Complexity Analysis of Highly Improved Hybrid Turbo Codes

M. Jose Raj* and Sharmini Enoch**

Abstract: Modern digital communication systems need efficient error correcting codes. Highly Improved Hybrid Turbo Code (HIHTC) is a low complex error correcting code with better Bit Error Rate (BER) which is comparable to Turbo Convolutional code (TCC). Rate 1/3 HIHTC shows a BER of 10^-4 for E_b/N_0 of 1.7 dB which is closer to the E_b/N_0 TCC. In this paper we compare the complexity of TCC, Improved Low Complexity Hybrid Turbo Code (ILCHTC), Low complexity Hybrid Turbo Code (LCHTC) with HIHTC. The overall decoder complexity of HIHTC is less than that of TCC, LCHTC and ILCHTC.

Index Terms: Zigzag-Hadamard (ZH), A Posteriori Probability (APP), Bit Error Rate (BER), Decoder Complexity (DC), Hybrid Turbo Code (HTC)

1. INTRODUCTION
A reliable and efficient error correcting code is essential for efficient communication. Since reflection of multiple channels and fading of signals lead to errors, Turbo convolution codes is the efficient error correcting code with better BER. But the decoder complexity of TCC is more. It needs more iteration to decode one bit. Even though MAP based algorithm is used in logarithmic domain the decoder complexity is more. Decoder complexity remains unchanged, even after puncturing is used to improve the code rate. HIHTC is the improved version of ILCHTC which is the combination of TCC and ZH codes. In HIHTC Complexity is very much reduced and BER is improved and approaches Shannon’s limit.

2. HADAMARD CODES
A Hadamard codeword is obtained from Hadamard matrix.

\[
H_1 = [+1]
\]

\[
n \times n(n = 2^r) \text{ Hadamard matrix } H_n \text{ in } [+1, -1] \text{ can be constructed recursively as }
\]

\[
H_n = \begin{bmatrix}
+H_{\frac{n}{2}} & +H_{\frac{n}{2}} \\
+H_{\frac{n}{2}} & -H_{\frac{n}{2}}
\end{bmatrix}
\]

The columns of Hadamard matrix are bi-orthogonal. Each Hadamard Codeword carries \(r+1\) information bits.

A normalized \(H_n\) has \(\frac{n(n-1)}{2}\) elements of \(-1\)s and \(\frac{n(n+1)}{2}\) elements of \(+1\)s. Every pair of rows and every pair of columns differs exactly in \(\frac{n}{2}\) places. For a normalized Hadamard Matrix of order \(\geq 2\) every row or column has \(\frac{n}{2}+1\)s and \(\frac{n}{2}-1\)s.

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3. **ZIGZAG HADAMARD CODE**

In ZH code the information bit sequence $D$ is first segmented into block $d_k = [d_k(1), d_k(2), \ldots, d_k(r)]$ is represented by white nodes.

Figure 3: Unpunctured Zigzag Hadamard code

Figure 4: Punctured Zigzag Hadamard code
Here the last parity bit of previous segment is used as the first input bit to the Hadamard encoder

The coded bits are
\[ C_k = H[C_k(0), C_k(1), \ldots, C_k(2^r - 1)] \]

\[ C_k(0) = C_{k-1}(2^r - 1) \] and \[ C_k(j) = d_k(j) \]

and \[ j = 1, 2, \ldots, \ldots, r \]

For convenience the first bit is taken as 0.

The black node is the common bit which is the first bit of a segment and last bit of a previous segment. The grey nodes are parity bits. The coded bit sequence is

\[ C_k = \{d_k, q_k, P_k\} \]

\[ q_k = C_k(0) \]

\[ P_k \rightarrow \text{Parity bits} \]

4. ENCODER DESCRIPTION

HIHTC encoder using Zigzag Hadamard code and Recursive Systematic Convolution (RSC) code as shown in Fig.

Initial data is directly given to encoder and then the interleaver \( \delta(m) \) interleaves the data and the data given to RSC and ZH encoder.

\[ ZH^M(K) = \sum_{j=1}^{i} d(h, k) + ZH^m(K - 1) \text{ mod } 2 \]

Random Interleaver is used in HIHTC.

5. HIHTC DECODER

From the received information bits Log Likely hood Ratio (LLR) is calculated and the LLR are arranged in array form of \( H \times K \). In soft in Soft out (SISO) decoding A Posteriori Probability (APP) algorithm is used.

Algorithm Process:
(i) Each row of the row is decoded using priori LLRS and is given as the input to convolution decoder. The output is taken as posteriori LLR.

(ii) The Damping Factor (DF) is applied to Posteriori LLR for gradual updating of LLRs Successfully.

(iii) Then each column of the array is decoded using A priori LLRs for ZH decoder.

(iv) The overall decoder is given to the iterative decoder.

\[
L(i) = \log \frac{P(C(i) = +1/x)}{P(C(i) = -1/x)}
\]

\[
= \log \frac{P(x(i) = +1)P(C(i) = +1)}{P(x(i) = -1)P(C(i) = -1)}
\]

\[
= \log \frac{\sum C \in \{\pm h^j\} : C(i) = +1P(y,c)}{\sum C \in \{\pm h^j\} : C(i) = -1P(y,c)}
\]

\[i = 0, 1, \ldots \ldots \ldots \ldots \ldots 2^r - 1\]

\[H(i, j) \text{ is the } (i, j)\text{th entry Hadamard matrix } H. \text{ For } C(i) = +1, \text{ we have } C = h^j \text{ with } H(i, j) = +1 \text{ otherwise } C = -h^j \text{ with } H(i, j) = -1 \text{ similarly for } C(i) = -1.\]

Assumptions shall be made that the coded bits are transmitted through AWGN channel with variance

\[\sigma^2 = \frac{N_o}{2}\]

The priori LLR \(Lapr\) \((i) = \log \frac{p(C(i) = +1)}{p(C(i) = -1)}\)

\[
L(\hat{U}) = \log \frac{\sum h^j \in \{H : H(i, j) = \pm i\} \exp(-\|h^j - x^2\| / 2\sigma^2)P(C(i) = \pm h^j)}{\sum h^j \in \{H : H(i, j) = \mp i\} \exp(-\|h^j - x^2\| / 2\sigma^2)P(C(i) = \mp h^j)}
\]

\[
= \log \frac{\sum h^j \in \{H : H(i, j) = \pm 1\}r(\pm h^j)}{\sum h^j \in \{H : H(i, j) = \mp 1\}r(\pm h^j)}
\]

Here

\[r(\pm h^j) = \exp \left( \frac{1}{2} < \pm h^j, \frac{2x}{\sigma^2} + Lapr > \right)\]

< ... > is inner product.

6. PERFORMANCE ANALYSIS

HIHTC, ILHTC, LCHTC and TCC are simulated with interleaver length \(N = 1452\) and RSC code with generator polynomial \(G = \left[ 1, \left( \frac{15}{13} \right) \right]_8\). For code Rate \(RC = \frac{1}{3}\) BER performance of HIHTC, ILCHTC, LCHTC and TCC are shown in Fig.
For Rate $\frac{1}{3}$ HIHTC $E_b/N_o$ is 1.7 dB at BER of which is nearest to Shannon’s limit. For ILCHTC at $10^{-5}$ BER the $E_b/N_o$ is 1.7 dB which is 0.2 dB higher than HIHTC and TCC shows $E_b/N_o$ of 1.5 dB at the BER of $10^{-5}$ which is 0.2 dB less than HIHTC.

7. COMPLEXITY ANALYSIS:

The number of multiplications and additions determine the computational complexity of decoders. For an ILCHTC with $L = H$ the trellis length is $N$ let $Q_s$ be the number of multiplications / Information Bit / Iteration (M/IB/I) and $Q_a$ be the number of Additions / Information Bit / Iteration (A/IB/I). When we use log-MAP algorithm for a decoder $Q_s$ and $Q_a$ shall be calculated as follows.

For TCC

$$Q_s = 8, S_r \ast P$$

$$Q_a = [16 S_r + 2 \ast P] - 2$$

For ILCHTC decoder

$$Q_s = \frac{L \ast (8 \ast S_r - 4)}{H}$$

$$Q_a = \left[\frac{16 S_r - 1}{H}\right] + \left[(5 + 5) / H\right] - 1$$

For HIHTC decoder

$$Q_s = \frac{L \ast (8 \ast S_r - 2)}{H}$$
\[ Q_+ = \left[ \frac{16 S - 1}{H} \right] + \left\lfloor \frac{3 + 2}{H} \right\rfloor - 1 \]

Table 1

<table>
<thead>
<tr>
<th>Decoder</th>
<th>R</th>
<th>Parameter</th>
<th>( Q^* )</th>
<th>( Q^+ )</th>
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<tr>
<td>TCC</td>
<td>1/2</td>
<td>M=2</td>
<td>120</td>
<td>256</td>
</tr>
<tr>
<td></td>
<td>1/3</td>
<td>M=2</td>
<td>120</td>
<td>256</td>
</tr>
<tr>
<td>LCTCC</td>
<td>1/2</td>
<td>M=2, H=3, L=2</td>
<td>40</td>
<td>96</td>
</tr>
<tr>
<td></td>
<td>1/3</td>
<td>M=2, H=3, L=2</td>
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<td>145</td>
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<tr>
<td>ILCHTCC</td>
<td>1/2</td>
<td>M=2, H=3, L=2</td>
<td>30</td>
<td>79</td>
</tr>
<tr>
<td></td>
<td>1/3</td>
<td>M=2, H=3, L=2</td>
<td>60</td>
<td>150</td>
</tr>
<tr>
<td>HIHTC</td>
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<td>M=2, H=3, L=2</td>
<td>25</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>1/3</td>
<td>M=2, H=3, L=2</td>
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<td>1409</td>
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Table 2

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<th>EB/No</th>
<th>EB/No</th>
<th>EB/No</th>
<th>EB/No</th>
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<tr>
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<td>5</td>
</tr>
<tr>
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<td>18</td>
<td>14</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

For a TCC decoder of rate \( \frac{1}{3} \), \( Q^*_1 = 120 \) and \( Q^*_3 = 256 \) and average number of iterations is 4 when \( E_b/N_o = 2 \). For the same code rate \( Q^*_1 = 50, \theta_1 = 140 \) and average number of iterations is 4 for \( E_b/N_o = 2 \). The complexity of HIHTC is 50 percent reduced as compared to TCC. The complexity of HIHTC is reduced 0.05 times as compared to ILCHTCC. At the same time the BER of HIHTC is better than ILCHTCC.

Hence the overall complexity of HIHTC is less than that of TCC, LCHTC and ILCHTC.

8. CONCLUSION

In this paper a novel error correcting code called HIHTC and its complexity is analyzed. The HIHTC uses the combination of convolution code and Zigzag Hadamard code. The BER of HIHTC approaches Shannon’s limit. In HIHTC for interleaver length \( N = 1452 \) BER of \( 10^{-5} \) is achieved at \( E_b/N_o = 1.7 \) dB which is 0.1 dB more than TCC and less than LCHTC and ILCHTC. The complexity is 0.5 times reduced as compared to TCC and 0.05 times as compared to ILCHTCC. At the same time BER of HIHTC is 0.2 dB less than ILCHTC at a reference of \( 10^{-5} \)BER for a rate \( \frac{1}{3} \) encoder.

References

[1] M. Jose Raj and Dr. Sharmini Enoch, “Performance Analysis of Highly Improved Hybrid Turbo Codes”.


