OPTIMAL ROBUST DESIGN FOR A TWIN ROTOR SYSTEM USING MULTI-OBJECTIVE GENETIC ALGORITHM TUNED MODEL PREDICTIVE CONTROLLER

Parvesh Kumar* and Dr. Shiv Narayan

Abstract: In this paper an optimal robust design for the twin rotor mimc (TRMS) system has been implemented with random measurement noise. TRMS has higher order and exhibit non-linearities. Here, tuning of model predictive controller’s (MPC) parameter for achieving robustness and tracking performance has been considered. To achieve the desire robust response and tracking performance, infinity norm of the sensitivity function and integral square of the error is minimized for both the inputs using multi-objective genetic algorithm (MOGA) tuned MPC. By tuning of the MPC parameters using MOGA, an optimal set of solutions are generated and the ideal solution is selected from the Pareto optimal set using level diagrams. From the simulation results it is clear that MOGA tuned MPC is robust to measurement noise applied at the output side as well as it also performs the proper tracking of the desired yaw angle and pitch angle in twin rotor mimc system.

Key Words: Model Predictive Control; Multi-objective Genetic Algorithm; Control horizon; Prediction horizon; Robust control; Multi Input Multi Output; Twin rotor system.

1. INTRODUCTION

Twin rotor MIMO system (TRMS) are widely used in the aerospace application. TRMS consist of two rotors one is for the horizontal moment and other for the vertical moment. These two rotors are attached on a beam for the counter balance, which ensures the safety of the helicopter. For controlling the TRMS mechanical parts and electrical circuitry are used together. In this paper multi-objective genetic algorithm (MOGA) tuned model predictive controller (MPC) is considered for the robust control of pitch and yaw angle in TRMS system. Robust control of the pitch and yaw angle is essential when disturbance or noise is present in the system. To assure robustness and tracking of the system, the controller design problem is formulated as infinity norm of sensitivity function as first objective and integral square error as the second objective of the system.

Model Predictive Control (MPC) is an optimal control strategy based on the mathematical optimization of the performance index. Moving horizon control (MHC) is one of the well known names of MPC and it is prominent for the control dynamical system eg. chemical plants, process control, gas pipeline control etc. At each control interval the controller optimizes the performance index of the plant by estimating the future response of the plant and future manipulated variables.

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MPC has become one of the best computer controlled algorithm that are currently used in industries, because of the computational technique of MPC that has improved the performance of the process [1], [2], [3], [4], [5]. The main advantage of the MPC is that it gives better response for multi input multi output (MIMO) system with large number of control variables. Generalized MPC for MIMO system using the state space interpretation is proposed by A. Gambier [1]. While tracking, error is minimized over prediction horizon with constraints on the input, states and the output of the system. The settling time required in a system must be less than the prediction horizon, because if it is greater than the prediction horizon the system will have oscillations in the response, for the further change in the input.

R. Shridhar et al. [6] explains the concept of tuning the SISO DMC parameters for unconstrained SISO DMC for the 1st order plus delay time (FOPDT) system of the process dynamics. Here FOPDT model approximation is used for the tuning rules such as Cohen-Coon, Integral time absolute error (ITAE), and integral absolute error (IAE) for PID implementations. MIMO system has been simulated by R. Galindo et al. [7] with uncertainty is explained by Wolfgang Ponweiser et al. [8] and F. Abdollahi et al. [9] develop a new technique based on the worst case minimization.

Andrea Richter et al. [10] explained the tuning parameter of different controllers that are used together in multi-loop control system by optimizing the different objective functions e.g. ISE, ITSE and ISTSE. The concept of the gain scheduling control strategy for the multivariable MPC is demonstrated by V.R.Ravi et al. [11]. The tuning of the multiple linear MPC for the Two Conical Tank Interacting Level System is done using real coded genetic algorithm (GA). One of the linear MPC controller output is selected as gain scheduling adaptive controller's output based on the current value of the measured process variable. A robust MPC controller design over infinite horizon for the online optimization has been formulated by V. Ghaffari et al [12], by posing it as worst case optimization problem using LMI. MPC is used for the proper tracking of the reference signal; optimization of the performance indices is solved using the gradient. Based on this gradient, a second order approximation of the economic function is obtained and used in the MPC optimization problem resulting in a convex optimization problem. Recursive feasibility and convergence to the optimal equilibrium point is ensured by D. Limon et al. [13].

By using the LQR design S. K. Pandey and V. Laxmi [14] has designed an optimal state feedback controller for nonlinear twin rotor mimo system to achieve the desired transient and steady state response but the paper doesn’t addresses the robustness of the system. Tuning of the MPC is done G. A. N. Junior et. al [15] design an optimal unconstrained MPC with model uncertainties for shell heavy oil fractionators process using particle swarm optimization (PSO) for the worst case control scenario. In [16], a hot water mixing tank is considered and a hybrid of fuzzy logic and genetic algorithm has been used for the optimum tuning of the MPC parameters to get the desired characteristics, this paper doesn’t address the robustness of the designed system.

In this paper, the design of the MPC controller is done for achieving the robust response of raw and pitch angle of the TRMS, when it is not attached to the helicopter. The design controller offers robust response and has been checked for varying inputs. Also the designed system presents an efficient noise rejection; a continuous random noise has been introduced in the system and the design control system efficiently rejects it and maintains the system stability.
2. MATHEMATICAL MODELING OF TRMS

The control model consists of two rotors one is for horizontal moment and other one is for vertical moment as shown in Figure 1. Generally TMRS models are non linear in nature, which means that TRMS has one or more non linear states in the system. To obtain the transfer function of the TMRS, model has to be linearised. Non linear equations of the system has been derived [14], [17] the approximate values of the parameters are chosen experimentally is shown in Table 1. For the vertical moment of TRMS, momentum equation is below

\[ I_1 \ddot{\psi} = M_1 - M_{FG} - M_{\psi\psi} - M_G \]  

(1)

A torque is induced in the TRMS by nonlinearity static characteristic \( M_1 \), which is produced by the rotor and can be estimated as polynomial of 2nd order. The representation of the nonlinearity is given below.

\[ M_1 = a_1 \tau_1^2 + b_1 \tau_1 \]  

(2)

\( M_{FG} \) is the gravitational momentum produces by the weight of the helicopter around the pivot point and it is expressed by Equation 3.

\[ M_{FG} = M_g \sin(\psi) \]  

(3)

\( M_{\psi\psi} \) is the friction force momentum in the TRMS, which is given by the following equation.

\[ M_{\psi\psi} = B_{\psi} \dot{\psi} + B_{2\psi} \text{sign}(\psi) \]  

(4)

Because of the coriolis force a torque is produces in the TRMS called gyroscopic momentum. When the main rotor changes its position in azimuth direction gyroscopic torque is resulted and described by the following equation.

\[ M_G = K_{\psi}\phi \cos(\psi) \]  

(5)

Here, 1st order transfer function is considered for the motor control circuit and electrical control circuit. Laplace transform of the motor momentum is given by the following equation.
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\[ \tau_1 = \frac{K_1}{T_{11}s + T_{10}} u_1 \]  

(6)

Equations 1-6 are the equations that are used for motion in the vertical plane. Correspondingly for the horizontal plane equation can be developed. The net torques produced by the horizontal plane motion is given by Equation 7.

\[ I_2 \ddot{\phi} = M_2 - M_{B\phi} - M_R \]  

(7)

Similarly \( M_2 \) also induces a momentum in the TMRS in the horizontal plane and is of second order polynomial. The representation of the nonlinearity \( M_2 \) is given by the following equation.

\[ M_2 = a_2\tau_2^2 + b_2\tau_2 \]  

(8)

Frictional torque for the horizontal plane is calculated in a similar way as in vertical plane and is given by

\[ M_{B\psi} = B_{\psi}\dot{\psi} + B_{2\phi} \text{sign}(\dot{\phi}) \]  

(9)

The momentum of the cross reaction (\( M_R \)) is a transfer function of 1\(^{st} \) order, represented by the following equation.

\[ M_R = \frac{k_1(T_2s + 1)}{(T_p s + 1)} \tau_1 \]  

(10)

Here, DC motor with electrical circuit is represented by a transfer function of 1\(^{st} \) order, given below as

\[ \tau_2 = \frac{k_2}{T_{21}s + T_{20}} u_2 \]  

(11)

The nonlinear equations 1-11 of TRMS as given above are linearized about the point \( X_0 \). The value of \( X_0 \) is given below.

\[ X_0 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \]

Vectors consisting of states and outputs are given below as

\[ X = [\psi \ \dot{\psi} \ \phi \ \dot{\phi} \ \tau_1 \ \tau_2 \ M_R]^T \]

\[ Y = [\psi \ \dot{\phi}]^T \]

Generalized state space representation of the plant is given by the following equation.

\[ \dot{x} = Ax + Bu \]

\[ y = Cx \]  

(12)

TRMS consists of 2 inputs and 2 outputs. Where \( \psi \) and \( \Phi \) are pitch angle and raw angle respectively. The corresponding linearized MIMO transfer function of the TRMS is given below:
3. **MPC FORMULATION**

Consider the system has \( n \) states, \( p \) inputs and \( q \) outputs. It is arduous to control each output individually, with no difference in the actual output and desired output in steady state, if number of inputs is less than number of outputs in system. Here design uses the predicted process output \((y)\), which optimizes objective functions, \( J_1 \) and \( J_2 \). Measurement noise \( v(k) \) is considered in the mathematical formulation of the problem.

\[
x_p(k+1) = A_p x_p(k) + B_p u(k)
\]

\[
y(k) = C_p x_p(k) + v(k)
\]

where \( x_p(k) \) is the state variable. \( v(k) \) is the measurement noise, which is random in nature.

where \( A_p, B_p \) and \( C_p \) are the matrices corresponding to Equation 14 and 15. The tracking cost function which penalizes the increment in the change in control action \((\Delta u)\) and error \((e)\), which is represented by \( J \).
\[ J = \sum_{j=0}^{j=N_r} \| y(k+j|k) - r(k+j|k) \|^2_{Q(i)} + \sum_{i=0}^{i=N_c-1} \| \Delta u(k+i|k) \|^2_{R(i)} \]  

(16)

Where

\[ e(k+j|k) = r(k+j|k) - y(k+j|k) \]  

(17)

\[ Y(k) = \begin{bmatrix} y(k) \\ y(k+1|k) \\ \vdots \\ y(k+N_p|k) \end{bmatrix}; \quad T(k) = \begin{bmatrix} r(k) \\ r(k+1|k) \\ \vdots \\ r(k+N_p|k) \end{bmatrix}; \quad \Delta U(k) = \begin{bmatrix} \Delta u(k|k) \\ \Delta u(k+1|k) \\ \vdots \\ \Delta u(k+N_c-1|k) \end{bmatrix}; \]

\[ Q = \begin{bmatrix} Q(0) & 0 & \ldots & 0 \\ 0 & Q(1) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & Q(N_p) \end{bmatrix}; \quad R = \begin{bmatrix} R(0) & 0 & \ldots & 0 \\ 0 & R(1) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & R(N_c-1) \end{bmatrix}; \]

\[ Y(k) = \Psi x(k) + \Upsilon u(k-1) + \Theta \Delta U(k) \]  

(18)

For the stable matrices \( \Upsilon \), \( \Psi \) and \( \Theta \). Define

\[ \varepsilon(k) = Y(k) - \Psi x(k) - \Upsilon u(k-1) \]  

(19)
4. ROBUST TUNING CRITERIA

Robust stability criteria for a TRMS system is formulated with the infinity norm of sensitivity function and to achieve the tracking performance of the system ISE is taken as the second objective function that is to be optimized simultaneously.

\[ J_1 = \|S(j \omega)\|_\infty \]  

\[ J_2 = \sum_{j=0}^{j=N_p} \| y(k + j \mid k) - r(k + j \mid k) \|^2 \]  

Optimization of the \( J_1 \) is done to make the system robust from external noise and disturbances and \( J_2 \) is optimized to achieve the desired tracking performance. Both the objective function are optimized simultaneously to achieve a robust TRMS using MPC.

5. MULTIOBJECTIVE GENETIC ALGORITHM

Real world applications are multi-objective in nature and these objectives are conflicting. In single objective and weighted multi-objective optimization, there exists a single solution, whereas in multi-objective, there exist a set of solutions exists. These optimal solutions are the best solution in the search space where no other solution exist that can give better results in the search space with all the objectives are taken into consideration. GA is an efficient optimization technique which is capable to handle multiple objectives simultaneously.

GA is a computational method inspired by evolution. It’s based on the Darwin’s theory of “survival of the fittest”. At first GA was introduced by Rechenberg and it was further developed by John Holland and some of his scholars and associates. GA is direct method to find the global best solution in the search space during optimization. The main process of GA consists of natural evolution: reproduction, selection and miscellany of the generation. Initially GA selects a set possible solution or individuals or chromosome which is used to create population. The steps involved in GA are selection, crossover, mutation and acceptance of the final solution. The fitness function value of each chromosome from new generation is calculate to find whether it is fitter than the previous generation chromosome or not. The entire process is repeated again and again until a global best solution is obtained.

The steps involved in the process of GA are shown in the flowchart given below [18]:

![Figure 2: Flowchart of MOGA](image-url)
6. Tuning of MPC Using MOGA

Optimization of the MPC parameters \([Q, R, N_c, N_p]\) is done by minimizing the objective function given in equation 20 and 21 using the MOGA. The lower bounds and the upper bounds of the MPC parameters are \([0.001, 0.001, 1, 11]\) and \([1, 1, 10, 20]\) respectively. The ranges of the controller parameters are chosen by running the optimization number of times to achieve the robustness and the tracking performances of the system. It is clear from the results obtained from the optimization that optimum results occur in the lower bounds and the upper bounds specified above. By defining the range for the optimization parameters, it will take less time to find the optimal solution and that solution will not lead to a solution which is local minimum or makes the system unstable.

<table>
<thead>
<tr>
<th>Genetic Algorithm Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>75</td>
</tr>
<tr>
<td>No. of Generations</td>
<td>1000</td>
</tr>
<tr>
<td>Tournament Size</td>
<td>2</td>
</tr>
<tr>
<td>Crossover</td>
<td>0.8</td>
</tr>
<tr>
<td>Migration Fraction</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The optimized values of the MPC parameters obtained using multi-objective genetic algorithm are \([Q, R, N_c, N_p]\) = \([0.7365, 0.0648, 4, 17]\) and the optimum values of the objective function obtained are \([||S_1(j\omega)||_{\infty}, ||S_2(j\omega)||_{\infty}, ISE_1, ISE_2]\) = \([1.0862e^{-04}, 8.9958e^{-04}, 0.0527, 0.2226]\). Where \(||S(j\omega)||_{\infty}\) is the sensitivity function and \(ISE\) is the integral square error. The values of the sensitivity function for both the inputs are less than 1, which makes the system robust in nature.

![Variable step response without random measurement noise](image)

Figure 3: Variable step response without random measurement noise of MOGA tuned MPC.
Figure 4: Variable step response with random measurement noise of MOGA tuned MPC.

Figure 5: Pareto front obtained using MOGA tuned MPC for $1^{st}$ input.
Figure 6: Pareto front obtained using MOGA tuned MPC for 2nd input.

Figure 7: Design objectives and controller parameters values obtained using MOGA tuned MPC.
Figure 8: Pareto set obtained using level diagram analysis.

Figure 9: Pareto front obtained using level diagram analysis.

Table 3: Optimized values of MPC parameters and Objective functions for MOGA tuned MPC.

| S. No. | Q   | R     | $N_c$ | $N_p$ | $||S_1(j\omega)||_\infty$ | $||S_2(j\omega)||_\infty$ | $ISE_1$ | $ISE_2$ |
|-------|-----|-------|-------|-------|----------------------------|----------------------------|----------|----------|
| 1.    | 0.5440 | 0.0757 | 6     | 14    | $2.4388\times10^{-8}$     | $8.3622\times10^{-4}$    | 0.1384   | 0.1136   |
| 2.    | 0.0660 | 0.6982 | 8     | 16    | $7.7402\times10^{-4}$     | $1.0895\times10^{-7}$    | 0.3972   | 0.4186   |
| 3.    | 0.8256 | 0.0355 | 6     | 12    | $1.3676\times10^{-4}$     | $7.9956\times10^{-4}$    | 0.3972   | 0.3186   |
| 4.    | 0.0716 | 0.6867 | 6     | 16    | $6.2040\times10^{-4}$     | $2.7779\times10^{-6}$    | 0.3186   | 0.4070   |
| 5.    | 0.0679 | 0.6960 | 7     | 15    | $7.8371\times10^{-4}$     | $2.0493\times10^{-6}$    | 0.3186   | 0.4070   |
| 6.    | 0.8860 | 0.2077 | 4     | 18    | $1.7283\times10^{-5}$     | $0.05$                   | 0.0343   | 0.6719   |
| 7.    | 0.7365 | 0.0648 | 4     | 17    | $1.0862\times10^{-4}$     | $8.9958\times10^{-4}$    | 0.0527   | 0.2226   |
| 8.    | 0.9010 | 0.2639 | 4     | 18    | $2.9325\times10^{-6}$     | $0.0444$                 | 0.0347   | 0.6275   |
| 9.    | 0.8254 | 0.2348 | 6     | 16    | $7.3704\times10^{-5}$     | $1.1519\times10^{-4}$    | 0.1528   | 0.1251   |
| 10.   | 0.9308 | 0.2728 | 4     | 19    | $1.9389\times10^{-4}$     | $0.0377$                 | 0.0337   | 0.6014   |

*Table 3 Contd...*
7. SIMULATION RESULTS

This paper delineates the tuning of the MPC controller parameters using MOGA for a robust TRMS system. Figure 3 and 4 shows the variable step response of the system with MOGA tuned MPC with and without measurement noise. From the Table III it is clear that all the solution obtained from MOGA tuned MPC for TRMS is robust in nature because the value of the infinity norm of the sensitivity is not less than 1 but it is significantly less values for both the variable inputs. Figure 8 and 9 shows the pareto set and pareto front obtained using level diagram analysis, where 2- norm of the controller parameters and objective functions are used to get the optimum solution from the set of solution obtained using MOGA tuned MPC for TRMS.

8. CONCLUSIONS

Almost all the engineering problem are multi-objectives in nature. In the real world applications noise and disturbances make the system strenuous to achieve the required design requirements. So it is required to make sure that the system is robust in nature. This paper explores the optimal tuning of the MPC parameters using MOGA to make TRMS robust as well as it satisfy the desired tracking performances. Here the optimization problem is formulated as minimization of the infinity of the sensitivity function and ISE for achieving the desired robustness and tracking performances of the raw and pitch angles of TRMS. From the results obtained in this paper, TRMS offers efficient robustness to the continuous random measurement noise inserted in the system. This makes sure the product quality and safety of the process.

References


