FUZZY SET THEORY APPROACH TO MINIMIZE THE RENTAL COST FOR SPECIALLY STRUCTURED TWO STAGE FLOW SHOP SCHEDULING

Deepak Gupta, Sameer Sharma & Shefali Aggarwal

Abstract: This paper pertain to a specially structured $n$-jobs, 2-machines flow shop scheduling in which the processing time and set up time of jobs are uncertain that is not known exactly and are in fuzzy environment. Here, we employ triangular fuzzy membership functions to describe uncertain processing times and set up times. Further, we consider that the average high ranking (AHR) of expected flow times are not random but bear a well defined relationship to one another. The present work is an attempt to develop a heuristic algorithm to minimize the rental cost of machines under the specified rental policy. A numerical given is provided to demonstrate the computational efficiency of proposed algorithm.

Keywords: Average high ranking, Fuzzy processing time, Fuzzy set up time, Rental policy, Specially structured flowshop scheduling, Utilization time.

1. INTRODUCTION

Scheduling is a enduring process where the existence of real time information frequently forces the review and modification of pre-established schedules. The real world is complex; complexity in the world generally arises from uncertainty. From this prospective, the concept of fuzzy environment is introduced in the theory of scheduling. The past few years have witnessed a rapid growth in the number and variety of applications of fuzzy logic (FL). In most applications, FL solution is a translation of a human solution which can model non linear functions of arbitrary complexity to a desired degree of accuracy. Zadeh introduced the term fuzzy logic in his seminal work “Fuzzy sets”, which described the mathematics of fuzzy set theory [20]. The permissiveness of fuzziness in the human thought process suggests that much of the logic behind thought processing is not traditional two-valued logic or even multivalued logic, but logic with fuzzy truths, fuzzy connectiveness and fuzzy rules of inference. As per literature review it has been found that processing time of jobs are not random but follow some well defined structural conditions. In such cases we can have different heuristic approach to find the algorithm(s) alternative and proficient as...
compared to the existing algorithm(s) to minimize the utilization time of the machines and hence their rental cost under a specified rental policy. Further, the majority of scheduling research assumes setup as negligible or part of processing time. This assumption adversely affects solution quality for many applications which required explicit treatment of setup. Such applications have motivated increasing interest to include setup considerations in scheduling theory. A flow shop scheduling problems has been one of the classical problems in production scheduling since Johnson [10] proposed the well known Johnson’s rule in the two stage flow shop scheduling problem. MacCahon and Lee [12] discussed the job sequencing with fuzzy processing time. Ishibuchi and Lee [9] addressed the formulation of fuzzy flowshop scheduling problem with fuzzy processing time. Hong and Chuang [8] developed a new triangular Johnson algorithm. Some of the noteworthy approaches are due to Bagga [1], Gupta, J. N. D [3], Yager [18], Shukla and Chen [14], Martin and Roberto [13], Yao and Lin [11], Singh and Gupta [15], Sanuja and Song [16], Singh, Sunita and Allawalia [17].

Gupta D., Sharma S., and Shashi [6] studied specially structured two stage flow shop scheduling to minimize the rental cost. In the present work we have introduced the concept of fuzziness in processing as well in independent setup time, when the expected processing time of the machines are not in random order but satisfies some well defined structural conditions. Here, we use triangular fuzzy numbers to describe the uncertain processing times and setup times. Fuzzy set theory in the form of approximate reasoning provides decision support and expert systems with powerful reasoning capabilities.

2. PRACTICAL SITUATION

Fuzzy set theory has emerged as a profitable tool for controlling and steering of systems and complex industrial processes, as well as for household and entertainment electronics. Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. Medical science can save the patient’s life but proper care leads to a faster recovery. Care giving techniques often require hi-tech, expensive medical equipment. Many of these equipments can even help in saving the life of critical patients. Most of these equipments are expensive & they are often needed for a few days or weeks thus buying them do not make much sense even if one can afford them. Many patients even lose their lives just because they can not afford to buy these products. For example, in the starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring
Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allow up gradation to new technology. Setup time includes work to prepare the machine, process or bench for product parts or the cycle. This includes obtaining tools, positioning work-in-process material, return tooling, cleaning up, setting the required jigs and fixtures, adjusting tools and inspecting material and hence significant.

3. FUZZY MEMBERSHIP FUNCTION
All information contained in a fuzzy set is described by its membership function. The triangular membership functions are used to represent fuzzy processing times and fuzzy setup times in our algorithm. The membership value of the $x$ denoted by $\mu_x$, $x \in R^n$, can be calculated according to the formula

\[\text{Figure 1: Triangular Membership Function}\]

Figure 1 shows the triangular membership function of a fuzzy set $\tilde{P}$, $\tilde{P} = (a, b, c)$. The membership value reaches the highest point at ‘$b$’, while ‘$a$’ and ‘$c$’ denote the lower bound and upper bound of the set $\tilde{P}$ respectively.

4. AVERAGE HIGH RANKING (A.H.R.)
To find the optimal sequence, the processing times and setup times of the jobs are calculated by using Yager’s(1981) average high ranking formula (AHR) $= h(A) = \frac{3b + c - a}{3}$.

5. FUZZY ARITHMETIC OPERATIONS
If $A_1 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1})$ and $A_2 = (m_{A_2}, \alpha_{A_2}, \beta_{A_2})$ be the two triangular fuzzy numbers, then

1. $A_1 + A_2 = (m_{A_1} + m_{A_2}, \alpha_{A_1} + \alpha_{A_2}, \beta_{A_1} + \beta_{A_2})$.
2. $A_1 - A_2 = (m_{A_1} - m_{A_2}, \alpha_{A_1} - \alpha_{A_2}, \beta_{A_1} - \beta_{A_2})$.
3. $kA_1 = (km_{A_1}, k\alpha_{A_1}, k\beta_{A_1})$; if $k > 0$.
4. $kA_1 = (km_{A_1}, k\alpha_{A_1}, k\beta_{A_1})$; if $k < 0$.

6. NOTATIONS
$S$ : Sequence of jobs 1, 2, 3,……, $n$
$M_j$ : Machine $j$, $j = 1$, 2
$M$ : Minimum makespan
7. RENTAL POLICY (P)

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required i.e. the first machine will be taken on rent in the starting of the processing the jobs and 2nd machine will be taken on rent at time when 1st job is completed on 1st machine.

8. PROBLEM FORMULATION

Let some job \(i (i = 1, 2, \ldots, n)\) are to be processed on two machines \(M_j (j = 1, 2)\) under the specified rental policy \(P\). Let \(a_{ij}\) be the processing time of \(i^{th}\) job on \(j^{th}\) machine and \(s_{ij}\) be the setup time of \(i^{th}\) job on \(j^{th}\) machine which are described by triangular fuzzy numbers. Let \(A_{ij}; i = 1, 2, 3, \ldots, n; j = 1, 2\) be the average high ranking (AHR) of expected flow time for all jobs on two machines \(M_1\) and \(M_2\) such that either \(A_{i1} \geq A_{j2}\) or \(A_{i1} \leq A_{j2}\) for all values of \(i, j\). Our aim is to find the sequence \(\{S_k\}\) of the jobs which minimize the rental cost of the machines.

<table>
<thead>
<tr>
<th>Jobs</th>
<th>Machine (M_1)</th>
<th>Machine (M_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a_{11})</td>
<td>(a_{12})</td>
</tr>
<tr>
<td>2</td>
<td>(a_{21})</td>
<td>(a_{22})</td>
</tr>
<tr>
<td>3</td>
<td>(a_{31})</td>
<td>(a_{32})</td>
</tr>
<tr>
<td>-</td>
<td>(s_{11})</td>
<td>(s_{12})</td>
</tr>
<tr>
<td>-</td>
<td>(s_{21})</td>
<td>(s_{22})</td>
</tr>
<tr>
<td>-</td>
<td>(s_{31})</td>
<td>(s_{32})</td>
</tr>
<tr>
<td>(n)</td>
<td>(a_{n1})</td>
<td>(a_{n2})</td>
</tr>
</tbody>
</table>
Mathematically, the problem is stated as:

Minimize \( R(S_k) = t_{n_1}(S_k) \times C_1 + U_2(S_k) \times C_2 \)

Subject to constraint: Rental Policy \( (P) \)

Our objective is to minimize rental cost of machines while minimizing the utilization time.

**Theorem 8.1:** If \( A'_{i_1} \leq A'_{j_2} \) for all \( i, j, i \neq j \), then \( k_1, k_2, \ldots, k_n \) is a monotonically decreasing sequence, where \( k_n = \sum_{i=1}^{n} A'_{i_1} - \sum_{i=1}^{n-1} A'_{j_2} \); \( A'_{i_1} = A_{i_1} - S_{i_2} \) and \( A'_{j_2} = A_{j_2} - S_{1} \).

**Solution:** Let \( A'_{i_1} \leq A'_{j_2} \) for all \( i, j, i \neq j \) i.e., max \( A'_{i_1} \leq \min A'_{j_2} \) for all \( i, j, i \neq j \).

Let \( k_n = \sum_{i=1}^{n} A'_{i_1} - \sum_{i=1}^{n-1} A'_{j_2} \).

Therefore, we have \( k_1 = A'_{i_1} \)

Also \( k_2 = A'_{i_1} + A'_{21} - A'_{12} = A'_{i_1} + (A'_{21} - A'_{12}) \leq A'_{i_1} \) \( \therefore A'_{21} \leq A'_{12} \)

\( \leq k_1 \leq k_2. \)

Now, \( k_3 = A'_{i_1} + A'_{21} + A'_{31} - A'_{12} - A'_{22} \)

\( = A'_{i_1} + A'_{21} - A'_{12} + (A'_{31} - A'_{22}) = k_2 + (A'_{31} - A'_{22}) \leq k_2 \) \( \therefore A'_{31} \leq A'_{22} \).

Therefore, \( k_3 \leq k_2 \leq k_1 \) or \( k_1 \geq k_2 \geq k_3. \)

Continuing in this way, we can have \( k_1 \geq k_2 \geq k_3 \geq \ldots \geq k_n \), a monotonically decreasing sequence.

**Corollary 1:** The total rental cost of machines is same for all the sequences.

**Proof:** The total elapsed time

\( T(S) = \sum_{i=1}^{n} A_{i_2} + \sum_{i=1}^{n-1} S_{i_2} + A_{1_1} = \text{Constant for all sequences.} \)

It implies that under rental policy \( P \), the total elapsed time remains constant. Therefore total rental cost of machines is same for all the sequences.

**Theorem 8.2:** If \( A'_{i_1} \geq A'_{j_2} \) for all \( i, j, i \neq j \), then \( k_1, k_2, \ldots, k_n \) is a monotonically increasing sequence, where \( k_n = \sum_{i=1}^{n} A'_{i_1} - \sum_{i=1}^{n-1} A'_{j_2} \); \( A'_{i_1} = A_{i_1} - S_{i_2} \) and \( A'_{j_2} = A_{j_2} - S_{1} \).

**Proof:** Let \( k_n = \sum_{i=1}^{n} A'_{i_1} - \sum_{i=1}^{n-1} A'_{j_2} \).

Let \( A'_{i_1} \geq A'_{j_2} \) for all \( i, j, i \neq j \) i.e., \( \min A'_{i_1} \geq \max A'_{j_2} \) for all \( i, j, i \neq j \).

Here \( k_1 = A'_{i_1} \).

\( k_2 = A'_{i_1} + A'_{21} - A'_{12} = A'_{i_1} + (A'_{21} - A'_{12}) \geq k_1 \) \( \therefore A'_{21} \geq A'_{12} \)
Therefore, \( k_2 \geq k_1 \).

Also, 
\[
\begin{align*}
  k_3 &= A'_{11} + A'_{21} + A'_{31} - A'_{12} - A'_{22} = A'_{11} + A'_{21} - A'_{12} + (A'_{31} - A'_{22}) \\
  &= k_2 + (A'_{31} - A'_{22}) \geq k_2 (\because A'_{31} \geq A'_{22})
\end{align*}
\]

Hence, \( k_3 \geq k_2 \geq k_1 \).

Continuing in this way, we can have \( k_1 \leq k_2 \leq k_3 \), … \( \leq k_n \), a monotonically increasing sequence.

**Corollary 2:** The total rental cost of machines is same for all the possible sequences.

**Proof:** The total elapsed time = \( T(S) \).
\[
\begin{align*}
  &= \sum_{i=1}^{n} A_{i2} + \sum_{i=1}^{n} S_{i2} + \max_{1 \leq i \leq n} \{ k_i \} \\
  &= \sum_{i=1}^{n} A_{i2} + \sum_{i=1}^{n} S_{i2} + k_n = \sum_{i=1}^{n} A_{i2} + \sum_{i=1}^{n} S_{i2} + \left( \sum_{i=1}^{n} A'_{1i} - \sum_{i=1}^{n} A'_{i2} \right) \\
  &= \sum_{i=1}^{n} A_{i2} + \sum_{i=1}^{n} S_{i2} + \left( \sum_{i=1}^{n} A_{1i} - \sum_{i=1}^{n} S_{i2} - \sum_{i=1}^{n} A_{i2} + \sum_{i=1}^{n} S_{i1} \right) \\
  &= \sum_{i=1}^{n} A_{1i} + \sum_{i=1}^{n} S_{i1} + A_{n2} = \text{Constant for all sequences.}
\end{align*}
\]

It implies that under rental policy \( P \) the total elapsed time is constant for all sequences for machine \( M_2 \). Therefore total rental cost of machines is same for all the sequences.

9. ALGORITHM

**Step 1:** Define the two fictitious machines \( G \) and \( H \) with processing time \( A_{i1} \) and \( A_{i2} \) defined as follows: \( A_{i1} = a_{i1} - s_{i2} \); \( A_{i2} = a_{i2} - s_{i1} \).

**Step 2:** Find the average high ranking (AHR) \( A'_{ij} \), \( i = 1, 2, ..., n \), \( j = 1, 2 \) of expected flow time for all jobs on two machines \( M_1 \) and \( M_2 \).

**Step 3:** Check the feasibility of solution, i.e. If \( A'_{1i} \geq A'_{j2} \) or \( A'_{j1} \leq A'_{i2} \) for all \( i, j, i \neq j \). If the condition holds then go to step 4 else the proposed algorithm is not applicable.

**Step 4:** Obtain the job \( J_1 \) (say) having maximum processing time on 1st machine.

**Step 5:** Obtain the job \( J_n \) (say) having minimum processing time on 2nd machine.

**Step 6:** If \( J_1 \neq J_n \), then put \( J_1 \) on the first position and \( J_n \) as the last position & go to step 9, Otherwise go to step 7.

**Step 7:** Take the difference of processing time of job \( J_1 \) on \( M_1 \) from job \( J_2 \) (say) having next maximum processing time on \( M_1 \). Call this difference as \( G_1 \). Also, Take the
difference of processing time of job $J_n$ on $M_2$ from job $J_{n-1}$ (say) having next minimum processing time on $M_2$. Call the difference as $G_2$.

**Step 8:** If $G_1 \leq G_2$ put $J_n$ on the last position and $J_1$ on 1st position otherwise put $J_1$ on 1st position and $J_{n-1}$ on the last position.

**Step 9:** Arrange the remaining $(n-2)$ jobs between 1st job & last job in any order, thereby we get the sequences $S_1, S_2, \ldots, S_r$.

**Step 10:** Compute the total completion time $CT(S_k)$ $k = 1, 2, \ldots, r$.

**Step 11:** Calculate utilization time $U_2$ of 2nd machine.

**Step 12:** Find rental cost $R(S_k) = t_n(S_k) \times C_1 + U_2 \times C_2$, where $C_1$ & $C_2$ are the rental cost per unit time of 1st & 2nd machine respectively.

**10. NUMERICAL ILLUSTRATION**

Consider 5 jobs, 2 machine flow shop problem with processing time and setup time described by triangular fuzzy numbers as given in the following table. The rental cost per unit time for machines $M_1$ and $M_2$ are 4 units and 3 units respectively. Our objective is to obtain optimal schedule to minimize the total rental cost of the machines, under the rental policy $P$.

<table>
<thead>
<tr>
<th>Jobs</th>
<th>Machine $M_1$</th>
<th>Machine $M_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>$a_{i1}$</td>
<td>$s_{i1}$</td>
</tr>
<tr>
<td>1</td>
<td>(13, 14, 16)</td>
<td>(5, 7, 8)</td>
</tr>
<tr>
<td>2</td>
<td>(15, 16, 18)</td>
<td>(2, 4, 5)</td>
</tr>
<tr>
<td>3</td>
<td>(9, 10, 25)</td>
<td>(4, 5, 6)</td>
</tr>
<tr>
<td>4</td>
<td>(12, 13, 15)</td>
<td>(2, 3, 4)</td>
</tr>
<tr>
<td>5</td>
<td>(10, 11, 14)</td>
<td>(5, 6, 7)</td>
</tr>
</tbody>
</table>

**Solution:** As per step 1: The expected flow times for the two machines $G$ and $H$ are

<table>
<thead>
<tr>
<th>Jobs</th>
<th>Machine $M_1$</th>
<th>Machine $M_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>$A_{i1}$</td>
<td>$A_{i2}$</td>
</tr>
<tr>
<td>1</td>
<td>(9, 20, 21)</td>
<td>(1, 15, 16)</td>
</tr>
<tr>
<td>2</td>
<td>(12, 21, 22)</td>
<td>(3, 11, 12)</td>
</tr>
<tr>
<td>3</td>
<td>(7, 14, 28)</td>
<td>(2, 13, 14)</td>
</tr>
<tr>
<td>4</td>
<td>(9, 18, 19)</td>
<td>(1, 8, 9)</td>
</tr>
<tr>
<td>5</td>
<td>(6, 18, 19)</td>
<td>(1, 14, 15)</td>
</tr>
</tbody>
</table>
As per step 2: AHR's of expected flow times are

<table>
<thead>
<tr>
<th>Jobs</th>
<th>Machine $M_1$</th>
<th>Machine $M_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>$A'_{i1}$</td>
<td>$A'_{i2}$</td>
</tr>
<tr>
<td>1</td>
<td>72/3</td>
<td>60/3</td>
</tr>
<tr>
<td>2</td>
<td>73/3</td>
<td>42/3</td>
</tr>
<tr>
<td>3</td>
<td>63/3</td>
<td>51/3</td>
</tr>
<tr>
<td>4</td>
<td>64/3</td>
<td>32/3</td>
</tr>
<tr>
<td>5</td>
<td>67/3</td>
<td>56/3</td>
</tr>
</tbody>
</table>

Here $A'_{i1} \geq A'_{j2}$ for all $i, j$. Also, $\text{Max} A'_{i1} = 73/3$ which is for job 2, i.e., $J_1 = 2$.

Min $A'_{j2} = 32/3$ which is for $4^{th}$ job, i.e., $J_n = 4$. i.e., $J_1 \neq J_n$.

Therefore $J_1 = 2^{nd}$ job will be on 1$^{st}$ position and $J_n = 4^{th}$ job will be on the last position.

Therefore, the optimal sequences are:

$$S_1 = 2 - 1 - 3 - 5 - 4,$$

$$S_2 = 2 - 1 - 5 - 3 - 4,$$

$$S_3 = 2 - 3 - 5 - 1 - 4,$$

The total elapsed time is same for all these possible 6 sequences $S_1, S_2, S_3, S_4, S_5, ... , S_6$.

The In-out table for any of these 6 sequences $S_1, S_2, S_3, ... , S_6$; say for $S_1 = 2 - 1 - 3 - 5 - 4$.

<table>
<thead>
<tr>
<th>Jobs</th>
<th>Machine $M_1$</th>
<th>Machine $M_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In-out</td>
<td>In-out</td>
</tr>
<tr>
<td></td>
<td>(0, 0, 0) – (15, 16, 18)</td>
<td>(15, 16, 18) – (20, 22, 26)</td>
</tr>
<tr>
<td></td>
<td>(17, 20, 23) – (30, 34, 39)</td>
<td>(30, 34, 39) – (36, 41, 48)</td>
</tr>
<tr>
<td></td>
<td>(35, 41, 47) – (44, 51, 72)</td>
<td>(44, 51, 72) – (50, 58, 81)</td>
</tr>
<tr>
<td></td>
<td>(48, 56, 78) – (58, 67, 92)</td>
<td>(58, 67, 92) – (64, 74, 101)</td>
</tr>
<tr>
<td></td>
<td>(63, 73, 99) – (75, 86, 114)</td>
<td>(75, 86, 114) – (78, 90, 120)</td>
</tr>
</tbody>
</table>

Therefore, the total elapsed time, $CT(S_1) = (78, 90, 120)$

Utilization time for $M_2$, $U_2(S_1) = (78, 90, 120) - (15, 16, 18) = (63, 108, 136)$.

Therefore the total rental cost for each of the sequence ($S_k$), $k = 1, 2, 3, ... , 6$ is $R(S_k) = (489, 668, 864)$. 


11. CONCLUSION

A heuristic algorithm to minimize the rental cost of the machines for a specially structured two stage flow shop scheduling is discussed irrespective of their total elapsed time and utilization time. The processing times and setup times of machine are in fuzzy environment and are represented by triangular fuzzy membership functions. The study may further be extended using trapezoidal fuzzy membership functions and various other constraints of flow shop scheduling problems.

REFERENCES


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