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EDGE ANTIMAGIC LABELING OF *N* COPIES OF GENERALIZED PETERSEN GRAPHS

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ABSTRACT: Let $G = (V, E)$ be a simple graph with v vertices and e edges. An (a, d)-*edge antimagic total labeling* of G is a bijection $f : V \cup E \to \{1, 2, \ldots, v + e\}$ so that the set of edge weights of all edges in G is $\{a, a + d, a + 2d, \ldots a + (e - 1)d\}$ where a, d are two fixed positive integers and G is called *edge antimagic total* **(EAT).**

f is called super (a, d)-edge antimagic labeling if $f(V) = \{1, 2, \ldots, v\}$ and in that case G is called *super edge antimagic total* **(SEAT).**

In this paper we study the edge antimagic labeling of N copies of generalized Petersen graphs. A generalized Petersen graph $P(n, m)$, $n \geq 3$, $1 \leq m < \frac{n}{2}$ is a 3-regular graph with $2n$ vertices $u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n$ and edges (u_i, v_i), (u_i, u_{i+1}), (v_i, v_{i+m}) for all $i \in \{1, 2, \ldots, n\}$, where the subscripts are taken modulo n. $P(5, 2)$ is the standard Petersen graph.

Keywords: Edge antimagic labeling, Copywise edge antimagic total, Generalized petersen graphs.

1. INTRODUCTION

A very popular concept of Graph theory is the concept of labeling of graphs which was introduced in the late 1960's. Graph labeling is an assignment of integers to the vertices or edges or both under certain conditions. A detailed survey of graph labelings is given in [2]. In this paper, we study the edge antimagic labeling of N copies of Generalized Petersen graphs where N is a finite positive integer. Magic labeling is treated as very important among other labelings since many graph – theoretic properties of some graphs can be studied by investigating their magic properties. They also play a vital role in Wireless networks, Circuit designs, Radars, etc.

Definition 1: Let $G = (V, E)$ be a simple graph with v vertices and e edges. An (a, d)-*edge antimagic total labeling* of G is a bijection $f : V \cup E \to \{1, 2, \ldots, v + e\}$ so that the set of edge weights of all edges in G is $\{a, a + d, a + 2d, \ldots, a + (e - 1)d\}$ where a, d are two fixed positive integers and G is called *edge antimagic total* **(EAT).**

f is called super (a, d)-edge antimagic labeling if $f(V) = \{1, 2, \ldots, v\}$ and in that case G is called *super edge antimagic total* **(SEAT).** Many results in edge antimagic labelings can be seen in [3] and [5].

Definition 2: Let $G = (V, E)$ be a simple graph with v vertices and e edges. Consider NG, the N copies of G where N is a finite positive integer. A bijection $g : V(NG) \cup E(NG) \to \{1, 2, \ldots, N(v + e)\}$ is called *copywise (a_t, d)-edge antimagic total labeling* if each of the t^{th} copy of NG becomes (a_t, d)-edge antimagic total separately for $t = 1, 2, \ldots, N$ and NG is called copywise (a_t, d)-edge antimagic total.

Definition 3: A generalized Petersen graph $P(n, m)$, $n \geq 3$, $1 \leq m < \frac{n}{2}$ is a 3-regular graph with $2n$ vertices $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n$ and edges (u_i, v_i), (u_i, u_{i+1}), (v_i, v_{i+m}) for all $i \in \{1, 2, \ldots, n\}$, where the subscripts are taken modulo n.

In [4] we can get (a, d)-edge antimagic labeling of generalized Petersen graphs.

2. Edge Antimagic Labeling of $NP(n, m)$

Theorem 1: Every generalized Petersen graph $P(n, m)$, $n \geq 3$, $1 \leq m < \frac{n}{2}$ has a super $(4n + 2, 1)$-edge antimagic total labeling.

Proof: Consider $G = (V, E) = P(n, m)$ with $v = 2n$ vertices and $e = 3n$ edges where $n \geq 3$, $1 \leq m < \frac{n}{2}$. Define $f : V \cup E \to \{1, 2, \ldots, v + e = 5n\}$ as follows:

$$f(u_i) = n - i + 1,\ f(v_i) = n + i \quad \text{for} \quad 1 \leq i \leq n$$

$$f(u_i u_{i+1}) = \begin{cases} 3n + 1 + i & \text{for} \quad 1 \leq i \leq n-1 \\ 3n + 1 & \text{for} \quad i = n \end{cases}$$

$$f(u_i v_i) = 4n + i,\ f(v_i v_{i+m}) = 3n + 1 - i \quad \text{for} \quad 1 \leq i \leq n$$

Then the edge weights of $P(n, m)$ are given by

$$W_1 = \{w(u_i u_{i+1})\} = f(u_i) + f(u_{i+1}) + f(u_i u_{i+1}) = \{5n + 2 - i, 1 \leq i \leq n\}$$

$$W_2 = \{w(u_i v_i)\} = f(u_i) + f(v_i) + f(u_i v_i) = \{6n + 1 + i, 1 \leq i \leq n\}$$

$$W_3 = \{w(v_i v_{i+m})\} = f(v_i) + f(v_{i+m}) + f(v_i v_{i+m}) = \{5n + i + 1, 1 \leq i \leq n\}.$$

Thus the set of edge weights is given by $W_1 \cup W_2 \cup W_3 = \{4n + 2, 4n + 3, \ldots, 5n, 5n + 1, \ldots 6n, 6n + 1, \ldots, 7n, 7n + 1\}$ which gives an $(4n + 2, 1)$-edge antimagic total labeling.

Theorem 2: For $n \geq 3$ and $1 \leq m < \frac{n}{2}$, $NP(n, m)$ is copywise edge antimagic total where the t^{th} copy of $NP(n, m)$ has a $(n(5N + 5t - 6) + 2, 1)$-edge antimagic total labeling.

Proof: Consider the labeling f given in theorem 1.Then the labeling $g^t : V(NP(n, m)) \cup E(NP(n, m)) \to \{1, 2, \ldots, 5Nn\}$ for the t^{th} graph (copy) of $NP(n, m)$ is given by

$$g^t(u_i) = f(u_i) + (t - 1)\, 5n$$

$$g^t(v_i) = f(v_i) + (t - 1)\, 5n$$

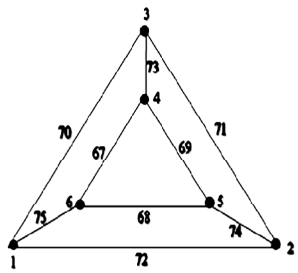
$$g^t(u_i v_i) = f(u_i v_i) + (N - t)\, 5n$$

$$g^t(u_i u_{i+1}) = f(u_i u_{i+1}) + (N - t)\, 5n$$

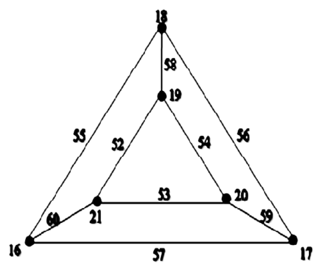
$$g^t(v_i v_{i+m}) = f(v_i v_{i+m}) + (N - t)\, 5n, \quad \text{where} \quad t = 1, 2, \ldots, N.$$

It can be easily verified that the set of edge weights of the t^{th} graph (copy) is given by $E_t = \{n(5N + 5t - 6) + 2, n(5N + 5t - 6) + 3, \ldots, n(5N + 5t - 3) + 1\}$, (i.e) each of the t^{th} copy of $NP(n, m)$ has a $(n(5N + 5t - 6) + 2,\ 1)$-edge antimagic total labeling. Hence $NP(n, m)$ is copywise edge antimagic total.

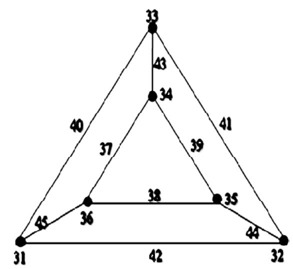
Example: Copywise edge antimagic total labeling of $5\ P(3, 1)$



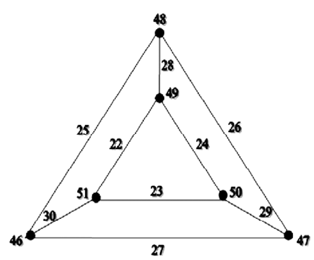
$E_1 = \{74, 75, \ldots, 82\}$



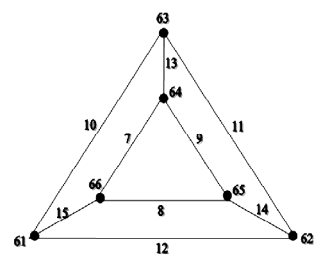
$E_2 = \{89, 90, \ldots, 97\}$



$E_3 = \{104, 105, \ldots, 112\}$



$E_4 = \{119, 120, \ldots, 127\}$



$E_5 = \{134, 135, \ldots, 142\}$

Theorem 3: For odd n, $n \geq 3$, every generalized Petersen graph $P(n, 1)$ has a $\left(\frac{9n+5}{2}, 2\right)$-edge antimagic total labeling.

Proof: Consider the labeling $f: V \cup E \to \{1, 2, \ldots, 5n\}$ such that

$$f(u_i) = \begin{cases} \dfrac{1}{2}(2n + 2 + i), & \text{for} \quad i \equiv 0 \pmod 2 \\ \dfrac{1}{2}(3n + 2 + i), & \text{for} \quad i \equiv 1 \pmod 2,\ i \neq n \\ n + 1, & \text{for} \quad i = n \end{cases}$$

$$f(v_i) = \begin{cases} \dfrac{1}{2}(7n + 3 + i), & \text{for} \quad i \equiv 0 \pmod 2,\ i \neq n - 1 \\ \dfrac{1}{2}(6n + 3 + i), & \text{for} \quad i \equiv 1 \pmod 2, \\ 3n + 1, & \text{for} \quad i = n - 1 \end{cases}$$

$$f(u_i u_{i+1}) = \begin{cases} 2n + 2 + i, & \text{for } \; i \neq n-1, n \\ 2n + 1, & \text{for } \; i = n-1, \\ 2n + 2, & \text{for } \; i = n \end{cases}$$

$$f(u_i v_i) = \begin{cases} 4n + 2 + i, & \text{for } \; i \neq n-1, n \\ 4n + 1, & \text{for } \; i = n-1, \\ 4n + 2, & \text{for } \; i = n \end{cases}$$

$$f(v_i v_{i+1}) = \begin{cases} 3 + i, & \text{for } \; i \neq n-2, n-1, n \\ 3 + i - n, & \text{for } \; i = n-2, n-1, n \end{cases}$$

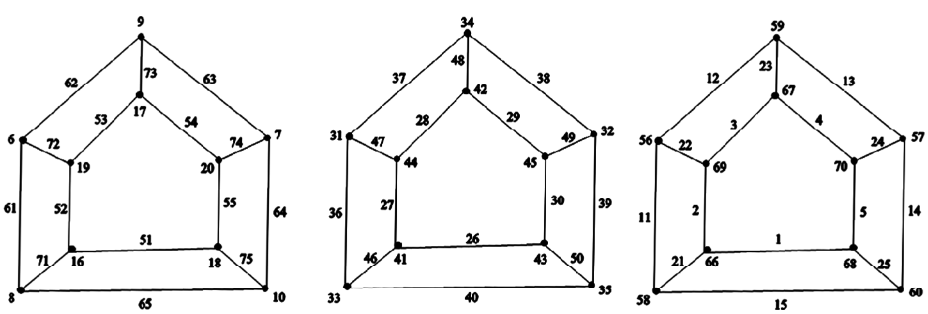
It can be easily verified that under this labeling f the set of edge-weights of $P(n, 1)$ is given by $\left\{\frac{9n+5}{2}, \frac{9n+5}{2} + 2, \ldots \frac{21n+1}{2}\right\}$.

Theorem 4: For odd n, $n \geq 3$, NP(n, 1) is copywise edge antimagic total where the t^{th} copy of N$P(n,1)$ has a $\left(\frac{(10N+10t-11)n+5}{2}, 2\right)$-edge antimagic total labeling.

Proof: Consider the labeling f for $P(n, 1)$ given in theorem 3. Then define the labeling g^t for the t^{th} graph (copy) of $NP(n, 1)$ as given in theorem 2 using this f.

It can be easily verified that the set of edge weights of the t^{th} graph (copy) is given by $E_t = \left\{\frac{(10N+10t-11)n+5}{2}, \frac{(10N+10t-11)n+5}{2} + 2, \ldots \frac{(10N+10t+1)n+1}{2}\right\}$ (i.e) each of the t^{th} copy of $NP(n, 1)$ has a $\left(\frac{(10N+10t-11)n+5}{2}, 2\right)$-edge antimagic total labeling. Hence N$P(n,1)$ is copywise edge antimagic total.

Example: Copywise edge antimagic total labeling of $3P(5, 1)$



$E_1 = \{75, 77, \ldots, 103\}$ $E_2 = \{100, 102, \ldots, 128\}$ $E_3 = \{125, 127, \ldots, 153\}$

Theorem 5 For odd n, $n \geq 3$, every generalized Petersen graph $P(n, 2)$ has a $\left(\frac{9n+5}{2}, 2\right)$-edge antimagic total labeling.

Proof: Define $f : V \cup E \to \{1, 2, \ldots, v + e = 5n\}$ as follows:

$$f(u_i) = \begin{cases} \frac{1}{2}(2n+2+i), & \text{for } i \equiv 0 \pmod 2 \\ \frac{1}{2}(3n+2+i), & \text{for } i \equiv 1 \pmod 2, i \neq n \\ n+1, & \text{for } i = n \end{cases}$$

$$f(u_i u_{i+1}) = \begin{cases} 2n+2+i, & \text{for } i \neq n-1, n \\ 2n+1, & \text{for } i = n-1, \\ 2n+2, & \text{for } i = n \end{cases}$$

$$f(v_i v_{i+2}) = \begin{cases} \frac{1}{2}(2n-2-i), & \text{for } i \equiv 0 \pmod 2 \\ \frac{1}{2}(n-2-i), & \text{for } i \equiv 1 \pmod 2, i \neq n-2, n \\ n, & \text{for } i = n-2 \\ n-1, & \text{for } i = n \end{cases}$$

Case 1: For $n \equiv 1 \pmod 4$

$$f(v_i) = \begin{cases} \frac{1}{4}(16n-i), & \text{for } i \equiv 0 \pmod 4, \\ \frac{1}{4}(13n-i), & \text{for } i \equiv 1 \pmod 4, \\ \frac{1}{4}(14n-i), & \text{for } i \equiv 2 \pmod 4, \\ \frac{1}{4}(15n-i), & \text{for } i \equiv 3 \pmod 4, \\ 4n, & \text{for } i = n. \end{cases}$$

$$f(u_i v_i) = \begin{cases} \frac{1}{4}(18n+2+i), & \text{for } i \equiv 0 \pmod 4, \\ \frac{1}{4}(17n+2+i), & \text{for } i \equiv 1 \pmod 4, \\ \frac{1}{4}(16n+2+i), & \text{for } i \equiv 2 \pmod 4, \\ \frac{1}{4}(19n+2+i), & \text{for } i \equiv 3 \pmod 4. \end{cases}$$

Case 2: For $n \equiv 3 \pmod 4$

$$f(v_i) = \begin{cases} \frac{1}{4}(16n - i), & \text{for } i \equiv 0 \pmod 4, \\ \frac{1}{4}(15n - i), & \text{for } i \equiv 1 \pmod 4, \\ \frac{1}{4}(14n - i), & \text{for } i \equiv 2 \pmod 4, \\ \frac{1}{4}(13n - i), & \text{for } i \equiv 3 \pmod 4, \\ 4n, & \text{for } i = n. \end{cases}$$

$$f(u_i v_i) = \begin{cases} \frac{1}{4}(18n + 2 + i), & \text{for } i \equiv 0 \pmod 4, \\ \frac{1}{4}(19n + 2 + i), & \text{for } i \equiv 1 \pmod 4, \\ \frac{1}{4}(16n + 2 + i), & \text{for } i \equiv 2 \pmod 4, \\ \frac{1}{4}(17n + 2 + i), & \text{for } i \equiv 3 \pmod 4. \end{cases}$$

The set of edge-weights of $P(n, 2)$ is given by $\left\{\frac{9n+5}{2}, \frac{9n+5}{2} + 2, \ldots \frac{21n+1}{2}\right\}$.

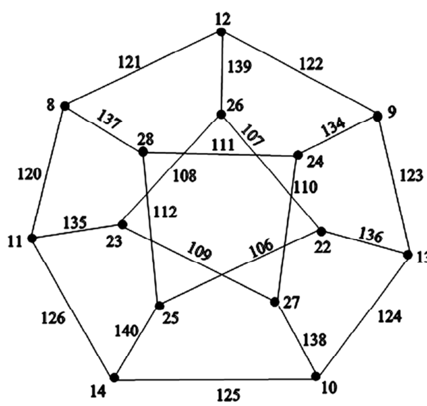
Thus $P(n, 2)$ has a $\left(\frac{9n+5}{2}, 2\right)$-edge antimagic total labeling.

Theorem 6: For odd n, $n \geq 3$, $NP(n, 2)$ is copywise edge antimagic total where the t^{th} copy of $NP(n, 2)$ has a $\left(\frac{(10N+10t-11)n+5}{2}, 2\right)$-edge antimagic total labeling.

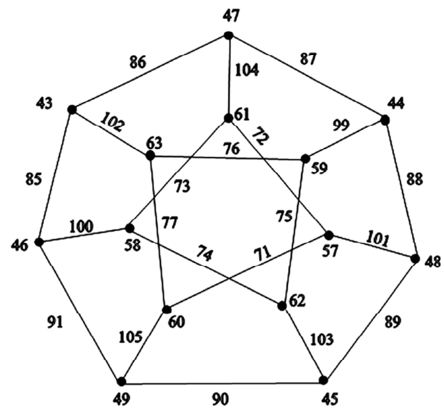
Proof: Construct the labeling g^t for the t^{th} graph (copy) of $NP(n, 2)$ same way as in theorem 2 by using the labeling f of theorem 5.

Then the set of edge weights of the t^{th} copy of $NP(n, 2)$ is given by $E_t = \left\{\frac{(10N+10t-11)n+5}{2}, \frac{(10N+10t-11)n+5}{2} + 2, \ldots \frac{(10N+10t+1)n+1}{2}\right\}$ (i.e) each of the t^{th} copy of $NP(n, 1)$ has a $\left(\frac{(10N+10t-11)n+5}{2}, 2\right)$-edge antimagic total labeling. Hence $NP(n, 2)$ is copywise edge antimagic total.

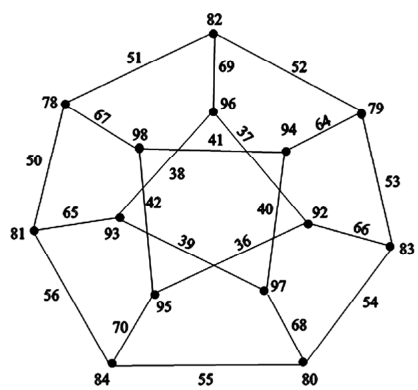
Example: Copywise edge antimagic total labeling of 4 $P(7, 2)$



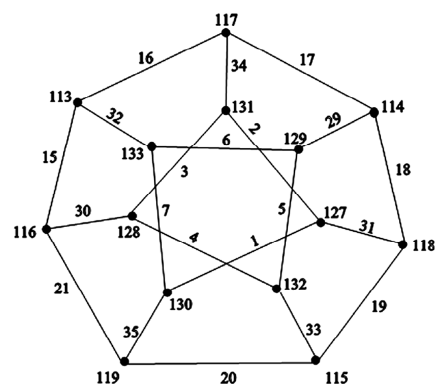
$E_1 = \{139, 141, \ldots, 179\}$



$E_2 = \{174, 176, \ldots, 214\}$



$E_3 = \{209, 211, \ldots, 249\}$



$E_4 = \{244, 246, \ldots, 284\}$

3. CONCLUSION

In this paper we have discussed about edge antimagic labelings of *N* copies of generalized Petersen graphs which are isomorphic to each other. In the case of non isomorphic disconnected graphs still the problem of constructing edge antimagic labelings remains open. Some more open problems in this area are given in [1].

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