Position Control of Two-link Flexible Manipulator using Low Chattering SMC Techniques

K. Lochan\textsuperscript{a}, and B. K. Roy\textsuperscript{b}

\textbf{Abstract:} In this paper an attempt is made to control a angular position of the two-link flexible manipulator with slowly varying parameter uncertainties. The lumped parameter model is used throughout the work for obtaining the manipulator dynamics. The problem of controlling the position is achieved by using two low chattering sliding mode control (SMC) techniques. The techniques are: Proportional integral (PI) SMC and asymptotic SMC. The importance of these techniques is that chattering is attenuated even in first order SMC and in presence of bounded unknown disturbances including uncertainties associated with the plant. The proposed technique are examined under bounded unknown disturbances and slowly time varying uncertainties in system parameters. Comparison of the performances of the two control techniques are also highlighted.

\textbf{Keywords:} Sliding Mode Control; PI SMC; Asymptotic SMC; Two-link flexible manipulator.

1. INTRODUCTION

Flexible manipulator (FM) has many advantages over rigid manipulators [1] though it has many inherent complexities and nonlinearities. Modeling and control of such manipulators is the main focal area of research. The objective of this paper is to control the FM in presence of disturbances and uncertainties.

Finite element method (FEM) [2], assumed modes method (AMM) [3], lumped parameter method [4] are the most widely used modeling techniques of FM. Position control [5] and trajectory tracking [6] are the main control problem for controlling the manipulators. Many control techniques are available for FM like PID [7], feedback control [8], observer based control [9], SMC [10], $H_\infty$ [11], intelligent control [12], etc.

SMC is the most useful control method because of many inherent advantages like finite time convergence and insensitive to parameter uncertainties and/or disturbances [13]. However, SMC technique has its own main drawback which is the phenomena of chattering. Several SMC techniques are proposed in literature for the reduction in chattering like Adaptive SMC [14], observer-based [15], state dependent gain method [16], use of hysteresis loop [17], using low pass filter [18], second order SMC [19], backstepping smc [22], terminal SMC [23, 24] etc. SMC techniques have been in use by some researchers for different control problems of two-link flexible manipulators (TLFM) [6, 10, 20, 23]. But controlling the position of TLFM in the presence of unknown disturbances and uncertainties with time varying parameters is a challenging task.

In this paper, two different SMC techniques are considered for the position control of two-link flexible manipulator. The proposed techniques worked satisfactorily in presence of unknown disturbances and uncertainties resulting in low chattering. PI and asymptotic sliding surfaces are designed in PI SMC and

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asymptotic SMC, respectively. The Lyapunov stability theory is used to prove the stability of the sliding surfaces. Comparison on the working performances of the techniques are highlighted in terms of settling time, links deflection, reaching time of the sliding surface, Integral square error and control input energies. The manipulator dynamics are designed by using the lumped parameter method. Controlling the position of the TLFM in the presence of bounded unknown disturbance and the time varying parameters using low chattering SMC techniques is the novelty of this paper.

The paper is organised as follows. Firstly, the dynamic modeling of the TLFM is presented in Section II. The description of PI SMC and asymptotic SMC are given in Section III and IV respectively. Section V deals with the results and discussions. Finally, conclusions are presented in Section VI.

2. SYSTEM DYNAMICS

Consider a planar two-link flexible manipulator with bending deformation in the plane of motion. Fig. 1 shows the two-link flexible manipulator. The coordinates frames are given as \((\hat{X}_0, \hat{Y}_0)\), the moving rigid body frame for link \(i\) as \((X_i, Y_i)\) and the moving flexible body frame of \(i^{th}\) link as \((\hat{X}_i, \hat{Y}_i)\). The hub is attached to the motor shaft which drives the link to reach the desired position.

The dynamic equations of the system is found by the Euler-Lagrange formulation is first obtained. The kinetic energy (KE) and potential energy (PE) of each link and hub are calculated from the given system. The total KE is given as the combination of the links KE and hubs KE.

\[
\text{KE}_{\text{total}} = \text{KE}_{L1} + \text{KE}_{L2} + \text{KE}_{H1} + \text{KE}_{H2} = \frac{1}{2} I_{eq1} \dot{\theta}_1^2 + \frac{1}{2} I_{eq2} \dot{\theta}_2^2 + \frac{1}{2} I_{L1} (\dot{\theta}_1 + \dot{\alpha}_1)^2 + \frac{1}{2} I_{L2} (\dot{\theta}_2 + \dot{\alpha}_2)^2
\]

(1)

\[
\text{PE}_{\text{total}} = \frac{1}{2} k_s \dot{\alpha}^2
\]

(2)

The Euler-Lagrange equation is given as

\[
\frac{d}{dt} \left( \frac{\delta L}{\delta \dot{\theta}} \right) - \frac{\delta L}{\delta \theta} = T - B_{eq} \dot{\theta}
\]

(3)

\[
\frac{d}{dt} \left( \frac{\delta L}{\delta \dot{\alpha}} \right) - \frac{\delta L}{\delta \alpha} = 0
\]

(4)

where \(\alpha\) and \(\theta\) are the two generalized coordinates and

\[
L = KE_{\text{total}} - PE
\]

(5)

\[\text{Figure 1: Schematic diagram of two-link flexible manipulator.}\]

Solving the equations (4) and (5), we get
\[ \dot{\theta}_1 = -p_1 \dot{\theta}_1 + p_2 \alpha_1 + p_3 V_m \]  
\[ \dot{\theta}_2 = -p_4 \dot{\theta}_2 + p_5 \alpha_2 + p_6 V_m \]  
\[ \ddot{\alpha}_1 = p_1 \dot{\theta}_1 - p_7 \alpha_1 - p_3 V_m \]  
\[ \ddot{\alpha}_2 = p_4 \dot{\theta}_2 - p_8 \alpha_2 - p_6 V_m \]  

where

\[ p_1 = \frac{\eta_m \eta_g k_t k_g^2 k_m B_{eq} R_m}{J_{eq1}}, \quad p_2 = \frac{k_s}{J_{eq1}}, \quad p_3 = \frac{\eta_m \eta_g k_t k_g^2 k_m B_{eq} R_m}{J_{eq2}}, \quad p_4 = \frac{k_s}{J_{eq2}}, \quad p_5 = \frac{\eta_m \eta_g k_t k_g^2 k_m B_{eq} R_m}{J_{eq1}}, \quad p_6 = \frac{\eta_m \eta_g k_t k_g^2 k_m B_{eq} R_m}{J_{eq2}} \]  

Table 1

<table>
<thead>
<tr>
<th>Mass of link 1, ( m_1 )</th>
<th>0.065 Kg</th>
<th>Coefficient of viscous damping, ( B_{eq} )</th>
<th>1.99</th>
<th>Link M. I, ( J_{arm1} )</th>
<th>0.00195 Kg m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of link 2, ( m_2 )</td>
<td>0.070 Kg</td>
<td>Efficiency of gear box, ( \eta_g )</td>
<td>0.9</td>
<td>Link M. I, ( J_{arm2} )</td>
<td>0.000933 Kg m²</td>
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<td>Length of link 1, ( L_1 )</td>
<td>0.3 m</td>
<td>Efficiency of motor, ( \eta_m )</td>
<td>0.69</td>
<td>Load Torque = ( T )</td>
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</tr>
<tr>
<td>Length of link 2, ( L_2 )</td>
<td>0.2 m</td>
<td>Constant of back e. m. f, ( K_m )</td>
<td>0.00767</td>
<td>Motor voltage= ( V_m )</td>
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<tr>
<td>Resistance of Armature, ( R_m )</td>
<td>2.6 ( \Omega )</td>
<td>Gear ratio, ( K_g )</td>
<td>70</td>
<td></td>
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<tr>
<td>Equivalent M. I at load, ( J_{eq1} )</td>
<td>0.099 Kg m²</td>
<td>Natural frequency, ( f_c )</td>
<td>3.2 Hz</td>
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<tr>
<td>Equivalent M. I at load, ( J_{eq2} )</td>
<td>0.092 Kg m²</td>
<td>Stiffness of the link, ( K_s )</td>
<td>2( \pi f_c )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Defining, \( x_1 = \theta_1, x_2 = \theta_2, x_3 = \alpha_1, x_4 = \alpha_2, x_5 = \dot{\theta}_1, x_6 = \dot{\theta}_2, x_7 = \dot{\alpha}_1, x_8 = \dot{\alpha}_2, u = V_m \) and using the above equations (6) to (9), manipulator dynamics can be represented as [21]:

\[ \dot{x}_1 = x_5 \]
\[ \dot{x}_2 = x_6 \]
\[ \dot{x}_3 = x_7 \]
\[ \dot{x}_4 = x_8 \]
\[ \dot{x}_5 = -b_1 x_5 + b_2 x_3 + b_3 u + d \]
\[ \dot{x}_6 = -b_4 x_6 - b_5 x_4 + b_6 u \]
\[ \dot{x}_7 = b_1 x_5 - b_7 x_3 - b_3 u \]
\[ \dot{x}_8 = b_4 x_6 - b_8 x_4 - b_6 u \]  

where \( b_1(t) = p_1 + d_1(t), b_2(t) = p_2 + d_2(t), b_3(t) = p_4 + d_4(t), b_4(t) = p_5 + d_5(t), b_5(t) = p_7 + d_7(t), b_6(t) = p_8 + d_8(t), \) and \( d_1(t), d_2(t), d_4(t), d_5(t), d_7(t), d_8(t) \) are uncertainties in the plant parameters \( p_1, p_2, p_4, p_5, p_7, p_8, \) respectively, i.e. slowly time varying parameters are considered. \( d \) is the plant disturbance. Numerical values of these parameters are given in the results and discussions section. Position control using PI SMC is discussed in the next section.

### 3. POSITION CONTROL OF TLFM USING PI SMC

This section describes the use of PI SMC technique for the position control of TLFM. The objective is to design a controller in such a manner that it causes the manipulator dynamics to follow the desired position. Thus, by considering the desired position \( x_d \) for the links, the error is defined as;
Here, a unit step angular position is considered as desired for \(x_{d1}\) and \(x_{d2}\). Designing of a SMC technique involves two steps. First is the design of sliding surface and in second step, we design the control law [13]. The objective of the first step is to bring the dynamics on the sliding surface and the second step is to maintain the trajectory on the sliding surface [13]. The PI sliding surface is considered as:

\[
es_1 = \hat{e}_1 + c_1 \int_0^t e_1(\tau) \, d\tau \quad \text{and} \quad s_2 = \hat{e}_2 + c_2 \int_0^t e_2(\tau) \, d\tau
\]

Now, the equivalent sliding surface is considered as

\[
s_p = s_1 + a_1 s_2 + a_2 s_3 + a_3 s_4
\]

where \(a_i(i = 1, 2, 3)\), \(c_i(i = 1, 2, 3, 4)\) are positive constants whose values depend on the choice of designer. The system operates on the sliding surface when it satisfies:

\[
s_p = 0, \dot{s}_p = 0
\]

To ensure the occurrence of sliding mode, control law (using (11), (12) and (15)) is proposed as;

\[
u_p = \left(-\frac{1}{k}\right) \left[ x_3(-b_7a_2 + b_2 + c_3a_2) + x_5(b_1a_2 - b_1) + x_6(-b_4a_1 - b_4a_3) + x_4(-b_6a_3 + c_4a_3 + c_5a_1) + c_1e_1 + a_1c_2e_2 \right] - \rho \, \text{sgn}(s_p)
\]

where \(k = b_6a_1 - a_3b_6 - b_3a_2 + a_3\), \(\rho\) are positive constants.

**Remark 1:** The control law \(u_p\) in (16) causes the system trajectories (11) to follow the desired position in presence of unknown disturbance and slow time varying parameters and to maintain on the sliding surface \(s_p(t)\).

In the next section we shall discuss the performance of the system using asymptotic SMC.

### 4. POSITION CONTROL OF TLFM USING ASYMPTOTIC SMC

In this section, asymptotic SMC is designed for describing the position control of manipulator dynamics in equation (17). The sliding surface is also designed in terms of control function derivative. In this case control input is the integral of high frequency switching function. So, the manipulator dynamics is modified and is described as

\[
\begin{align*}
\dot{x}_1 &= x_5 \\
\dot{x}_2 &= x_6 \\
\dot{x}_3 &= x_7 \\
\dot{x}_4 &= x_8 \\
\dot{x}_5 &= -b_4 x_5 + b_2 x_3 + b_3 u \\
\dot{x}_6 &= -b_4 x_6 - b_5 x_4 + b_6 u \\
\dot{x}_7 &= b_1 x_5 - b_7 x_3 - b_3 u \\
\dot{x}_8 &= b_4 x_6 - b_9 x_4 - b_6 u \\
\dot{u} &= v
\end{align*}
\]

Now, the sliding variables are described as
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\[
\begin{align*}
s_1 &= \dot{e}_1 + c_{1a} e_1 \\
s_2 &= \dot{e}_2 + c_{2a} e_2 \\
s_3 &= \dot{x}_3 + c_{3a} x_3 \\
s_4 &= \dot{x}_4 + c_{4a} x_4
\end{align*}
\]

Here, the equivalent sliding surface is considered as

\[
s_A = s_1 + a_{1a} s_2 + a_{2a} s_3 + a_{3a} s_4
\]

where \(a_{il} (i = 1, 2, 3)\), \(c_{il} (i = 1, 2, 3, 4)\) are positive constants whose values are defined by the user. For low chattering, the auxiliary sliding variable is defined as

\[
\sigma = \dot{s}_A + \bar{c} s_A
\]

Now our aim is to design \(v\) that provides finite time convergence of \(\sigma \to 0\), and an ideal sliding mode to occur in the sliding surface \(\sigma = 0\) as

\[
\sigma = \dot{s}_A + \bar{c} s_A = 0
\]

Control law \(v\) that derives the \(\sigma\) to zero in finite time is defined as

\[
v = \left(-\frac{1}{\sigma}\right) [Ax_7 + Bx_8 + CC_1 + DD_1 + EE_1 + FF_1 + \bar{c} (c_1 x_5 + a_1 c_2 x_6)] - \rho \text{sgn} (\sigma)
\]

where \(A = -b_7 a_2 + b_2 + c_3 a_2 \bar{c}\), \(B = -b_8 a_3 + c_4 a_3 \bar{c} + c_5 a_1 + c_4 a_3 \bar{c}\), \(C = b_1 a_2 - b_4 + c_1 + \bar{c}\), \(D = -a_1 b_4 + a_1 c_2 + a_3 b_4 + a_1 \bar{c}\), \(E = a_2 c_3 + a_2 \bar{c}\), \(F = a_3 c_4 + a_3 \bar{c}\), \(G = b_3 + a_3 b_6 - a_2 b_3 - a_3 b_6\), \(C_1 = -b_1 x_5 + b_2 x_3 + b_3 u\), \(D_1 = -b_4 x_6 + b_5 x_4 + b_6 u\), \(E_1 = b_1 x_5 - b_7 x_3 - b_3 u\), \(F_1 = b_4 x_6 - b_8 x_4 - b_6 u\).

Remark 2: The control law \(v\) in (22) of asymptotic sliding mode makes both \(\sigma\) and \(\bar{\sigma}\) \(\to 0\) together with convergence of states to desired position as time increases, even in presence of unknown disturbance and slowly time varying parameters.

5. RESULTS AND DISCUSSIONS

We used the fourth order Runge Kutta method for solving the dynamics of (11), (17) and simulating the results with step time \(h=10^{-3}\) in MATLAB. The parameter values are: \(p_1 = 2.0796\), \(p_2 = 7.955\), \(p_3 = 1.29\), \(p_4 = 2.237\), \(p_5 = 4.09\), \(p_6 = 1.39\), \(p_7 = 412\), \(p_8 = 408\). The values of parameter uncertainties are: \(d_1 = 0.2 \sin(5t)\), \(d_2 = 0.7 \sin(5t)\), \(d_4 = 0.2(5t)\), \(d_5 = 0.4 \sin(5t)\), \(d_7 = 41.2 \sin(5t)\), \(d_6 = 40.8 \sin(5t)\) and \(d = 0.1 \sin(x_1) \sin(x_2) \sin(x_3) \sin(x_4) \sin(x_5) \sin(x_6) \sin(x_7) \sin(x_8)\). The initial conditions for the analysis with PI SMC are chosen as \(x(0) = (0, 0, 0, 0, 0, 0, 0, 0, 0)^T\), and the value of different constants are: \(c_1 = 7\), \(c_2 = 2\), \(\rho = 5\), \(c_3 = 3\), \(c_4 = 0.5\), \(a_1 = 0.6\), \(a_2 = 0.3\), \(a_3 = 0.25\). The initial conditions for the analysis with asymptotic SMC are chosen as \(x(0) = (0, 0, 0, 0, 0, 0, 0, 0, 0)^T\) and the value of different constants are: \(c_{1a} = 7\), \(c_{2a} = 2\), \(\rho = 7\), \(c_{3a} = 3\), \(c_{4a} = 3\), \(a_{1a} = 0.6\), \(a_{2a} = 0.3\), \(a_{3a} = 0.25\), \(\bar{c} = 2.21\).

5.1 POSITION CONTROL WITH PI SMC

Fig.2 (a, b) shows the position of states \((x_1, x_2)\) i.e angle of the first joint and second joint. Fig. 2 (c, d) shows the position of states \((x_3, x_4)\) i.e deflection of the first and second link. The velocities of the joints angle and links deflection are shown in Fig. 3 (a, b) and Fig. 3 (c, d), respectively. The time responses of the sliding surface and control input are shown in Fig. 4 (a) and Fig. 4 (b), respectively. It is observed from Fig. 2 (a, b) that first and second joint angles reached the desired position in 1.33 s and 1.28 s, respectively. The deflection of the links are very small as shown in Fig. 2 (c, d). It is also apparent from Fig. 3 that velocities of the joints angle and links deflection tend to zero.
5.1 POSITION CONTROL WITH PI SMC

Fig. 2 (a, b) shows the position of states \( x_1, x_2 \) i.e., angle of the first joint and second joint. Fig. 2 (c, d) shows the position of states \( x_3, x_4 \) i.e., deflection of the first and second link. The velocities of the joints angle and links deflection are shown in Fig. 3 (a, b) and Fig. 3 (c, d), respectively. The time responses of the sliding surface and control input are shown in Fig. 4 (a) and Fig. 4 (b), respectively. It is observed from Fig. 2 (a, b) that first and second joint angles reached the desired position in 1.33 s and 1.28 s, respectively. The deflection of the links are very small as shown in Fig. 2 (c, d). It is also apparent from Fig. 3 that velocities of the joints angle and links deflection tend to zero.

Figure 2: Link position with PI SMC (a) of first link (b) of second link. Link deflection (c) of the first link (d) of the second link with PI SMC.

Figure 3: Velocity with PI SMC (a) of the first joint (b) of the second joint. Velocity of the (c) first link deflection (d) second link deflection with PI SMC.

Figure 4: Time response of sliding surface with PI SMC. (b) Time response of control input required with PI SMC.

Sliding surface in Fig. 4 (a) reaches zero (converge) in s but the required control input in this case (PI SMC) is comparatively high. The next subsection deals with the results and discussions of position control using asymptotic SMC.
5.2 Position Control With Asymptotic SMC

The angular position of the first link and second link with settling time 3.05 s with their deflections are given in Fig. 5 (a, b) and Fig. 5 (c, d), respectively, which tends to zero. Their respective velocities are also shown in Fig. 6 (a,b) and Fig. 6 (c, d). Fig. 7 (a) and Fig. 7 (b) gives the sliding surface and control input of the controller with asymptotic SMC.

5.3 Comparisons between PI-SMC and Asymptotic SMC

It is clear that the joints with PI SMC reaches the desired position in less time as compared to the asymptotic SMC. Similarly, comparison of the states i.e. deflection of both the links showed that the deflection of the links with asymptotic SMC is small as compared to PI SMC. The performance indices of the flexible manipulator are given in Table 2. Moreover, to check the control input required, the energies are also calculated and compared.

![Graphs showing links position and deflection](image1.png)

Figure 5: Links position (a) of first link (b) of second link with asymptotic SMC. (c) deflection of first link (d) deflection of second link with asymptotic SMC.

![Graphs showing joint angle velocity](image2.png)

Figure 6: Velocity of the joint angle (a) the first joint (b) the second joint with asymptotic SMC. Velocity (c) of the first link (d) of the second link with asymptotic SMC.
Table 2
Comparisons on The Performance of PI SMC and Asymtotic SMC.

<table>
<thead>
<tr>
<th>S. L.</th>
<th>Performance Indices</th>
<th>PI SMC</th>
<th>Asymptotic SMC</th>
<th>S.L</th>
<th>Performance Indices</th>
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<tr>
<td>1.</td>
<td>Settling time of $\theta_1$</td>
<td>1.33s</td>
<td>3.15s</td>
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<td>Sliding surface reaching time</td>
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<td></td>
<td>Settling time of $\theta_2$</td>
<td>1.28s</td>
<td>3.05s</td>
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<td>Integral Square Error (ISE)</td>
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<td>$a_1$ High deflection magnitudes</td>
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<td>Small deflection magnitudes</td>
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<td>Control input energy</td>
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<td>$a_2$  Large deflection magnitudes</td>
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<td>Small deflection magnitudes</td>
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<td></td>
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<td></td>
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6. CONCLUSIONS

Two low chattering SMC control techniques for position tracking and tip deflection control of a two-link flexible manipulator are presented. The control techniques are been developed on the basis of low chattering in sliding surfaces and in control input. The techniques are implemented within the simulation environment with bounded unknown disturbances and slowly time varying uncertainties in parameter of TLFM. The performances of the control techniques are evaluated. A comparative analysis of the two control techniques reveal that PI SMC technique results in better performance than asymptotic SMC in respect of settling time of hub angular position, sliding surface reaching time and in intergral square error. However, the links deflection and control input energy are less in case as of asymptotic SMC.

References


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