbols or the shifted factorials, since $(1)_n = n!$ $(n\in N_0)$

$$(\lambda)_n=\frac{\Gamma(\gamma+n)}{\Gamma(\gamma)}=\begin{cases}1 & (n=0;\lambda\in\mathbb{C}-\{0\})\\ \gamma(\gamma+1)......(\gamma+n-1) & (n\in\mathbb{N};\lambda\in\mathbb{C})\end{cases} \tag{1.4}$$

Srivastava and Tomovski [10] studied and generalized the Mittag-Leffer type function $E^{\gamma}_{\alpha,\beta}(z)$ is defined as follows

$$E^{\gamma,k}_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{\gamma_{kn}}{\Gamma(\alpha n+\beta)}\frac{z^n}{n!}$$

$$(\alpha,\beta,\gamma\in\mathbb{C};\Re(\alpha)>0,\Re(\beta)>0,\Re(\gamma)>0) \tag{1.5}$$

which, in the special case when

$$k=q(q\in(0,1)\cup\mathbb{N}) \tag{1.6}$$

and $\min\{\Re(\alpha),\Re(\beta)\}\geq 0$ was considered earlier by Shukla and Prajapati [16].

A multivariable analogue of Mittag-Leffer function defined in (1.3) has also been studied by Gautam [17] and Saxena et al. [18] in the following form

$$E^{\gamma_j}_{(\rho_j),\gamma}[z_1,.....,z_r]=E^{(\gamma_1........\gamma_r)}_{(\rho_1.......\rho_r),\lambda}[z_1,......,z_r]$$

$$=\sum_{k_1,........,k_r=0}^{\infty}\frac{(\gamma)_{k_1}.......(\gamma)_{k_r}}{\Gamma(\lambda+k_1\rho_1+....+k_r\rho_r)}\frac{z^{k_1}....z^{k_r}}{k_1!....k_r!} \tag{1.7}$$

Where $\lambda,\gamma_j,\rho_j\in\mathbb{C};\Re(\rho_j)>0;j=1,2,......,r.$

If we take $\rho_1=\rho_2=......=\rho_r=1$, then the above equation (1.7) reduces to the following confluent hypergeometric Series (see [19]):

$$\Phi_2^{(r)}\left[\gamma_1,\ldots..\gamma_r;\lambda;z_1,\ldots..,z_r\right]=\frac{1}{\Gamma(\lambda)}\sum_{k_1,\ldots..,k_r=0}^{\infty}\frac{\prod_{i=1}^{r}(\gamma_i)_{k_i}}{(\lambda)_{k_1+\ldots\ldots.k_r}}\frac{z^{k_1}\ldots.z^{k_r}}{k_1!\ldots..k_r!} \tag{1.8}$$

Where

$$\lambda,\gamma_j,z_j\in\mathbb{C};(j=1,2,\ldots..,r)\text{ and max}.\left\{|z_r|,\ldots..,|z_r|\right\}<1;\lambda\notin\mathbb{Z}_0^-$$

A mild generalization of multivariable analogue of Mittag-Leffer function (1.7) is also due to Saxena et al.defined as [18]

$$E_{(\rho_j),\lambda}^{(\gamma_j),(l_j)}[z_1,\ldots\ldots,z_r]=\sum_{k_1\ldots\ldots..k_r=0}^{\infty}\frac{(\gamma)_{k_1l_1,\ldots\ldots.}(\gamma)_{k_rl_r}}{\Gamma(\lambda+k_1\rho_1+\ldots.+k_r\rho_r)}\frac{z^{k_1}\ldots.z^{k_r}}{k_1!\ldots.k_r!}, \tag{1.9}$$

Where

$$\lambda\in\mathbb{C}\setminus\mathbb{Z}_0^-;\gamma_j,\rho_j\in\mathbb{C};\Re(\rho_j)>0;\Re(\gamma_j)>0;l_j\in N(j=1,\ldots\ldots,r).$$

2. FRACTIONAL KINETIC EQUATIONS

The fractional differential equation between rate of change of the reaction, the destruction rate and the production rate was established by Haubold and Mathai [2] given as follows: The solutions of the fractional kinetic equations in this section are obtained in terms of the generalized

$$\frac{dN}{dt}=-d(N_t)+p(N_t) \tag{2.1}$$

Where N = N(t) the rate of reaction, d = d(N) the rate of destruction p = p(N) the rate of production and N_t denotes the function defined by $N(t^*)=N(t-t^*)t^*>0$

$$\frac{dN}{dt}=-c_iN_i \tag{2.2}$$

With the initial condition that $N_i(t=0)=N_0$ is the number density of the species i at time t = 0 and $c_i>0$. If we remove the index i and integrate the standard kinetic equation (2.2), we have

$$N(t)-N_0=-c_0D_t^{-1}N(t) \tag{2.3}$$

Where ${}_0D_t^1$ is the special case of the Riemann-Liouville integral operator ${}_0D_t^{v}$ defined as

$${}_0D_t^{v}f(t)=\frac{1}{\Gamma(v)}\int_0^t(t-s)^{v-1}f(s)ds,\ (\mathrm{t}>0,\ \mathrm{R}(v)>0 \tag{2.4}$$

The fractional generalization of the standard kinetic equation (2.3) is given by Haubold and Mathai [2] as follows:

$$N(t) - N_0 = -c^v {}_0D_t^{-1} N(t) \tag{2.5}$$

and obtained the solution of (2.5) as follows:

$$N(t) = N_0 \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(vk+1)} (ct)^{vk} \tag{2.6}$$

Further, [5] considered the following fractional kinetic equation:

$$N(t) - N_0 f(t) = -c^v {}_0D_t^{-v} N(t) \qquad (\Re(v) > 0) \tag{2.7}$$

Where N(t) denotes the number density of a given species at time t, $N_0 = N(0)$ is the number density of that species at time t = 0; c is a constant and $f \in L(0,\infty)$. By applying the Laplace transform to (2.7) (see[4])

$$L\{N(t);p\} = N_0 \frac{F(P)}{1 + c^v p^{-v}} = N_0 \left(\sum_{n=0}^{\infty} (-c^v)^n p^{-vn} \right) F(p) \quad (n \in N_0 \, \frac{c}{p} < 1) \tag{2.8}$$

Where the Laplace transforms [6] is given by

$$F(p) = L\{N(t);p\} = \int_0^{\infty} e^{-pt} f(t)dt, \qquad (\Re(p) > 0) \tag{2.9}$$

In this section, we investigated the solutions of the generalized fractional kinetic equations by considering generalized Multi index Bessel function.

Remark 1. The solutions of the fractional kinetic equations in this section are obtained in terms of the generalized Mittag-Leffler function $E_{\alpha,\beta}(x)$ (Mittag-Leffer [3]), which is defined as:

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z_n}{\Gamma(\alpha n + \beta)}, \qquad \Re(\alpha) > 0, \Re(\beta) > 0. \tag{2.10}$$

Theorem 1: If $d > 0$, $a > 0$, $v > 0$, $\lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-; \gamma_j, \rho_j \in \mathbb{C};$

$$\Re(\rho_j) > 0; \Re(\gamma_j) > 0; l_j \in N (j = 1, \dots, r).$$

such that $a \neq d$, then the solution of the following fractional kinetic equation:

$$N(t) - N_0 E_{(\rho_j),\lambda}^{(\gamma_j),(l_j)} [(z_1 d^v t^v), \dots, (z_r d^v t^v)] = -a^v {}_0D_t^{-v} N(t) \tag{2.11}$$

is given by following formula

$$N(t) = N_0 \sum_{k_1,\dots\dots,k_r=0}^{\infty} \frac{(\gamma)_{k_1 l_1}, \dots\dots, (\gamma)_{k_r l_r}}{\Gamma(\lambda + k_1\rho_1 + \dots + k_r\rho_r)} \frac{(z_1 t^v d^v)^{k_1} \dots (z_r t^v d^v)^{k_r}}{k_1! \dots k_r!}$$

$$\times \Gamma(v(k_1 + \dots + k_r) + 1) \times E_{v, v(k_1 + \dots + k_r)+1}(-a^v t^v). \tag{2.12}$$

Proof. Laplace transform of Riemann-Liouville fractional integral operator is given by (Erdelyi etal. [1], Srivastava and Saxena [7]):

$$L\left\{ {}_0D_t^{-v} f(t); p \right\} = p^{-v} F(p) \tag{2.13}$$

Where F(p) is defined in (2.9). Now, applying Laplace transform on (2.11) gives,

$$L\left\{ N(t); p \right\} = N_0 L\left\{ E_{(\rho_j),\lambda}^{(\gamma_j),(l_j)} [(z_1 d^v t^v),...,\right.$$
$$\left.(z_r d^v t^v)]; p \right\} - a^v L\left\{ {}_0D_t^{-v} N(t); p \right\} \tag{2.14}$$

$$N(p) = N_0 \left(\int_0^\infty e^{-pt} \sum_{k_1,......,k_r=0}^{\infty} \frac{(\gamma)_{k_1 l_1},........,(\gamma)_{k_r l_r}}{\Gamma(\lambda + k_1\rho_1 + ... + k_r\rho_r)} \frac{(z_1 d^v t^v)....(z_r d^v t^v)}{k_1!.........k_r!} dt \right) \tag{2.15}$$
$$-a^v p^{-v} N(p)$$

Interchanging the order of integration and summation in (2.15), we have

$$N(p) + a^v p^{-v} N(p) = N_0 \sum_{k_1,.......,k_r=0}^{\infty} \frac{(\gamma)_{k_1 l_1},.......,(\gamma)_{k_r l_r}}{\Gamma(\lambda + k_1\rho_1 + ... + k_r\rho_r)}$$
$$\frac{(z_1 d^v)^{k_1}....(z_r d^v)^{k_r}}{k_1!.....k_r!} \times \int_0^\infty e^{-pt} t^{v(k_1+.....+k_r)} dt \tag{2.16}$$

$$= N_0 \sum_{k_1,.......,k_r=0}^{\infty} \frac{(\gamma)_{k_1 l_1},.......,(\gamma)_{k_r l_r}}{\Gamma(\lambda + k_1\rho_1 + ... + k_r\rho_r)}$$
$$\times \frac{(z_1 d^v)^{k_1}....(z_r d^v)^{k_r}}{k_1!.....k_r!} \frac{\Gamma(v(k_1 + ... + k_r) + 1)}{p^{v(k_1+.......+k_r)+1}} \tag{2.17}$$

this leads to

$$N(p) = N_0 \sum_{k_1,.......,k_r=0}^{\infty} \frac{(\gamma)_{k_1 l_1},.......,(\gamma)_{k_r l_r}}{\Gamma(\lambda + k_1\rho_1 + ... + k_r\rho_r)} \frac{(z_1 d^v)^{k_1}....(z_r d^v)^{k_r}}{k_1!.....k_r!}$$
$$\times \Gamma(v(k_1 + ... + k_r) + 1) \times \left\{ p^{-(v(k_1+......+k_r)+1)} \sum_{l=0}^{\infty} \left[-\left(\frac{p}{a}\right)^{-v} \right]^l \right\} \tag{2.18}$$

Taking Laplace inverse of (2.18), and by using

$$L^{-1}\left\{ p^{-v}; t \right\} = \frac{t^{v-1}}{\Gamma(v)}, \ (\Re(v) > 0) \tag{2.19}$$

we have

$$L^{-1}\{N(p)\} = N_0 \sum_{k_1,\ldots\ldots,k_r=0}^{\infty} \frac{(\gamma)_{k_1 l_1},\ldots\ldots,(\gamma)_{k_r l_r}}{\Gamma(\lambda + k_1\rho_1 + \ldots.. + k_r\rho_r)} \frac{(z_1 d^v)^{k_1} \ldots.(z_r d^v)^{k_r}}{k_1!\ldots..k_r!}$$

$$\times \Gamma(v(k_1 + \ldots.. + k_r) + 1) \times L^{-1}\left\{\sum_{l=0}^{\infty} (-1)^l a^{vl} p^{-[v(k_1+\ldots\ldots+k_r)+1]}\right\} \tag{2.20}$$

i.e.

$$N(t) = N_0 \sum_{k_1,\ldots\ldots,k_r=0}^{\infty} \frac{(\gamma)_{k_1 l_1},\ldots\ldots,(\gamma)_{k_r l_r}}{\Gamma(\lambda + k_1\rho_1 + \ldots.. + k_r\rho_r)} \frac{(z_1 d^v)^{k_1} \ldots.(z_r d^v)^{k_r}}{k_1!\ldots..k_r!}$$

$$\times \Gamma(v(k_1 + \ldots.. + k_r) + 1) \times \left\{\sum_{l=0}^{\infty} (-1)^l a^{vl} \frac{t^{v(k_1+\ldots\ldots+k_r+l)}}{\Gamma(v(k_1 + \ldots.. + k_r + l) + 1)}\right\} \tag{2.21}$$

$$N(t) = N_0 \sum_{k_1,\ldots\ldots,k_r=0}^{\infty} \frac{(\gamma)_{k_1 l_1},\ldots\ldots,(\gamma)_{k_r l_r}}{\Gamma(\lambda + k_1\rho_1 + \ldots.. + k_r\rho_r)} \frac{(z_1 d^v)^{k_1} \ldots.(z_r d^v)^{k_r}}{k_1!\ldots..k_r!}$$

$$\times \Gamma(v(k_1 + \ldots.. + k_r) + 1) \times t^{v(k_1+\ldots\ldots+k_r)} \left\{\sum_{l=0}^{\infty} (-1)^l \frac{(a^v t^v)^l}{\Gamma(v(k_1 + \ldots.. + k_r + l) + 1)}\right\} \tag{2.22}$$

Using (2.10)

$$N(t) = N_0 \sum_{k_1,\ldots..,k_r=0}^{\infty} \frac{(\gamma)_{k_1 l_1,}\ldots..(\gamma)_{k_r l_r}}{\Gamma(\lambda + k_1\rho_1 + \ldots.. + k_r\rho_r)} \frac{(z_1 t^v d^v)^{k_1} \ldots.(z_r t^v d^v)^{k_r}}{k_1!\ldots..k_r!}$$

$$\times \Gamma(v(k_1 + \ldots.. + k_r) + 1) \times E_{v,v(k_1+\ldots\ldots.+k_r)+1}(-a^v t^v) \tag{2.23}$$

Hence the required result.

Theorem 2. If $d > 0$, $a > 0$, $v > 0$, $\lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-; \gamma_j, \rho_j \in \mathbb{C};$

$$\Re(\rho_j) > 0; \Re(\gamma_j) > 0; l_j \in N(j = 1,\ldots,r).$$

then the solution of the following fractional kinetic equation:

$$N(t) - N_0 E_{(\rho_j),\lambda}^{(\gamma_j),(l_j)}[(z_1 d^v t^v),\ldots,(z_r d^v t^v)] = -d^v{}_0D_t^{-v} N(t) \tag{2.24}$$

is given by following formula

$$N(t) = N_0 \sum_{k_1,\ldots..,k_r=0}^{\infty} \frac{(\gamma)_{k_1 l_1},\ldots\ldots,(\gamma)_{k_r l_r}}{\Gamma(\lambda + k_1\rho_1 + \ldots.. + k_r\rho_r)} \frac{(z_1 t^v d^v)^{k_1} \ldots.(z_r t^v d^v)^{k_r}}{k_1!\ldots..k_r!}$$

$$\times \Gamma(v(k_1 + \ldots.. + k_r) + 1) \times E_{v,v(k_1+\ldots\ldots.+k_r)+1}(-d^v t^v) \tag{2.25}$$

Theorem 3. If $d > 0$, $v > 0, \lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-; \gamma_j, \rho_j \in \mathbb{C};$

$$\Re(\rho_j) > 0; \Re(\gamma_j) > 0; l_j \in N (j = 1, ..., r), then$$

the solution of the following fractional kinetic equation

$$N(t) - N_0 E_{(\rho_j),\lambda}^{(\gamma_j),(l_j)}[(z_1 t), ..., (z_r t)] = -d^v {}_0D_t^{-v} N(t) \tag{2.26}$$

is given by following formula

$$N(t) = N_0 \sum_{k_1 ....k_r = 0}^{\infty} \frac{(\gamma)_{k_1 l_1, ...,} (\gamma)_{k_r l_r}}{\Gamma(\lambda + k_1\rho_1 + ... + k_r\rho_r)} \frac{(z_1 t)^{k_1} ....(z_r t)^{k_r}}{k_1 !.....k_r !}$$

$$\times \Gamma(v(k_1 + ... + k_r) + 1) \times E_{v,(k_1 + ... + k_r)+1}(-d^v t^v) \tag{2.27}$$

Proof. The proof of the Theorem 2 and 3 are similar as that of Theorem 1, therefore we omit the details.

3. SPECIAL CASES

By assigning the suitable values to the parameters, we have the following particular cases.

If we choose $l_j = 1 (j = 1, ..., r)$ the established results in Theorems 1, 2 and 3 reduces to the following form:

Corollary 1. If $d > 0$, $a > 0$, $v > 0, \lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-; \gamma_j, \rho_j \in \mathbb{C};$

$$\Re(\rho_j) > 0; \Re(\gamma_j) > 0; (j = 1, ..., r),$$

such that a ≠ d, then the solution of the following fractional kinetic equation:

$$N(t) - N_0 E_{(\rho_j),\lambda}^{(\gamma_j)}[(z_1 d^v t^v), ..., (z_r d^v t^v)] = -a^v {}_0D_t^{-v} N(t) \tag{3.1}$$

is given by following formula

$$N(t) = N_0 \sum_{k_1 ....k_r = 0}^{\infty} \frac{(\gamma)_{k_1, ...,} (\gamma)_{k_r}}{\Gamma(\lambda + k_1\rho_1 + ... + k_r\rho_r)} \frac{(z_1 t^v d^v)^{k_1} ....(z_r t^v d^v)^{k_r}}{k_1 !.....k_r !}$$

$$\times \Gamma(v(k_1 + ... + k_r) + 1) \times E_{v,v(k_1 + ... + k_r)+1}(-a^v t^v) \tag{3.2}$$

Corollary 2. If $d > 0$, $v > 0, \lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-; \gamma_j, \rho_j \in \mathbb{C};$

$$\Re(\rho_j) > 0; \Re(\gamma_j) > 0 (j = 1, ..., r), then$$

the solution of the following fractional kinetic equation:

$$N(t) - N_0 E_{(\rho_j),\lambda}^{(\gamma_j)}[(z_1 d^v t^v), ..., (z_r d^v t^v)] = -d^v {}_0D_t^{-v} N(t) \tag{3.3}$$

is given by following formula

$$N(t) = N_0 \sum_{k_1.......k_r=0}^{\infty} \frac{(\gamma)_{k_1,.......,}(\gamma)_{k_r}}{\Gamma(\lambda + k_1\rho_1 + ... + k_r\rho_r)} \frac{(z_1 t^v d^v)^{k_1} ....(z_r t^v d^v)^{k_r}}{k_1 !.....k_r !}$$

$$\times \Gamma(v(k_1 + ... + k_r) + 1) \times E_{v,v(k_1+.......+k_r)+1}(-d^v t^v) \tag{3.4}$$

Corollary 3. If $d > 0$, $v > 0$, $\lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-; \gamma_j, \rho_j \in \mathbb{C};$

$$\Re(\rho_j) > 0; \Re(\gamma_j) > 0 \ (j = 1,...,r),$$

then the solution of the following fractional kinetic equation:

$$N(t) - N_0 E_{(\rho_j),\lambda}^{(\gamma_j),(l_j)}[(z_1 t),...,(z_r t)] = -d^v {}_0D_t^{-v} N(t) \tag{3.5}$$

is given by following formula

$$N(t) = N_0 \sum_{k_1.......k_r=0}^{\infty} \frac{(\gamma)_{k_1,.......,}(\gamma)_{k_r}}{\Gamma(\lambda + k_1\rho_1 + ... + k_r\rho_r)} \frac{(z_1 t)^{k_1} ....(z_r t)^{k_r}}{k_1 !.....k_r !}$$

$$\times \Gamma(v(k_1 + ... + k_r) + 1) \times E_{v,(k_1+.......+k_r)+1}(-d^v t^v) \tag{3.6}$$

If we choose $\rho_1 = ... = \rho_r = 1$, the established results in Theorems 1 2 and 3 reduces to the following form:

Corollary 4. If $d > 0$, $a > 0$, $v > 0, \lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-; \gamma_j, \rho_j \in \mathbb{C};$

$$\Re(\rho_j) > 0; \Re(\gamma_j) > 0; l_j \in N(j = 1,...,r),$$

such that $a \neq d$, then the solution of following fractional kinetic equation's:

$$N(t) - N_0 \Phi_2^{(r)}[\gamma_1,...,\gamma_r; \lambda; (z_1 d^v t^v),.....,(z_r d^v t^v)] = -a^v {}_0D_t^{-v} N(t) \tag{3.7}$$

Is given by following formula:

$$N(t) = \frac{N_0}{\Gamma(\lambda)} \sum_{k_1,.....,k_r=0}^{\infty} \frac{\prod_{i=1}^{r} (\gamma_i)_{k_i}}{(\lambda)_{k_1+.....+k_r}} \frac{(z_1 t^v d^v)^{k_1} ....(z_r t^v d^v)^{k_r}}{k_1 !.....k_r !}$$

$$\times \Gamma(v(k_1 + ... + k_r) + 1) \times E_{v,v(k_1+.......+k_r)+1}(-a^v t^v) \tag{3.8}$$

Corollary 5. If $d > 0$, $v > 0, \lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-; \gamma_j, \rho_j \in \mathbb{C};$

$$\Re(\rho_j) > 0; \Re(\gamma_j) > 0; l_j \in N(j = 1,...,r),$$

then the solution of the following fractional kinetic equation:

$$N(t) - N_0 \Phi_2^{(r)}[\gamma_1,...,\gamma_r; \lambda; (z_1 d^v t^v),.....,(z_r d^v t^v)] = -d^v {}_0D_t^{-v} N(t) \tag{3.9}$$

is given by following formula:

$$N(t) = \frac{N_0}{\Gamma(\lambda)} \sum_{k_1,\ldots,k_r=0}^{\infty} \frac{\prod_{i=1}^{r} (\gamma_i)_{k_i}}{(\lambda)_{k_1+\ldots+k_r}} \frac{(z_1 t^v d^v)^{k_1} \ldots (z_r t^v d^v)^{k_r}}{k_1! \ldots k_r!}$$

$$\times \Gamma(v(k_1+\ldots+k_r)+1) \times E_{v,v(k_1+\ldots+k_r)+1}(-d^v t^v) \tag{3.10}$$

Corollary 6. If $d > 0,\ v > 0, \lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-; \gamma_j, \rho_j \in \mathbb{C};$

$\Re(\rho_j) > 0; \Re(\gamma_j) > 0; l_j \in N(j = 1, \ldots, r)$, then

the solution of the following fractional kinetic equation:

$$N(t) - N_0 \Phi_2^{(r)}[\gamma_1, \ldots, \gamma_r; \lambda; (z_1 t), \ldots, (z_r t)] = -d^v\, {}_0D_t^{-v} N(t) \tag{3.11}$$

is given by following formula:

$$N(t) = \frac{N_0}{\Gamma(\lambda)} \sum_{k_1,\ldots,k_r=0}^{\infty} \frac{\prod_{i=1}^{r} (\gamma_i)_{k_i}}{(\lambda)_{k_1+\ldots+k_r}} \frac{(z_1 t)^{k_1} \ldots (z_r t)^{k_r}}{k_1! \ldots k_r!}$$

$$\times \Gamma(v(k_1+\ldots+k_r)+1) \times E_{v,(k_1+\ldots+k_r)+1}(-d^v t^v) \tag{3.12}$$

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