Robust Reliable Controls for Uncertain Neutral Systems with Input Time-varying Delay

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ABSTRACT

The robust reliable control for uncertain neutral systems with time-varying input delays is investigated in the first part of this paper. The design of reliable control is based on the failure of sensor or actuator. Hence the failure condition should be considered for designing the feedback controls (for example; failures are caused due to the temperature and humidity of environment, factor of used time, saturation and hysteresis of actuator). Hence designing a practical and useful feedback control is a very important work. The reliable controls (IQC performance, $H_\infty$, guaranteed cost controls) for switched systems with interval time-varying delays are investigated in the second part of this paper. This paper provides an extensive understand for switched systems and has a significant meaning for designing the switching signal to achieve the requirement of system performance.

1. INTRODUCTION

In practical systems, the failures of sensor and actuator will destroy the stability and system performance. Hence reliable controls have been introduced to tolerate some failures for sensors/actuators and maintains the system stability and performance. Many approaches had been proposed to design reliable controls for the outages of sensors and actuators [1]-[5]. In [6-10], the reliable control of time-delay systems has been studied. In [1], the Hamilton-Jacobi equation approach is used to design a reliable control for nonlinear systems. In [3], an algebraic Riccati equation approach is presented to guarantee the closed-loop stability and $H_\infty$ performance in some admissible component failures. In [5], an LMI approach is provided to guarantee the $H_\infty$ performance for nonlinear system in some admissible component outages. On the other hand, the system models contain always some uncertain elements and nonlinearities; these uncertainties and nonlinearities may be due to unknown additive noise, environmental influence, poor plant knowledge, and limitations of actuators or sensors. Hence a robust reliable control technology is to be developed to stabilize the uncertain linear or nonlinear systems with sensor and actuator failures [1], [5]-[10]. In [6], matrix and linear matrix inequalities have been proposed to design reliable controls with IQC performance for uncertain systems with input delay. In [7], reliable control problem for uncertain nonlinear systems with multiple time delays has been solved by proposing some matrix and linear matrix inequalities. In [9]-[10], the reliable controls for uncertain time-delay systems had been designed via a modified Riccati equation approach. In [11], a reliable control for uncertain fuzzy dynamic systems with time-varying delays has been designed via the LMI approach.

Over the past few decades, the $H_\infty$ control problem for uncertain systems with disturbance inputs has been an active topic in control system theory and application [1], [4]-[5], [9], [12]-[13]. The $H_\infty$ control is
proposed to reduce the effect of the disturbance input on the regulated output to within a prescribed level. There are many approaches for dealing the $H_{\infty}$ control problem. Riccati equation approach was proposed to design $H_{\infty}$ control for time-delay system [12]. The Riccati equation and Hamilton-Jacobi equation approaches have difficulties in finding feasible solutions and minimizing the $H_{\infty}$-norm bound ($\gamma$). In [4], the LMI approach has been used to design a reliable $H_{\infty}$ control for a given $H_{\infty}$-norm bound $\gamma$. Reliable control with IQC performance is a generalization of the $H_{\infty}$ control problem [6], [8]. In [6], a reliable control for uncertain systems with input delay has been considered. In [8], a robust reliable control for uncertain systems with input and state delays has been considered.

In the first part this paper, the LMI approach will be used to design reliable control with IQC performance for uncertain neutral systems with both state and input time-varying delays. Switched system is a class of hybrid systems which exhibits the switching feature between multi-models, which is usually used to approximate many practical nonlinear systems. Some examples for switched systems are automated highway systems, constrained robotics, power systems and power electronics, transmission and stepper motors. A practical example for switched system is PWM-driven boost converter system. In the second part of this paper, the reliable control with IQC performance, reliable $H_{\infty}$ control, and reliable guaranteed cost control for uncertain switched systems with interval time-varying delay are considered. The additional nonnegative inequalities and the Leibniz-Newton formula [22]-[24] will be used to improve the conservativeness and find delay-dependent stabilization results.

2. RELIABLE CONTROL FOR UNPERTURBED NEUTRAL SYSTEMS WITH IQC PERFORMANCE

Consider the following neutral system with state and input time-varying delays:

$$
\ddot{x}(t) = A_0 x(t) + A_1 x(t-h(t)) + A_2 \dot{x}(t) + B_2 u(t) + B_n w(t) + f(x(t), x(t-h(t)), \dot{x}(t)) \quad \forall \theta \in [-H, 0],
$$

$$
y(t) = C_1 x(t), t \geq 0,
$$

$$
z(t) = C_0 x(t) + C_2 \dot{x}(t-h(t)) + C_3 \dot{x}(t) + D_2 u(t) + D_n w(t), t \geq 0,
$$

$$
x(t) = \phi(t), \quad t \in [-H, 0],
$$

where $x \in \mathbb{R}^n$, $x_i$ is state at time $t$ defined by $x_i(\theta) := x(t + \theta), \quad \forall \theta \in [-H, 0]$, $u(t) \in \mathbb{R}^m$ is the control input of actuator or sensor fault, $w \in \mathbb{R}$ is the disturbance input, $y \in \mathbb{R}$ is the measurable output, $z \in \mathbb{R}$ is the regulated output. $A_i \in \mathbb{R}^{n \times n}, \quad i = 0, 1, 2, \quad B_i \in \mathbb{R}^{n \times m}, \quad B_n \in \mathbb{R}^{n \times d}, \quad C_i \in \mathbb{R}^{m \times n}, \quad i = 0, 1, 2, \quad C_n \in \mathbb{R}^{m \times n}, \quad D_i \in \mathbb{R}^{n \times d}, \quad D_n \in \mathbb{R}^{n \times d}$ are some given constant matrices with $\text{rank}(C_i) = r$. The time-varying delays satisfy $0 \leq h(t) \leq \bar{h}, \quad 0 \leq \eta(t) \leq \bar{\eta}, \quad \bar{h}, \quad \bar{\eta}$, and $\tau$ are nonnegative constants with $H = \max\{\bar{h}, \bar{\eta}, \tau\}$. The initial vector $\phi$ is a differentiable function on $[-H, 0]$. At first, the fault of control input for actuator (or sensor) is described as follows:

$$
u(t) = Ru(t),
$$

where $R$ is the actuator fault matrix with

$$
R = \text{diag}[r_1, r_2, \ldots, r_m], \quad 0 \leq \underline{r}_i \leq r_i \leq \bar{r}_i, \quad \bar{r}_i \geq 1, \quad i = 1, 2, \ldots, m,
$$

$\underline{r}_i$ and $\bar{r}_i$, $i = 1, 2, \ldots, m$, are some given constants. $r_i = 0$ means that $i$th actuator or sensor completely fails, $r_i = 1$ means that $i$th actuator or sensor is normal.

Define
\[ R_0 = \text{diag}\left[r_{i0}, r_{20}, \cdots, r_{m0}\right], \quad r_{i0} = \frac{\bar{r}_i + r_i}{2}, \quad (2c) \]

\[ R_i = \text{diag}\left[r_{i1}, r_{21}, \cdots, r_{m1}\right], \quad r_{i1} = \frac{\bar{r}_i - r_i}{2}. \quad (2d) \]

Hence the matrix \( R \) can be rewritten as

\[ R = R_0 + R_i \cdot \Delta J \quad (2e) \]

where

\[ \Delta J = \text{diag}\left[j_1, j_2, \cdots, j_m\right], \quad -1 \leq j_i \leq 1 \]

The function \( f(x(t), x(t-h(t)), \dot{x}(t-\tau)) \) satisfies the following condition:

\[ f^T(x(t), x(t-h(t)), \dot{x}(t-\tau)) f(x(t), x(t-h(t)), \dot{x}(t-\tau)) \leq x^T(t) \Gamma^T \Gamma x(t) + x^T(t-h(t)) \Lambda^T \Lambda x(t-h(t)) + x^T(t-\tau) \Theta^T \Theta \dot{x}(t-\tau), \quad (2f) \]

where \( \Gamma, \Lambda, \) and \( \Theta \) are some given matrices.

**Definition 2.1:** (See [8]) Consider the system (1), with (2) and \( u(t) = -Kx(t) \); let the following conditions be satisfied:

(i) With \( w(t) = 0 \), the closed-loop system (1), with (2) and \( u(t) = -Kx(t) \) is globally asymptotically stable.

(ii) With the zero initial condition (i.e. \( \phi = 0 \)), the signals \( w(t) \) and \( z(t) \) are bounded by

\[
\int_{0}^{\infty} z(t)^T \Pi \cdot \begin{bmatrix} z(t) \\ w(t) \end{bmatrix} dt = \int_{0}^{\infty} \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix} \begin{bmatrix} z(t) \\ w(t) \end{bmatrix} dt \leq 0, \quad \forall \ w \in L_2[0, \infty), \ w \neq 0, \quad (3) \]

where \( \Pi \) is a given matrix, with \( \Pi_{11} > 0 \) and \( \Pi_{22} < 0 \). In this condition, the system (1) with (2) is said to satisfy the IQC performance defined by \( \Pi \) and the control law \( u(t) = -Kx(t) \) is said to be a reliable control with IQC performance.

**Remark 2.1.** If we choose \( \Pi_{11} = I, \Pi_{12} = 0, \Pi_{22} = -\gamma^2 \cdot I \), the condition (3) can be written as

\[ \int_{0}^{\infty} \|z(t)\|^2 dt \leq \gamma^2 \cdot \int_{0}^{\infty} \|w(t)\|^2 dt. \]

This yields a standard \( H_\infty \) control problem. The parameter \( \gamma > 0 \) is the \( H_\infty \) norm bound for the reliable \( H_\infty \) state feedback control \( u(t) = -Kx(t) \) (see [5]). Hence the reliable control with IQC performance can be seen as a generalized reliable \( H_\infty \) control.

The following two lemmas will be used to design a reliable state feedback control with IQC performance.

**Lemma 2.1.** (See [6] and [8]) Let \( U, V, W, X \) be real matrices of appropriate dimensions with \( X \) satisfying \( X = X^T \). Then

\[ X + UVW + W^T V^T U^T < 0, \quad \text{for all } V^T V \leq I, \]

if and only if there exists a scalar \( \varepsilon > 0 \) such that...
\[
X + \varepsilon \cdot UU^T + \varepsilon^{-1} \cdot W^T W = X + \varepsilon^{-1} \cdot (\varepsilon \cdot U) (\varepsilon \cdot U)^T + \varepsilon^{-1} \cdot (W)^T (W) < 0.
\]

**Lemma 2.2.** (Schur Complement, [15]). For a given symmetric matrix \( S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} \), with \( S_{11} = S_{11}^T \), \( S_{22} = S_{22}^T \), the following conditions are equivalent:

(i) \( S < 0 \),

(ii) \( S_{22} < 0, S_{11} - S_{12}^T S_{22}^{-1} S_{12} < 0 \).

Now, the reliable state feedback control \( u(t) = -Kx(t) \) will be designed from the following result.

**Theorem 2.1.** For a constant \( \rho > 0 \), let there exist two constants \( \varepsilon > 0, \sigma > 0 \), some \( n \times n \) positive-definite symmetric matrices \( \overline{P}, \overline{Q}_i, i = 1, 2, ..., \overline{R}_{11}, \overline{R}_{12}, \overline{S}_{11}, \overline{S}_{12} \), some \( 5n \times 5n \) positive-definite symmetric matrices \( \overline{R}_{11}, \overline{R}_{12}, \overline{S}_{11}, \overline{S}_{12} \), and some matrices \( \overline{R}_{12}, \overline{S}_{12}, \overline{S}_{12} \in \mathbb{R}^{5n \times n} \), such that following LMI conditions are satisfied:

\[
\begin{bmatrix}
\overline{R}_{11} & \overline{R}_{12} \\
* & \overline{R}_{22}
\end{bmatrix} > 0,
\begin{bmatrix}
\overline{R}_{11} & \overline{S}_{12} \\
* & \overline{S}_{22}
\end{bmatrix} > 0,
\begin{bmatrix}
\overline{S}_{11} & \overline{S}_{12} \\
* & \overline{S}_{22}
\end{bmatrix} > 0,
\begin{bmatrix}
\overline{S}_{11} & \overline{S}_{12} \\
* & \overline{S}_{22}
\end{bmatrix} > 0,
\]

(4a)

\[
\overline{R}_{11} > R_{11}, \overline{S}_{11} > S_{11}, \overline{Q}_2 > R_{22}, \overline{Q}_2 > R_{22}, \overline{Q}_4 > S_{22}, \overline{Q}_4 > S_{22}, \]

(4b)

\[
\Omega(A_0, A_1, A_2, B_0, B_1, C_0, C_1, C_2, D_0, D_1) =
\]

\[
\left[\begin{array}{ccccccccccc}
\Omega_{11} & \Omega_{12} & 0 & \Omega_{14} & 0 & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} & \Omega_{110} & 0 & \Omega_{115} & 0 & 0 \\
* & 0 & 0 & 0 & \Omega_{26} & 0 & \Omega_{26} & 0 & \Omega_{210} & 0 & 0 & 0 & \Omega_{214} & 0 \\
* & * & \Omega_{35} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & 0 & \Omega_{46} & 0 & \Omega_{46} & 0 & \Omega_{610} & 0 & \Omega_{612} & 0 & 0 & 0 \\
* & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Omega_{65} & 0 & \Omega_{40} & 0 & \Omega_{410} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Omega_{56} & 0 & \Omega_{55} & 0 & \Omega_{510} & 0 & 0 & 0 & \Omega_{515} \\
* & * & * & * & * & * & \Omega_{56} & 0 & \Omega_{40} & 0 & \Omega_{610} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \Omega_{60} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Omega_{60} & 0 & \Omega_{610} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Omega_{60} & 0 & 0 & 0 & 0 \\
\end{array}\right]
\]

(4c)

where

\[
\overline{A}_{11} = \overline{R}_{12} \overline{A}_1 + \overline{A}_1^T \overline{R}_{12} + \overline{R}_{12} \overline{A}_1 + \overline{A}_1^T \overline{R}_{12} + \overline{S}_{12} \overline{A}_2 + \overline{A}_2^T \overline{S}_{12} + \overline{S}_{12} \overline{A}_2 + \overline{A}_2^T \overline{S}_{12} + \overline{\eta}_1 \overline{S}_{11} + \overline{h} \cdot \overline{R}_{11},
\]

\[
\overline{A}_1 = \begin{bmatrix} I & -I & 0 & 0 & 0 \end{bmatrix}, \overline{\Delta}_1 = \begin{bmatrix} 0 & I & -I & 0 & 0 \end{bmatrix},
\]

\[
\overline{A}_2 = \begin{bmatrix} I & 0 & 0 & -I & 0 \end{bmatrix}, \overline{\Delta}_2 = \begin{bmatrix} 0 & 0 & 0 & I & -I \end{bmatrix}.
\]
\begin{align*}
\Omega_{11} &= \overline{P}A_{11} + A_0\overline{P} + \overline{Q}_1 + \overline{Q}_3, \quad \Omega_{12} = A_1\overline{P}, \quad \Omega_{14} = -B_uR_0\hat{K}, \quad \Omega_{16} = \rho \cdot \overline{P}A_{01}^T, \quad \Omega_{17} = A_2\overline{P}, \quad \\
\Omega_{48} &= B_w + \overline{P}C_{01} + \overline{Q}_1, \quad \Omega_{19} = \varepsilon \cdot I, \quad \Omega_{130} = \overline{P}C_{01}^T, \quad \Omega_{211} = \sigma \cdot B_uR_1, \quad \Omega_{113} = \overline{P}\Gamma^{-T}, \\
\Omega_{28} &= \rho \cdot \overline{P}A_{11}^T, \quad \Omega_{28} = \overline{P}C_{11}^T, \quad \Omega_{210} = \overline{P}C_{11}^T, \quad \Omega_{2314} = \rho \cdot A_{12}^T, \quad \Omega_{33} = -\overline{Q}_1, \\
\Omega_{46} &= -\rho \cdot \hat{K}^T R_0^T B_w^T, \quad \Omega_{48} = -\hat{K}^T R_0^T D_w^T, \quad \Omega_{410} = -\hat{K}^T R_0^T D_w^T, \quad \Omega_{412} = -\hat{K}^T, \\
\Omega_{55} &= -\overline{Q}_1, \quad \Omega_{66} = \overline{h} \cdot \overline{Q}_2 + \overline{\eta} \cdot \overline{Q}_4 + \overline{Q}_5 - 2\rho \cdot \overline{P}, \quad \Omega_{67} = \rho \cdot A_1\overline{P}, \quad \Omega_{68} = \rho \cdot B_w, \\
\Omega_{69} &= \rho \cdot \varepsilon \cdot I, \quad \Omega_{611} = \sigma \cdot \rho \cdot B_uR_1, \quad \Omega_{77} = -\overline{Q}_5, \quad \Omega_{78} = \overline{P}C_{21}^T \Pi_{12}, \\
\Omega_{710} &= \overline{P}C_{21}^T, \quad \Omega_{715} = \overline{P}d^T, \quad \Omega_{86} = \Pi_{22} + D_w^T \Pi_{12} + \Pi_{12}^T D_w, \quad \Omega_{810} = D_w^T, \\
\Omega_{811} &= \sigma \cdot \Pi_{12} D_w R_1, \quad \Omega_{1010} = -\Pi_{11}^{-1}, \quad \Omega_{1011} = \sigma \cdot D_w R_1, \\
\Omega_{99} &= \Omega_{1313} = \Omega_{1414} = \Omega_{1515} = -\varepsilon \cdot I, \quad \Omega_{1111} = \Omega_{1212} = -\sigma \cdot I.
\end{align*}

Then, the system (1) with (2) is asymptotically stabilizable via the reliable control \(u(t) = -Kx(t) = -\hat{K}P^{-1}x(t)\) with IQC performance.

**Proof.** Define the Lyapunov function as

\[
V(x_i) = x^T(t)Px(t) + \int_{t-h}^{t} x^T(s)Q_3x(s)ds + \int_{t-h}^{t-h} (s-(t-h))\dot{x}^T(s)Q_2\dot{x}(s)ds
\]

where \(P = \overline{P}^{-1} > 0, Q_i = \overline{P}^{-1}\overline{Q}_i\overline{P}^{-1} > 0, i = 1, 2, \ldots, 5\). The time derivative of \(V(x_i)\) in (5), along the trajectories of the system (1), with (2) and \(u(t) = -Kx(t)\), is given by

\[
\dot{V}(x_i) = A_0x(t) + A_1x(t-h(t)) + A_2\dot{x}(t - \tau) + B_uu^T(t - \eta(t)) + B_ww(t) + f^T P x(t)
\]

where \(f\) is the abbreviation of \(f(x(t), x(t-h(t), \dot{x}(t-\tau))\). By the Leibniz-Newton formulas, we have

\[
\int_{t-h}^{t} \dot{x}(s)ds = x(t) - x(t-h),
\]

(7a)

\[
\int_{t-h}^{t} \dot{x}(s)ds = x(t) - x(t-h),
\]

(7b)
\[
\int_{t-\tau}^{t} \dot{x}(s) \, ds = x(t) - x(t-\tau).
\]

Define
\[
X^T = \begin{bmatrix}
x^T(t) & x^T(t-h(t)) & x^T(t-\eta(t)) & x^T(t-\bar{\eta})
\end{bmatrix}.
\]

From (4a), we have
\[
\int_{t-h(t)}^{t} \left[ \begin{array}{c}
X \\
\dot{x}(s)
\end{array} \right]^T \left[ \begin{array}{cc}
R_{11} & R_{12} \\
* & R_{22}
\end{array} \right] \left[ \begin{array}{c}
X \\
\dot{x}(s)
\end{array} \right] \, ds = h(t) \cdot X^T R_{11} X + 2X^T R_{12} [x(t) - x(t-h(t))]+ \int_{t-h(t)}^{t} \dot{x}(s) R_{22} \dot{x}(s) \, ds \geq 0,
\]

(8a)
\[
\int_{t-\bar{\eta}}^{t} \left[ \begin{array}{c}
X \\
\dot{x}(s)
\end{array} \right]^T \left[ \begin{array}{cc}
\hat{R}_{11} & \hat{R}_{12} \\
* & \hat{R}_{22}
\end{array} \right] \left[ \begin{array}{c}
X \\
\dot{x}(s)
\end{array} \right] \, ds = (\bar{\eta} - h(t)) \cdot X^T \hat{R}_{11} X + 2X^T \hat{R}_{12} [x(t) - x(t-\eta(t))]+ \int_{t-\bar{\eta}}^{t} \dot{x}(s) \hat{R}_{22} \dot{x}(s) \, ds \geq 0,
\]

(8b)
\[
\int_{t-\eta(t)}^{t} \left[ \begin{array}{c}
X \\
\dot{x}(s)
\end{array} \right]^T \left[ \begin{array}{cc}
S_{11} & S_{12} \\
* & S_{22}
\end{array} \right] \left[ \begin{array}{c}
X \\
\dot{x}(s)
\end{array} \right] \, ds = \eta(t) \cdot X^T S_{11} X + 2X^T S_{12} [x(t) - x(t-\eta(t))]+ \int_{t-\eta(t)}^{t} \dot{x}(s) S_{22} \dot{x}(s) \, ds \geq 0,
\]

(8c)
\[
\int_{t-\bar{\eta}}^{t} \left[ \begin{array}{c}
X \\
\dot{x}(s)
\end{array} \right]^T \left[ \begin{array}{cc}
\hat{S}_{11} & \hat{S}_{12} \\
* & \hat{S}_{22}
\end{array} \right] \left[ \begin{array}{c}
X \\
\dot{x}(s)
\end{array} \right] \, ds = (\bar{\eta} - \eta(t)) \cdot X^T \hat{S}_{11} X + 2X^T \hat{S}_{12} [x(t) - x(t-\bar{\eta})]+ \int_{t-\eta}^{t} \dot{x}(s) \hat{S}_{22} \dot{x}(s) \, ds \geq 0,
\]

(8d)

where
\[
\hat{P} = \text{diag} \left[ P^{-1} \quad P^{-1} \quad P^{-1} \quad P^{-1} \quad P^{-1} \quad P^{-1} \right],
\]
\[
\begin{bmatrix}
R_{11} & R_{12} \\
* & R_{22}
\end{bmatrix} = \hat{P} \begin{bmatrix}
\bar{R}_{11} & \bar{R}_{12} \\
* & \bar{R}_{22}
\end{bmatrix} \hat{P} > 0,
\]
\[
\begin{bmatrix}
\hat{R}_{11} & \hat{R}_{12} \\
* & \hat{R}_{22}
\end{bmatrix} = \hat{P} \begin{bmatrix}
\bar{R}_{11} & \bar{R}_{12} \\
* & \bar{R}_{22}
\end{bmatrix} \hat{P} > 0,
\]
\[
\begin{bmatrix}
S_{11} & S_{12} \\
* & S_{22}
\end{bmatrix} = \hat{P} \begin{bmatrix}
\bar{S}_{11} & \bar{S}_{12} \\
* & \bar{S}_{22}
\end{bmatrix} \hat{P} > 0,
\]
\[
\begin{bmatrix}
\hat{S}_{11} & \hat{S}_{12} \\
* & \hat{S}_{22}
\end{bmatrix} = \hat{P} \begin{bmatrix}
\bar{S}_{11} & \bar{S}_{12} \\
* & \bar{S}_{22}
\end{bmatrix} \hat{P} > 0.
\]

From condition (2f), we have
\[
x^T(t) \Gamma^T \Gamma x(t) + x^T(t-h(t)) \Lambda^T \Lambda x(t-h(t)) + \dot{x}^T(t-\tau) \Theta^T \Theta \dot{x}(t-\tau) - f^T f \geq 0.
\]

(9)

From the system (1) with (2a) and \( u(t) = -Kx(t) \), we have
\[
\dot{V}(x) + \begin{bmatrix}
z(t) \\
w(t)
\end{bmatrix}^T \begin{bmatrix}
\Pi_{11} & \Pi_{12} \\
\Pi_{12} & \Pi_{22}
\end{bmatrix} \begin{bmatrix}
z(t) \\
w(t)
\end{bmatrix}
\leq \begin{bmatrix}
A_0 x(t) + A_1 x(t-h(t)) + A_2 \dot{x}(t-\tau) - B_R K x(t-\eta(t)) + B_w w(t) + f
\end{bmatrix}^T P x(t)
\]
\[
+ x^T(t) \begin{bmatrix}
P_0 & P_1 & P_2 \\
P_1 & P_1 & P_2 \\
P_2 & P_1 & P_2
\end{bmatrix} \begin{bmatrix}
A_0 x(t) + A_1 x(t-h(t)) + A_2 \dot{x}(t-\tau) - B_R K x(t-\eta(t)) + B_w w(t) + f
\end{bmatrix}
\]
\[
+ x^T(t) Q_1 x(t) - x^T(t-h(t)) Q_1 x(t-h(t)) + h \cdot \dot{x}^T(t) Q_2 \dot{x}(t) - \int_{t-h}^{t} \dot{x}(s) Q_2 \dot{x}(s) \, ds
\]
+ x^T(t)Q_{3x}(t) - x^T(t - \eta)Q_{3x}(t - \eta) + \eta \cdot \dot{x}^T(t)Q_{4x}(t) - \int_{t-\eta}^{t} \dot{x}^T(s)Q_4 \dot{x}(s)ds \\
+ \dot{x}^T(t)Q_{2x}(t) - \dot{x}^T(t - \tau)Q_{2x}(t - \tau) \\
+ [C_0x(t) + C_1x(t - h(t)) + C_2x(t - \tau) - D_uRKx(t - \eta(t)) + D_w w(t)]^T \Pi_{11} \\
[C_0x(t) + C_1x(t - h(t)) + C_2x(t - \tau) - D_uRKx(t - \eta(t)) + D_w w(t)] \\
+ 2w^T(t)\Pi_{12}[C_0x(t) + C_1x(t - h(t)) + C_2x(t - \tau) - D_uRKx(t - \eta(t)) + D_w w(t)] \\
+ w^T(t)\Pi_{22}w(t) \\
+ h(t) \cdot X^T R_{11} X + 2X^T R_{12}[x(t) - x(t - h(t))] + \int_{t-h(t)}^{t} \dot{x}^T(s)R_{22} \dot{x}(s)ds \\
+ (\eta - h(t)) \cdot X^T \hat{R}_{11} X + 2X^T \hat{R}_{12}[x(t - h(t)) - x(t - \eta)] + \int_{t-\eta}^{t} \dot{x}^T(s)\hat{R}_{22} \dot{x}(s)ds \\
+ \eta(t) \cdot X^T S_{11} X + 2X^T S_{12}[x(t - \eta(t)) - x(t - \eta)] + \int_{t-\eta}^{t} \dot{x}^T(s)S_{22} \dot{x}(s)ds \\
+ (\eta - \eta(t)) \cdot X^T \hat{S}_{11} X + 2X^T \hat{S}_{12}[x(t - \eta(t)) - x(t - \eta)] + \int_{t-\eta}^{t} \dot{x}^T(s)\hat{S}_{22} \dot{x}(s)ds \\
- 2\rho \cdot \dot{x}^T(t)P\dot{x}(t) \\
+ \rho \cdot \dot{x}^T(t)P[A_0x(t) + A_1x(t - h(t)) + A_2x(t - \tau) - B_uRKx(t - \eta(t)) + B_w w(t) + f] \\
+ \rho \cdot [A_0x(t) + A_1x(t - h(t)) + A_2x(t - \tau) - B_uRKx(t - \eta(t)) + B_w w(t) + f]^T P \cdot \dot{x}(t) \\
+ e^{-1} \cdot (\dot{x}^T(t)\Gamma^T \Gamma x(t) + \dot{x}^T(t - h(t))\Lambda^T \Lambda x(t - h(t)) + \dot{x}^T(t - \tau)\Theta^T \Theta x(t - \tau) - f^T f) \\
= Z^T \Sigma Z - \int_{t-h(t)}^{t} \dot{x}^T(s)(Q_2 - R_{22}) \dot{x}(s)ds - \int_{t-\eta}^{t} \dot{x}^T(s)(Q_2 - \hat{R}_{22}) \dot{x}(s)ds \\
- \int_{t-\eta(t)}^{t} \dot{x}^T(s)(Q_4 - S_{22}) \dot{x}(s)ds - \int_{t-\eta}^{t} \dot{x}^T(s)(Q_4 - \hat{S}_{22}) \dot{x}(s)ds \\
- h(t) \cdot X^T (\hat{R}_{11} - R_{11}) X - \eta(t) \cdot X^T (\hat{S}_{11} - S_{11}) X, \quad (10a)

where

\[ Z^T = \begin{bmatrix} x^T(t) & x^T(t - h(t)) & x^T(t - \eta) & x^T(t - \eta) & \dot{x}^T(t) & \dot{x}^T(t - \tau) & w^T(t) & f^T \end{bmatrix}, \]

\[ \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & 0 & \Sigma_{14} & \Sigma_{16} & \Sigma_{17} & \Sigma_{18} & P \\ 0 & \Sigma_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Sigma_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Sigma_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Sigma_{55} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Sigma_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Sigma_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Sigma_{88} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -e^{-1} \cdot I \end{bmatrix} + \begin{bmatrix} \Delta_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \]
\[
\begin{bmatrix}
C_0^T \\
C_1^T \\
0 \\
-K^T R^T D_u^T \\
0 \\
C_2^T \\
D_w^T \\
0
\end{bmatrix}
\left(\Pi_{11}\right)^{-1}
\begin{bmatrix}
C_0 & C_1 & 0 & -D_u R K & 0 & 0 & C_2 & D_w & 0
\end{bmatrix}
, \tag{10b}
\]

\[
\Delta_{11} = R_{12} \Delta_1 + \Delta_1^T R_{12}^T + \hat{R}_{12} \hat{\Delta}_1 + \hat{\Delta}_1^T \hat{R}_{12}^T + S_{12} \Delta_2 + \Delta_2^T S_{12}^T + \hat{S}_{12} \hat{\Delta}_2 + \hat{\Delta}_2^T \hat{S}_{12}^T + \bar{\eta} \cdot \hat{S}_{11} + \bar{\eta} \cdot \hat{R}_{11},
\]

\[
\Delta_1 = [I - I 0 0 0], \quad \hat{\Delta}_1 = [0 I - I 0 0],
\]

\[
\Delta_2 = [I 0 0 -I 0], \quad \hat{\Delta}_2 = [0 0 0 I - I].
\]

\[
\Sigma_{11} = A_0^T P + PA_0 + Q_1 + Q_3 + \varepsilon^{-1} \cdot \Gamma^T \Gamma, \quad \Sigma_{12} = PA_1, \quad \Sigma_{14} = -PB_u R K, \quad \Sigma_{16} = \rho \cdot A_0^T P,
\]

\[
\Sigma_{17} = PA_2, \quad \Sigma_{18} = PB_w + C_0^T \Pi_{12}, \quad \Sigma_{22} = \varepsilon^{-1} \cdot \Lambda^T \Lambda, \quad \Sigma_{26} = \rho \cdot A_1^T P, \quad \Sigma_{28} = C_1^T \Pi_{12},
\]

\[
\Sigma_{33} = -Q_4, \quad \Sigma_{46} = -\rho \cdot K^T R^T B_u^T P, \quad \Sigma_{48} = -K^T R^T D_u^T \Pi_{12}, \quad \Sigma_{55} = -Q_3,
\]

\[
\Sigma_{66} = \bar{\eta} \cdot Q_2 + \bar{\eta} \cdot Q_4 + Q_3 - 2 \rho \cdot P, \quad \Sigma_{67} = \rho \cdot PA_2, \quad \Sigma_{68} = \rho \cdot PB_w, \quad \Sigma_{77} = -Q_5 + \varepsilon^{-1} \cdot \theta^T \theta,
\]

\[
\Sigma_{78} = C_1^T \Pi_{12}, \quad \Sigma_{88} = \Pi_{22} + D_u^T \Pi_{12} + \Pi_{12} D_w.
\]

Premultiplying and postmultiplying the matrix \(\Sigma\) in (10b) by

\[
\text{diag}\left[P^{-1} \quad P^{-1} \quad P^{-1} \quad P^{-1} \quad P^{-1} \quad P^{-1} \quad P^{-1} \quad \varepsilon \cdot I\right]>0
\]

with \(\bar{P} = P^{-1}\), we can obtain the following matrix with (3e), \(\hat{K} = K \bar{P}\), \(\bar{Q}_i = \bar{P} Q_i \bar{P}\), \(i = 1, \cdots, 5\),

\[
\bar{\Sigma} = \left[
\begin{array}{cccccccc}
\Sigma_{11} & \Sigma_{12} & 0 & \Sigma_{14} & 0 & \Sigma_{16} & \Sigma_{17} & \Sigma_{18} & \varepsilon \cdot I \\
* & \Sigma_{22} & 0 & 0 & 0 & \Sigma_{26} & 0 & \Sigma_{28} & 0 \\
* & * & \Sigma_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & 0 & 0 & \Sigma_{46} & 0 & \Sigma_{48} & 0 \\
* & * & * & * & \Sigma_{55} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Sigma_{66} & \Sigma_{67} & \Sigma_{68} & \rho \cdot \varepsilon \cdot I \\
* & * & * & * & * & * & \Sigma_{77} & \Sigma_{78} & 0 \\
* & * & * & * & * & * & * & \Sigma_{88} & 0 \\
* & * & * & * & * & * & * & * & -\varepsilon \cdot I \\
\end{array}
\right] + \left[
\begin{array}{c}
\Delta_{11} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{array}
\right].
\]
\[
\begin{bmatrix}
\bar{P}C_0 \\
\bar{P}C_1 \\
0 \\
-C^T \bar{K}R^TD_w^T \\
0 \\
0 \\
\bar{P}C_2 \\
D_w^T \\
\end{bmatrix}
= \left(\Pi_{11}^{-1}\right)^{-1}
\begin{bmatrix}
C_q\bar{P} & C_1\bar{P} & 0 & -D_wR\hat{K} & 0 & 0 & C_2\bar{P} & D_w^T & 0
\end{bmatrix},
\tag{11}
\]

\[
\bar{\Delta}_1 = \bar{R}_{12}\bar{\Delta}_1 + \bar{\Delta}_1\bar{R}_{12}^T + \bar{R}_{12}\bar{\Delta}_1 + \bar{\Delta}_1\bar{R}_{12}^T + \bar{S}_{12}\bar{\Delta}_2 + \bar{\Delta}_2\bar{S}_{12}^T + \bar{\Delta}_1\bar{S}_{12}^T + \bar{\Delta}_2\bar{S}_{12}^T + \bar{\eta}\cdot \bar{S}_{11} + \bar{h}\cdot \bar{R}_{11},
\]

\[
\bar{\Delta}_2 = [I \quad -I \quad 0 \quad 0 \quad 0], \quad \bar{\Delta}_2 = [0 \quad I \quad -I \quad 0 \quad 0],
\]

\[
\Sigma = \begin{bmatrix}
\Sigma_{11} & \Sigma_{12} & 0 & \Sigma_{14} & 0 & \Sigma_{16} & \Sigma_{17} & \Sigma_{18} & \varepsilon \cdot I & \bar{P}C_0^T \\
* & \Sigma_{22} & 0 & 0 & 0 & \Sigma_{26} & 0 & \Sigma_{28} & 0 & \bar{P}C_1^T \\
* & * & \Sigma_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \Sigma_{46} & 0 & \hat{\Sigma}_{48} & 0 & \hat{\Sigma}_{410} \\
* & * & * & * & \Sigma_{55} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Sigma_{66} & \Sigma_{67} & \Sigma_{68} & \rho \cdot \varepsilon \cdot I & 0 \\
* & * & * & * & * & \Sigma_{77} & \Sigma_{78} & 0 & \bar{P}C_2^T \\
* & * & * & * & * & * & \Sigma_{88} & 0 & D_w^T & 0 \\
* & * & * & * & * & * & * & \cdot & \cdot \cdot & \cdot \\
* & * & * & * & * & * & * & * & -\varepsilon \cdot I & 0 \\
* & * & * & * & * & * & * & * & * & -\Pi_{11}^{-1}
\end{bmatrix}
\]

Define the matrix...
\[
\begin{bmatrix}
\Delta_{11} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & D_u R_1 \\
\end{bmatrix}
+ \begin{bmatrix}
B_u R_1 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
B_u R_1 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
D_u R_1 \\
\end{bmatrix},
\]

(12)

where

\[
\hat{\Sigma}_{44} = -B_u R_0 \hat{K}, \hat{\Sigma}_{46} = -\rho \hat{K}^T R_0^T B_u^T, \hat{\Sigma}_{48} = -\hat{R}_0^T R_u^T \Pi_{12}^T, \hat{\Sigma}_{410} = -\hat{R}_0^T R_u^T D_u^T.
\]

By Lemmas 2.1-2.2, we get that \( \Sigma < 0 \) in (4c) is equivalent to \( \hat{\Sigma} < 0 \) in (12). By Lemma 2.2, \( \hat{\Sigma} < 0 \) is equivalent to \( \Sigma < 0 \) in (11). Condition \( \Sigma < 0 \) is also equivalent to \( \Sigma < 0 \) in (10b). From (8), (9), (10a), and \( \Sigma < 0 \) with \( \hat{w}(t) = 0 \), there exists a constant \( \lambda > 0 \) satisfying

\[
\dot{V}(x(t))|_{w(t) = 0} + z^T(t) \Pi_{11} z(t) \leq -\lambda \| x(t) \|^2.
\]

With \( \Pi_{11} > 0 \), we obtain the following condition:

\[
\dot{V}(x(t))|_{w(t) = 0} \leq -\lambda \| x(t) \|^2.
\]

Hence, the closed system (1)-(2) with \( u(t) = -K x(t) = -\hat{K} \hat{P}^{-1} x(t) \) and \( w(t) = 0 \) is asymptotically stable (see [16]-[17]).

Integrating the function in (10a) from 0 to \( \infty \) and from \( \Sigma < 0 \), we have

\[
\lim_{t \to \infty} V(x(t)) - V(x_0) + \int_0^\infty \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^T & \Pi_{22} \end{bmatrix} \begin{bmatrix} z(t) \\ w(t) \end{bmatrix} dt \leq 0.
\]

With the zero initial condition \( (x_0 = 0) \), we have

\[
V(x_0) = 0, \lim_{t \to \infty} V(x(t)) \geq 0,
\]

and

\[
\int_0^\infty \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^T & \Pi_{22} \end{bmatrix} \begin{bmatrix} z(t) \\ w(t) \end{bmatrix} dt \leq 0, \quad \forall \ w \in L_2[0, \infty), w \neq 0.
\]

By the Definition 2.1, the system (1)-(2) satisfies IQC performance.
If the constraints of time-varying delays are given by:
\[ 0 \leq h(t) \leq \bar{h}, \quad 0 \leq \eta(t) \leq \bar{\eta}, \quad \dot{h}(t) \leq h_D < 1, \quad \dot{\eta}(t) \leq \eta_D < 1, \]
(13)
where \( \bar{h}, \bar{\eta}, h_D, \) and \( \eta_D \) are some given constants. Then Theorem 2.1 can be rewritten as the following result:

**Corollary 2.1.** For a constant \( \rho > 0 \), let there exist two constants \( \varepsilon > 0, \sigma > 0, \) some \( n \times n \) positive-definite symmetric matrices \( \bar{P}, \bar{Q}_i, \) \( i = 1, 2, \cdots, 7, \) \( \bar{R}_{11}, \bar{R}_{12}, \bar{S}_{11}, \bar{S}_{12} \), some \( 5n \times 5n \) positive-definite symmetric matrices \( \bar{R}_{11}, \bar{R}_{12}, \bar{S}_{11}, \bar{S}_{12} \), and some matrices \( \bar{R}_{12}, \bar{R}_{11}, \bar{S}_{12}, \bar{S}_{11} \in \mathcal{R}_{5n \times 5n}, \) \( \bar{K} \in \mathcal{R}_{5n \times 5n} \), such that LMI conditions (4a)-(4b) and the following condition are satisfied:

\[
\begin{align*}
\hat{\Omega}(A, A_t, A_B, B_B, B_C, C_0, C_1, C_2, D_B, D_C) &=
\end{align*}
\]

\[
\begin{bmatrix}
\hat{\Omega}_{11} & \Omega_{12} & 0 & \Omega_{14} & 0 & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} & \Omega_{110} & \Omega_{111} & 0 & \Omega_{113} & 0 & 0 \\
* & \hat{\Omega}_{22} & 0 & 0 & 0 & \Omega_{26} & 0 & \Omega_{28} & 0 & \Omega_{239} & 0 & 0 & 0 & 0 & \Omega_{214} & 0 \\
* & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \hat{\Omega}_{44} & 0 & \Omega_{46} & 0 & \Omega_{48} & 0 & \Omega_{440} & 0 & \Omega_{4412} & 0 & 0 & 0 & 0 \\
* & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Omega_{66} & \Omega_{67} & \Omega_{68} & \Omega_{69} & 0 & \Omega_{610} & 0 & 0 & 0 & 0 & \Omega_{6115} & 0 \\
* & * & * & * & \Omega_{77} & \Omega_{78} & 0 & \Omega_{739} & 0 & \Omega_{760} & 0 & 0 & 0 & 0 & \Omega_{781} & 0 \\
* & * & * & * & * & \Omega_{88} & 0 & \Omega_{800} & \Omega_{811} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Omega_{99} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Omega_{1010} & \Omega_{1011} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Omega_{1111} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \Omega_{1212} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Omega_{1313} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Omega_{1414} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & \Omega_{1515} & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\end{align*}
\]

where

\[
\begin{align*}
\hat{\Omega}_{11} &= PA_0^T + A_0 \bar{P} + \bar{Q}_1 + \bar{Q}_3 + \bar{Q}_6 + \bar{Q}_7, \quad \hat{\Omega}_{22} = -(1 - h_D) \cdot \bar{Q}_6, \quad \hat{\Omega}_{44} = -(1 - \eta_D) \cdot \bar{Q}_7,
\end{align*}
\]

\[
\begin{bmatrix}
\bar{A}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} < 0,
\]
(14)
other matrices $\Omega_{ij}$, $i, j = 1, 2, \cdots, 15$, and $\bar{A}_{ij}$ are defined in Theorem 2.1. Then, the system (1) with (2) and (13) is asymptotically stabilizable via the reliable control $u(t) = -Kx(t) = -\hat{K}\bar{P}^{-1}x(t)$ with IQC performance.

**Proof.** Define the Lyapunov functional as follows:

$$
\bar{V}(x_t) = V(x_t) + \int_{t-h(t)}^{t} x^T(s)Q_0x(s)ds + \int_{t-n(t)}^{t} x^T(s)Q_\eta x(s)ds,
$$

where $V(x_t)$ is defined in (5), $Q_i = \bar{P}^{-1}Q_i \bar{P}^{-1} > 0$, $i = 6, 7$. This proof is same as Theorem 2.1.

3. RELIABLE CONTROL FOR UNCERTAIN NEUTRAL SYSTEMS WITH IQC PERFORMANCE

Consider the uncertain neutral system with state and input time-varying delays:

$$
\dot{x}(t) = A_0(t)x(t) + A_1(t)x(t-h(t)) + A_2(t)\dot{x}(t-\tau) + B_u(t)u(t) + B_w(t)w(t),
$$

$$
x(t) = \phi(t), \ t \in [-H, 0],
$$

$$
z(t) = C_0(t)x(t) + C_1(t)x(t-h(t)) + C_2(t)\dot{x}(t-\tau) + D_u(t)u(t) + D_w(t)w(t), \ t \geq 0,
$$

where

$$
A_0(t) = A_0 + \Delta A_0(t), \ A_1(t) = A_1 + \Delta A_1(t), \ A_2(t) = A_2 + \Delta A_2(t), \ B_u(t) = B_u + \Delta B_u(t), \ B_w(t) = B_w + \Delta B_w(t),
$$

$$
C_0(t) = C_0 + \Delta C_0(t), \ C_1(t) = C_1 + \Delta C_1(t), \ C_2(t) = C_2 + \Delta C_2(t), \ D_u(t) = D_u + \Delta D_u(t), \ D_w(t) = D_w + \Delta D_w(t),
$$

$A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w,$ are some given constant matrices, $\Delta A_0(t), \Delta A_1(t), \Delta A_2(t), \Delta B_u(t), \Delta B_w(t), \Delta C_0(t), \Delta C_1(t), \Delta C_2(t), \Delta D_u(t), \Delta D_w(t),$ are some time-varying functions satisfying

$$
\begin{bmatrix}
\Delta A_0(t) & \Delta A_1(t) & \Delta A_2(t) & \Delta B_u(t) & \Delta B_w(t) \\
\Delta C_0(t) & \Delta C_1(t) & \Delta C_2(t) & \Delta D_u(t) & \Delta D_w(t)
\end{bmatrix} = \begin{bmatrix} M_x \\ M_z \end{bmatrix}F(t)\begin{bmatrix} N_0 & N_1 & N_2 & N_3 & N_4 \end{bmatrix},
$$

where $M_x, M_z, N_i, i = 0, 1, 2, 3, 4,$ are some given constant matrices, $F(t)$ is real time-varying function with appropriate dimensions and bounded as follows:

$$
F(t)^T \times F(t) \leq I, \ \forall \ t \geq 0.
$$

With the result of Theorem 2.1 and by comparing system (1) and system (15), a result to design the robust reliable control $u(t) = -Kx(t)$ with IQC performance for system (15) is presented.

**Theorem 3.1.** For a constant $\rho > 0$, there exist some constants $\varepsilon > 0, \sigma > 0, \mu > 0$, some $n \times n$ positive-definite symmetric matrices $\bar{P}, \bar{Q}_i, i = 1, 2, \cdots, 5$, $\bar{R}_{12}, \bar{R}_{12}$, $\bar{S}_{22}, \bar{S}_{22}$, some $5n \times 5n$ positive-definite symmetric matrices $\bar{R}_{11}, \bar{R}_{11}, \bar{S}_{11}, \bar{S}_{11}$, and some matrices $\bar{R}_{12}, \bar{R}_{12}, \bar{S}_{12}, \bar{S}_{12} \in \mathbb{R}^{5\times n}$, $\hat{K} \in \mathbb{R}^{m\times n}$, such that that (4a)-(4b) and following LMI conditions are satisfied:

$$
\begin{bmatrix}
\Omega(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w) & \Lambda_1 \\
* & \Lambda_2
\end{bmatrix} < 0,
$$

(16a)
where \( \Omega(\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2, \mathbf{B}_u, \mathbf{B}_w, \mathbf{C}_0, \mathbf{C}_1, \mathbf{C}_2, \mathbf{D}_u, \mathbf{D}_w) \) is defined in (4c) and

\[
\Lambda_1 = \begin{bmatrix}
\mu \cdot M_x & \overline{P} N_0^T \\
0 & \overline{P} N_1^T \\
0 & 0 \\
0 & -\hat{K}^T \mathbf{R}_0^T N_3^T \\
0 & 0 \\
\mu \cdot \rho \cdot M_x & 0 \\
0 & \overline{P} N_2^T \\
\mu \cdot \Pi_{12}^T M_z & N_4^T \\
0 & 0 \\
\mu \cdot M_z & 0 \\
0 & \sigma \cdot \mathbf{R}_1^T N_3^T \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix},
\]

(16b)

\[
\Lambda_2 = \begin{bmatrix}
-\mu \cdot I & 0 \\
0 & -\mu \cdot I
\end{bmatrix}.
\]

(16c)

Then, the system (15) with (2) is asymptotically stabilizable by the reliable control \( u(t) = -Kx(t) = -\hat{K}\mathbf{P}^{-1}x(t) \) with IQC performance.

**Proof:** From the systems (1) and (15) with Theorem 2.1, a sufficient condition to design the reliable control with IQC performance for the system (15) is given by

\[
\Omega(\mathbf{A}_0(t), \mathbf{A}_1(t), \mathbf{A}_2(t), \mathbf{B}_u(t), \mathbf{B}_w(t), \mathbf{C}_0(t), \mathbf{C}_1(t), \mathbf{C}_2(t), \mathbf{D}_u(t), \mathbf{D}_w(t))
\]

\[
= \Omega(\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2, \mathbf{B}_u, \mathbf{B}_w, \mathbf{C}_0, \mathbf{C}_1, \mathbf{C}_2, \mathbf{D}_u, \mathbf{D}_w) + \Xi(t)\mathbf{E}^T + \Xi^T(t)\mathbf{E}^T,
\]

where

\[
\Xi^T = \begin{bmatrix}
\mathbf{M}_x^T & 0 & 0 & 0 & \rho \cdot \mathbf{M}_x^T & 0 & \mathbf{M}_z^T \Pi_{12} & 0 & \mathbf{M}_z^T & 0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
\Xi^T = \begin{bmatrix}
\mathbf{N}_0 \overline{P} & \mathbf{N}_1 \overline{P} & 0 & -\mathbf{N}_4 \mathbf{R}_0 \hat{K} & 0 & 0 & \mathbf{N}_2 \overline{P} & \mathbf{N}_4 & 0 & 0 & \sigma \cdot \mathbf{N}_4 \mathbf{R}_1 & 0 & 0 & 0
\end{bmatrix}.
\]

By Lemmas 2.1 and 2.2, condition (16) is equivalent to

\[
\Omega(\mathbf{A}_0(t), \mathbf{A}_1(t), \mathbf{A}_2(t), \mathbf{B}_u(t), \mathbf{B}_w(t), \mathbf{C}_0(t), \mathbf{C}_1(t), \mathbf{C}_2(t), \mathbf{D}_u(t), \mathbf{D}_w(t)) < 0.
\]

The proof is completed.

**Corollary 3.1.** For a constant \( \rho > 0 \), let there exist some constants \( \varepsilon > 0, \sigma > 0, \mu > 0 \), some \( n \times n \) positive-definite symmetric matrices \( \overline{P}, \overline{Q}_i, i = 1, 2, \ldots, 7, \overline{R}_{22}, \overline{R}_{22}, \overline{S}_{22}, \overline{S}_{22} \), some \( 5n \times 5n \) positive-
definite symmetric matrices $\overline{R}_{ii}, \overline{R}_{ij}, \overline{S}_{ii}, \overline{S}_{ij}$, and some matrices $\overline{R}_{12}, \overline{R}_{22}, \overline{S}_{12}, \overline{S}_{22} \in \mathbb{R}^{5n \times n}$, $\hat{K} \in \mathbb{R}^{m \times n}$, such that LMI conditions (4a)-(4b) and the following condition are satisfied:

$$\begin{bmatrix}
\tilde{Q}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w) \\
\star \\
\Lambda_1 \\
\Lambda_2
\end{bmatrix} < 0,$$

where $\tilde{Q}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w)$ is defined in (14), $\Lambda_1$ and $\Lambda_2$ are defined in (16b)-(16c). Then, the system (15) with (2) and (13) is asymptotically stabilizable via the reliable control $u(t) = -Kx(t) = -\hat{K}P^{-1}x(t)$ with IQC performance.

In many practical systems, the actuator or sensor is working either in normal or completely faulty conditions (see [8] and [10]). When the actuator or sensor has some failures, the possible fault matrices $R$ in (2a) can be rewritten as

$$R_i = \text{diag}(r_{i1}, r_{i2}, \cdots, r_{im}), r_{ji} = 0 \text{ or } r_{ji} = 1, \quad j = 1, 2, \cdots, m, \quad i = 1, 2, \cdots, N,$$

where $r_{ji}, j = 1, 2, \cdots, m, i = 1, 2, \cdots, N$, are some given constants, but $r_{ji} = 0$, for all $j = 1, 2, \cdots, m$ and some $i = 1, 2, \cdots, N$ (no input) are not allowed.

With the results of Theorems 2.1-2.2, the robust reliable control $u(t) = -Kx(t)$ with IQC performance for system (15) with (17) is provided in the following result.

**Theorem 3.2.** For a constant $\rho > 0$, let there exist two constants $\rho > 0, \mu > 0$, some $n \times n$ positive-definite symmetric matrices $\overline{P}, \overline{Q}_i, i = 1, 2, \cdots, 5$, $\overline{R}_{12}, \overline{R}_{22}, \overline{S}_{12}, \overline{S}_{22}$, some $5n \times 5n$ positive-definite symmetric matrices $\overline{R}_{ii}, \overline{R}_{ij}, \overline{S}_{ii}, \overline{S}_{ij}$, and some matrices $\overline{R}_{12}, \overline{R}_{22}, \overline{S}_{12}, \overline{S}_{22} \in \mathbb{R}^{5n \times n}$, $\hat{K} \in \mathbb{R}^{m \times n}$, such that that (4a)-(4b) and following LMI conditions are satisfied:

$$\begin{bmatrix}
\tilde{Q}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w, R_i) \\
\star \\
\tilde{\Lambda}_1 \\
\tilde{\Lambda}_2
\end{bmatrix} < 0, \quad i = 0, 1, 2, \cdots, N,$$

where $\tilde{\Lambda}_2$ is defined in (16c), $R_0 = I$, and

$$\tilde{\Lambda}_ii = \begin{bmatrix}
\mu \cdot M_x & \overline{P}N^T_0 \\
0 & \overline{P}N^T_1 \\
0 & 0 \\
0 & -\hat{K}^TR_i^TN^T_3 \\
0 & 0 \\
\mu \cdot \rho \cdot M_x & 0 \\
0 & \overline{P}N^T_2 \\
\mu \cdot \Pi_{12}M_z & N^T_4 \\
0 & 0 \\
\mu \cdot M_z & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}, \quad (18b)$$
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\[ \tilde{\Omega}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w, R) = \]

\[
\begin{bmatrix}
\Omega_{11} & \Omega_{12} & 0 & \tilde{\Omega}_{14} & 0 & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} & \Omega_{110} & \tilde{\Omega}_{111} & 0 & 0 \\
* & 0 & 0 & 0 & 0 & \Omega_{26} & 0 & \Omega_{28} & 0 & \Omega_{210} & 0 & \tilde{\Omega}_{212} & 0 \\
* & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & 0 & 0 & \tilde{\Omega}_{46} & 0 & \tilde{\Omega}_{48} & 0 & \tilde{\Omega}_{410} & 0 & 0 & 0 \\
* & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Omega_{66} & \Omega_{57} & \Omega_{68} & \Omega_{69} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Omega_{77} & \Omega_{78} & 0 & \Omega_{710} & 0 & 0 & \tilde{\Omega}_{713} \\
* & * & * & * & * & * & * & \Omega_{88} & 0 & \Omega_{810} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Omega_{99} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Omega_{1010} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \tilde{\Omega}_{1111} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & \tilde{\Omega}_{1212} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & \tilde{\Omega}_{1313}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\tilde{\Omega}_{111} = \bar{P} \Gamma^T, \tilde{\Omega}_{212} = \bar{P} \Lambda^T, \tilde{\Omega}_{713} = \bar{P} \Theta^T, \tilde{\Omega}_{1111} = \tilde{\Omega}_{1212} = \tilde{\Omega}_{1313} = -\varepsilon \cdot I, \\
\tilde{\Omega}_{14} = -B_u R \tilde{K}, \tilde{\Omega}_{46} = -\rho \cdot \tilde{K}^T R_i^T B_u^T, \tilde{\Omega}_{48} = -\tilde{K}^T R_i^T D_u^T \Pi_{12}, \tilde{\Omega}_{410} = -\tilde{K}^T R_i^T D_w^T,
\end{bmatrix}
\]

other matrices \( \Omega_{ij} \), \( i, j = 1, 2, \ldots, 15 \), and \( \bar{\Lambda}_{11} \) are defined in Theorem 2.1. Then, the system (15) with (2a), (2f), (17) is asymptotically stabilizable by the reliable control \( u(t) = -Kx(t) = -\tilde{K}\bar{P}^{-1}x(t) \) with IQC performance.

**Proof.** Under the vertex conditions in (17), we have the following convex combination for the matrix \( R \) in (2a):

\[
R = \sum_{i=0}^{N} \alpha_i \cdot R_i, \quad \forall \sum_{i=0}^{N} \alpha_i = 1.
\]

From the conditions (18), we have

\[
\begin{bmatrix}
\tilde{\Omega}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w, R) & \bar{\Lambda}_1 \\
* & \bar{\Lambda}_2
\end{bmatrix}
\]
= \sum_{i=0}^{N} \alpha_i \begin{bmatrix} \tilde{\Omega}(A_0, A_1, A_2, B_w, C_0, C_1, C_2, D_u, D_w, R_i) \\ * \\ \Lambda_1 \\ \Lambda_2 \end{bmatrix} < 0.

From the spirit of Theorems 2.1 and 3.1, this will imply that the asymptotic stabilization for the reliable control of the system with IQC performance under normal and failure situations has been achieved.

**Corollary 3.2.** For a constant \( \rho > 0 \), let there exist two constants \( \varepsilon > 0 \), \( \mu > 0 \), some \( n \times n \) positive-definite symmetric matrices \( \bar{P}, \bar{Q} \), \( i = 1, 2, \ldots, 7 \), \( \bar{R}_{22}, \bar{S}_{22} \), some \( 5n \times 5n \) positive-definite symmetric matrices \( \bar{\bar{R}}_{11}, \bar{\bar{R}}_{11}, \bar{\bar{S}}_1, \bar{\bar{S}}_1 \), and some matrices \( \bar{\bar{R}}_{12}, \bar{\bar{S}}_{12}, \bar{\bar{S}}_{12} \in \mathbb{R}^{5n \times n} \), \( \bar{K} \in \mathbb{R}^{n \times n} \), such that LMI conditions (4a)-(4b) and the following condition are satisfied:

\[
\begin{bmatrix}
\tilde{\Omega}(A_0, A_1, A_2, B_w, C_0, C_1, C_2, D_u, D_w, R_i) \\
\end{bmatrix} < 0, \quad i = 0, 1, 2, \ldots, N,
\]

where \( \tilde{\Omega}(A_0, A_1, A_2, B_w, C_0, C_1, D_w, R_i) \) is defined by

\[
\tilde{\Omega}(A_0, A_1, A_2, B_w, C_0, C_1, D_w, R_i) =
\]

\[
\begin{bmatrix}
\tilde{\Omega}_{11} & \tilde{\Omega}_{12} & 0 & \tilde{\Omega}_{14} & 0 & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} & \Omega_{410} & \tilde{\Omega}_{411} & 0 & 0 \\
* & \tilde{\Omega}_{22} & 0 & 0 & 0 & \Omega_{26} & 0 & \Omega_{28} & 0 & \Omega_{410} & 0 & \tilde{\Omega}_{212} & 0 \\
* & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \tilde{\Omega}_{44} & 0 & \tilde{\Omega}_{46} & 0 & \tilde{\Omega}_{48} & 0 & \tilde{\Omega}_{410} & 0 & 0 & 0 \\
* & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Omega_{66} & \Omega_{67} & \Omega_{68} & \Omega_{69} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Omega_{77} & \Omega_{78} & 0 & \Omega_{710} & 0 & 0 & \tilde{\Omega}_{713} \\
* & * & * & * & * & * & * & \Omega_{88} & 0 & \Omega_{410} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \Omega_{99} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \Omega_{1010} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \tilde{\Omega}_{111} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \tilde{\Omega}_{1212} & 0 & \tilde{\Omega}_{1313} \\
* & * & * & * & * & * & * & * & * & \tilde{\Omega}_{1313} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\tilde{\Lambda}_1 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}.
\]
\( \tilde{\Omega}_{11} = P A_0^T + A_0 P + Q_1 + Q_3 + Q_6 + Q_7 \), \( \tilde{\Omega}_{12} = -(1 - h_p) \cdot Q_6 \), \( \tilde{\Omega}_{44} = -(1 - \eta_p) \cdot Q_7 \),

other notations are defined in Theorem 3.1, \( \tilde{\Lambda}_1 \) and \( \Lambda_2 \) are defined in (18b) and (16c), respectively. Then, the system (15) with (2a), (2f), (13), (17) is asymptotically stabilizable via the reliable control \( u(t) = -Kx(t) = -\hat{K}P^{-1}x(t) \) with IQC performance.

4. NUMERICAL EXAMPLE

Consider the uncertain neutral system (15) with the following parameters:

\[
A_0 = \begin{bmatrix}
-1 & 1 & 0 & 0 \\
-1 & 0 & -0.5 & 0 \\
0 & 0 & 1 & 1 \\
0.5 & 0 & 0.5 & -0.1
\end{bmatrix},
\quad A_1 = \begin{bmatrix}
-0.3 & 0.2 & 0.3 & 0.1 \\
0 & 0.2 & 0.3 & 0.3 \\
0.1 & -0.2 & 0.2 & 0.2 \\
0.2 & 0.1 & 0.2 & 0.3
\end{bmatrix},
\]

\[
A_2 = \alpha \cdot \begin{bmatrix}
0.1 & 0.1 & 0.1 & 0.1 \\
0 & 0.05 & -0.1 & 0.1 \\
0.1 & -0.1 & 0.1 & 0.1 \\
0.1 & 0.1 & 0.1 & 0.1
\end{bmatrix},
\quad B_u = \begin{bmatrix}
2 & 1 \\
-1 & 1 \\
2 & 2 \\
-1 & 1
\end{bmatrix},
\quad B_w = \begin{bmatrix}
0.4 & 0.3 \\
0.5 & -0.5 \\
1 & -0.5 \\
0.5 & 0.3
\end{bmatrix},
\]

\[
C_0 = \begin{bmatrix}
0 & 0.2 & 0 & 0.8 \\
0 & 0 & 0 & 0.5
\end{bmatrix},
\quad C_1 = C_2 = 0,
\quad D_u = \begin{bmatrix}
0.8 & 0 \\
1 & 0.7
\end{bmatrix},
\quad D_w = \begin{bmatrix}
0.4 & 0 \\
0 & 0.5
\end{bmatrix},
\]

\[
M_x = \begin{bmatrix}
0.1 \\
0 \\
0.2 \\
0
\end{bmatrix},
\quad M_z = \begin{bmatrix}
0.1 \\
0 \\
0.1 \\
0
\end{bmatrix},
\quad N_0 = \begin{bmatrix}
0.2 & 0.4 & 0.3 & 0.2
\end{bmatrix},
\quad N_1 = \begin{bmatrix}
0.1 & 0.1 & 0.2 & 0.1
\end{bmatrix},
\quad \Gamma = \Lambda = \theta = 0,
\]

\[N_2 = \beta \cdot \begin{bmatrix}
0.1 & 0.2 & 0.1 & 0.2
\end{bmatrix},
\quad N_3 = \begin{bmatrix}
0.1 & 0.2
\end{bmatrix},
\quad N_4 = \begin{bmatrix}
0.2 & 0.1
\end{bmatrix},
\]

\[\tau > 0, \quad \eta_D = 0, \quad \overline{\eta} = 0.128, \quad h_D = 0, \quad \overline{h} = 2. \quad (20a)
\]

The obtained results with \( \alpha = \beta = 0.1 \) in this paper are formulated in the following.

(a) Consider the failure condition in (2), \( 0.6 \leq r_1 \leq 1, \ 0.9 \leq r_2 \leq 1.1, \) and the IQC performance bound

\[
\Pi = \begin{bmatrix}
\Pi_{11} & \Pi_{12} \\
\Pi_{12}^T & \Pi_{22}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0.1 & 0.2 \\
0 & 1 & 0.2 & 0.1 \\
0.1 & 0.2 & -2 & 0.1 \\
0.2 & 0.1 & 0.1 & -2
\end{bmatrix}. \quad (20b)
\]

From (2c)-(2d), we have
\[ R_0 = \begin{bmatrix} 0.8 & 0 \\ 0 & 1 \end{bmatrix}, \quad R_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}. \]

By using Corollary 3.1 with \( \rho = 0.22 \), the system (15) with (2) and (20) is asymptotically stabilizable by the robust reliable control
\[
 u(t) = -\hat{K}\overline{P}^{-1}x(t) = \begin{bmatrix} 0.2084 & -0.7022 & 0.3597 & 0.1186 \\ 0.2418 & 0.6306 & 0.7472 & 0.7154 \end{bmatrix}x(t)
\]
with IQC performance bound in (20b).

(b) Consider the fault matrix \( R_i = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \) in (17) (the input \( u_i(t) \) may completely fail in the future).

From Corollary 3.2 with \( \rho = 0.27 \), the system (15) with (17) and (20) is asymptotically stabilizable by the robust reliable control
\[
 u(t) = -\hat{K}\overline{P}^{-1}x(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4247 & -0.6866 & 2.3032 & 0.7418 \end{bmatrix}x(t)
\]
with IQC performance bound in (20b).

Even when in the special condition \( \alpha = \beta = 0 \), the LMI conditions in our past results in [8] have no feasible solution for the above two cases. The results in [8] cannot be used for any \( \tau > 0, \Gamma, \Lambda, \theta, \alpha, \beta \neq 0 \).

5. RELIABLE STATIC OUTPUT FEEDBACK CONTROL

If the state feedback control is replaced by static output control \( u(t) = -Gy(t) \) with \( G \in \mathbb{R}^{m \times r} \), then the previous obtained result can be rewritten as follows:

**Theorem 5.1.** For a constant \( \rho > 0 \), let there exist two constants \( \varepsilon > 0, \sigma > 0 \), some \( n \times n \) positive-definite symmetric matrices \( \overline{P}, \overline{Q}_i, i = 1, 2, \cdots, 5, \overline{R}_{22}, \overline{S}_{22}, \overline{S}_{22} \), some \( 5n \times 5n \) positive-definite symmetric matrices \( \overline{R}_{11}, \overline{R}_{11}, \overline{S}_{11}, \overline{S}_{11} \), and some matrices \( \overline{R}_{12}, \overline{R}_{12}, \overline{S}_{12}, \overline{S}_{12} \in \mathbb{R}^{5m \times n}, \hat{G} \in \mathbb{R}^{p \times r}, \hat{P} \in \mathbb{R}^{r \times r} \), such that following LMI and LME conditions are satisfied:

\[
 C_y\overline{P} = \hat{P}C_y, \\
 \begin{bmatrix} \overline{R}_{11} & \overline{R}_{12} \\ * & \overline{R}_{22} \end{bmatrix} > 0, \quad \begin{bmatrix} \overline{S}_{11} & \overline{S}_{12} \\ * & \overline{S}_{22} \end{bmatrix} > 0, \quad \begin{bmatrix} \overline{S}_{11} & \overline{S}_{12} \\ * & \overline{S}_{22} \end{bmatrix} > 0, \quad \begin{bmatrix} \overline{S}_{11} & \overline{S}_{12} \\ * & \overline{S}_{22} \end{bmatrix} > 0, \\
 \overline{R}_{11} > \overline{R}_{11}, \overline{S}_{11} > \overline{S}_{11}, \overline{Q}_2 > \overline{R}_{22}, \overline{Q}_2 > \overline{R}_{22}, \overline{Q}_4 > \overline{S}_{22}, \overline{Q}_4 > \overline{S}_{22}, \\
 \Omega(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w) = 0.
\]
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\[
\begin{align*}
\Omega &= \begin{bmatrix}
\Omega_{11} & \Omega_{12} & 0 & \hat{\Omega}_{14} & 0 & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} & \Omega_{110} & \Omega_{111} & 0 & \Omega_{113} & 0 & 0 \\
* & 0 & 0 & 0 & \Omega_{26} & 0 & \Omega_{28} & 0 & \Omega_{210} & 0 & 0 & 0 & \Omega_{214} & 0 \\
* & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & 0 & \hat{\Omega}_{46} & 0 & \hat{\Omega}_{48} & 0 & \hat{\Omega}_{410} & 0 & \hat{\Omega}_{412} & 0 & 0 & 0 \\
* & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Omega_{66} & \Omega_{67} & \Omega_{68} & \Omega_{69} & 0 & \Omega_{611} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Omega_{77} & \Omega_{78} & 0 & \Omega_{710} & 0 & 0 & 0 & 0 & 0 & \Omega_{715} \\
* & * & * & * & * & * & \Omega_{88} & 0 & \Omega_{830} & \Omega_{841} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \Omega_{99} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Omega_{1010} & \Omega_{1011} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Omega_{1111} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Omega_{1212} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Omega_{1313} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Omega_{1414} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & \Omega_{1515} & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\end{align*}
\]

\[
\bar{\Delta} &= \begin{bmatrix}
\Delta_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} < 0,
\]

where

\[
\begin{align*}
\hat{\Omega}_{14} &= -B_u R_0 \hat{G} C_y, \quad \hat{\Omega}_{46} = -\rho \cdot C^T_G \hat{G}^T R_0^T B_u^T, \quad \hat{\Omega}_{48} = -C^T_G \hat{G}^T R_0^T D_u^T \Pi_{12}, \quad \hat{\Omega}_{410} = -C^T_G \hat{G}^T R_0^T D_u^T, \\
\end{align*}
\]

Other notations are same as Theorem 2.1. Then, the system (1) with (2) is asymptotically stabilizable via the reliable control \( u(t) = -G y(t) = -\hat{G} \hat{p}^{-1} y(t) \) with IQC performance.

**Remark 5.1.** The LMI and LME conditions in Theorem 5.1 could be solved by some efficient computer optimization programs; such as Scilab (see http://scilabs.inria.fr/). Hence the standard LMI procedure can be directly employed to find a feasible solution of these conditions. The main purpose for equality constraint in (6d) is that convert the non-convex problem into convex one. The assumption \( \text{rank}(C_y) = r \) in system (1) will guarantee that the matrix \( \hat{p} \) is nonsingular for any \( P > 0 \) in Theorem 5.1.

Other results can be rewritten in the similar ways of Theorems 3.1-3.2 and Corollaries 2.1, 3.1-3.2. By the proposed methods in this paper, the observed-based control with IQC performance and previous results for guaranteed cost control can be developed in the similar way; see [18]-[24].
6. RELIABLE CONTROL FOR UNPERTURBED SWITCHED SYSTEMS WITH IQC PERFORMANCE

Consider the following switched system with interval time-varying delay:

\[
\begin{align*}
\dot{x}(t) &= A_{i}\sigma_{i} x(t) + A_{i}\sigma_{i} x(t - h(t)) + B_{i}\sigma_{i} u(t) + B_{i}\sigma_{i} w(t), \quad t \geq 0, \\
z(t) &= C_{i}\sigma_{i} x(t) + C_{i}\sigma_{i} x(t - h(t)) + D_{i}\sigma_{i} u(t) + D_{i}\sigma_{i} w(t), \quad t \geq 0, \\
x(t) &= \phi(t), \quad t \in [-h_{M}, 0],
\end{align*}
\]

where \( x \in \mathbb{R}^{n}, x_{i} \) is state at time \( t \) defined by \( x_{i}(\theta) := x(t + \theta), \quad \forall \theta \in [-H, 0] \), \( u \in \mathbb{R}^{m} \) is the control input of actuator or sensor fault, \( w \in \mathbb{R}^{i} \) is the disturbance input, \( z \in \mathbb{R}^{v} \) is the regulated output. Switching signal \( \sigma \) may depend on \( t \) or \( x \) and takes its values in the finite set \( \mathcal{N} \). Interval time-varying delay \( h(t) \) satisfies \( 0 \leq h_{m} \leq h(t) \leq h_{M}, \quad \dot{h}(t) \leq h_{d}, \quad h_{m}, \quad h_{d} \) are given. \( A_{i}, A_{i} \in \mathbb{R}^{n \times n}, \quad B_{i} \in \mathbb{R}^{n \times m}, \quad B_{i} \in \mathbb{R}^{m \times d}, \quad C_{i}, C_{i} \in \mathbb{R}^{q \times n}, \quad D_{i} \in \mathbb{R}^{q \times d}, \quad D_{i} \in \mathbb{R}^{d \times i}, \quad i \in \mathcal{N} \), are some given constant matrices. The initial vector \( \phi \) is a differentiable function on \([-h_{M}, 0]\). The fault of control input for actuator (or sensor) is described as follows:

\[
u(t) = Ru(t),
\]

where \( R \) is the actuator fault matrix with

\[
R = \text{diag}
\begin{bmatrix}
    r_{1}, r_{2}, \cdots, r_{m}
\end{bmatrix}, \quad 0 \leq \underline{r}_{i} \leq r_{i} \leq \bar{r}_{i}, \quad \bar{r}_{i} \geq 1, \quad i = 1, 2, \cdots, m
\]

\[
R_{0} = \text{diag}
\begin{bmatrix}
    r_{0}, r_{20}, \cdots, r_{m0}
\end{bmatrix}, \quad \underline{r}_{0} = \frac{\bar{r}_{i} + r_{i}}{2},
\]

\[
R_{i} = \text{diag}
\begin{bmatrix}
    r_{i0}, r_{2i}, \cdots, r_{mi}
\end{bmatrix}, \quad \bar{r}_{i} = \frac{\bar{r}_{i} - \underline{r}_{i}}{2}.
\]

Hence the matrix \( R \) can be rewritten as

\[
R = R_{0} + R_{1} \cdot \Delta J,
\]

where

\[
\Delta J = \text{diag}
\begin{bmatrix}
    j_{1}, j_{2}, \cdots, j_{m}
\end{bmatrix}, \quad -1 \leq j_{i} \leq 1.
\]

Now we define the functions \( \lambda_{i}(t, \sigma), \quad \forall i \in \mathcal{N} \), as follows:

\[
\lambda_{i}(t, \sigma) = \begin{cases} 
1, & \sigma = i, \\
0, & \text{otherwise}, \quad i \in \mathcal{N},
\end{cases}
\]

The state feedback control is proposed by

\[
u(t) = -K_{j}x(t), \quad \text{if} \quad \sigma(t, x(t)) = j,
\]

where \( K_{j} \in \mathbb{R}^{m \times n}, \quad j \in \mathcal{N}, \) are some matrices which will be designed from our developed results. The final state feedback control can be rewritten as follows:
u(t) = -\sum_{j=1}^{N} \lambda_j(t, \sigma) \cdot K_j x(t), \quad t \geq 0. \tag{23c}

The switched system in (21) can be rewritten as follows:

\[
\dot{x}(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \lambda_i(t, \sigma) \cdot \lambda_j(t, \sigma) \cdot \left\{A_{0i} x(t) + A_{1i} x(t-h(t)) - B_{wi} RK_j x(t) + B_{wi} w(t) \right\}
\]

\[
= \sum_{i=1}^{N} \lambda_i(t, \sigma) \cdot \left\{A_{0i} x(t) + A_{1i} x(t-h(t)) - B_{wi} RK_i x(t) + B_{wi} w(t) \right\}, \quad t \geq 0, \tag{23d}
\]

\[
x(t) = \phi(t), \quad t \in [-H, 0], \tag{23e}
\]

\[
z(t) = \sum_{i=1}^{N} \lambda_i(t, \sigma) \cdot \left\{C_{0i} x(t) + C_{1i} x(t-h(t)) - D_{wi} RK_i x(t) + D_{wi} w(t) \right\}, \quad t \geq 0, \tag{23f}
\]

where \( \lambda_i(t, \sigma) \) is defined in (23a) and \( \sum_{i=1}^{N} \lambda_i(t, \sigma) = 1 \), \( \lambda_i^2(t, \sigma) = \lambda_i(t, \sigma) \), and \( \lambda_i(t, \sigma) \cdot \lambda_j(t, \sigma) = 0 \), \( i \neq j \), \( \forall \ t \geq 0 \).

**Definition 6.1.** (See [8]) Consider the system (21), with (22) and \( u(t) = -Kx(t) \); let the following conditions be satisfied:

(iii) With \( w(t) = 0 \), the closed-loop system (21) with (22) and \( u(t) = -Kx(t) \) is globally asymptotically stable.

(iv) With the zero initial condition (i.e. \( \phi = 0 \)), the signals \( w(t) \) and \( z(t) \) are bounded by

\[
\int_0^\ell z(t)^T \cdot \Pi \cdot z(t) \cdot dt = \int_0^\ell z(t)^T \cdot \left[ \begin{array}{c} \Pi_{11} \Pi_{12} \\ \Pi_{21} \Pi_{22} \end{array} \right] \cdot z(t) \cdot w(t) \cdot dt \leq 0, \quad \forall \ w \in L_2(0, \infty), \ w \neq 0, \tag{24}
\]

where \( \ell > 0 \) is a constant, \( \Pi \) is a given matrix with \( \Pi_{11} > 0 \) and \( \Pi_{22} < 0 \). In this condition, the system (21) with (22) is said to satisfy the IQC performance defined by \( \Pi \) and the control law \( u(t) = -Kx(t) \) is said to be a reliable control with IQC performance.

**Remark 6.1:** If we choose \( \Pi_{11} = I, \ \Pi_{12} = 0, \ \Pi_{22} = -\gamma^2 \cdot I \), the condition (24) can be written as

\[
\int_0^\ell z(t)^2 \cdot dt \leq \gamma^2 \cdot \int_0^\ell w(t)^2 \cdot dt.
\]

This yields a standard \( H_\infty \) control problem. The parameter \( \gamma > 0 \) is the \( H_\infty \) norm bound for the reliable \( H_\infty \) state feedback control \( u(t) = -Kx(t) \) (see [5], [14]). Hence the reliable control with IQC performance can be seen as a generalized reliable \( H_\infty \) control.

Now, the reliable state feedback control (23b) will be designed from the following result.

**Theorem 6.1.** Suppose for a constant \( \rho > 0 \), there exist some \( n \times n \) positive-definite symmetric matrices \( \hat{Q}_i, \ i = 0, 1, 2, \ldots, 5 \), \( \hat{P}_{j_{22}}, \ j = 1, 2, 3 \), some \( 4n \times 4n \) positive-definite symmetric matrices \( \hat{P}_{j_{11}}, \ j = 1, 2, 3 \), and some \( 4n \times n \) matrices \( \hat{P}_{j_{12}}, \ j = 1, 2, 3 \), \( \hat{K}_l \in \mathbb{R}^{n \times n} \), constants \( \epsilon_i > 0, \ l = 1, 2, \ldots, N \), such that following LMI conditions are satisfied:
\[ \hat{P}_j = \begin{bmatrix} \hat{P}_{j11} & \hat{P}_{j12} \\ \hat{P}_{j21} & \hat{P}_{j22} \end{bmatrix} > 0, \quad j = 1, 2, 3, \quad \hat{P}_{211} > P_{111}, \quad \hat{P}_{311} > P_{211}, \quad \hat{Q}_4 > \hat{P}_{122}, \quad \dot{Q}_5 > \hat{P}_{222}, \quad Q_3 > \hat{P}_{322}, \quad (25a) \]

\[ \tilde{\Sigma}_i = \begin{bmatrix} \tilde{\Sigma}_{11i} & 0 & \tilde{\Sigma}_{13i} & 0 & \tilde{\Sigma}_{15i} & \tilde{\Sigma}_{17i} & \tilde{\Sigma}_{18i} & \tilde{\Sigma}_{19i} \\ \ast & \tilde{\Sigma}_{22i} & 0 & 0 & 0 & 0 & 0 & 0 \\ \ast & \ast & \tilde{\Sigma}_{33i} & 0 & \tilde{\Sigma}_{35i} & \tilde{\Sigma}_{36i} & \tilde{\Sigma}_{37i} & 0 \\ \ast & \ast & \ast & \tilde{\Sigma}_{44i} & 0 & 0 & 0 & 0 \\ \ast & \ast & \ast & \ast & \tilde{\Sigma}_{55i} & \tilde{\Sigma}_{56i} & 0 & \tilde{\Sigma}_{58i} \\ \ast & \ast & \ast & \ast & \ast & \tilde{\Sigma}_{66i} & \tilde{\Sigma}_{67i} & \tilde{\Sigma}_{68i} \\ \ast & \ast & \ast & \ast & \ast & \ast & \tilde{\Sigma}_{77i} & \tilde{\Sigma}_{78i} \\ \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\varepsilon_i \cdot I \end{bmatrix} + \begin{bmatrix} \tilde{\Omega}_i & 0_{(4\omega \varepsilon M)} \\ \ast & 0_{(5\omega \varepsilon M)} \end{bmatrix} < 0, \quad i = 1, 2, \ldots, N, \quad (25b) \]

where

\[ \tilde{\Sigma}_{11i} = A_{i1i} \hat{Q}_0 + \dot{\hat{Q}}_0 A^T_{i0i} - B_{m1i} R_{1i} \hat{K}_i - \dot{\hat{K}}_i R^T_{i1i} B^T_{i0i} + \dot{\hat{Q}}_1, \quad \tilde{\Sigma}_{13i} = A_{i1i} \hat{Q}_0 + \tilde{\Sigma}_{15i} = \rho \cdot (A_{i0i} \hat{Q}_0 - B_{m1i} R_{1i} \hat{K}_i)^T, \]

\[ \tilde{\Sigma}_{16i} = B_{wi} + C_{i0i} \dot{\hat{Q}}_0 - D_{wi} R_{1i} \hat{K}_i \tilde{\Pi}_{12}, \quad \tilde{\Sigma}_{17i} = \dot{\hat{Q}}_0 C^T_{i0i} - \dot{\hat{K}}_i R^T_{i1i} D^T_{i0i}, \quad \tilde{\Sigma}_{18i} = \varepsilon_i \cdot B_{wi} R_{1i}, \quad \tilde{\Sigma}_{19i} = -\dot{\hat{Q}}_1, \]

\[ \tilde{\Sigma}_{22i} = -\left( \dot{\hat{Q}}_0 - \dot{\hat{Q}}_2 \right), \quad \tilde{\Sigma}_{33i} = \left( 1 - h_i \right) \left( \dot{\hat{Q}}_2 - \dot{\hat{Q}}_3 \right), \quad \tilde{\Sigma}_{35i} = \rho \cdot \dot{\hat{Q}}_0 A^T_{i1i}, \quad \tilde{\Sigma}_{36i} = \dot{\hat{Q}}_0 C^T_{i1i} \tilde{\Pi}_{12}, \quad \tilde{\Sigma}_{37i} = \dot{\hat{Q}}_0 C^T_{i1i}, \quad \tilde{\Sigma}_{44i} = -\dot{\hat{Q}}_3, \]

\[ \tilde{\Sigma}_{55i} = h_m Q_1 + \left( h_M - h_m \right) Q_2 - 2 \rho \cdot Q_0, \quad \tilde{\Sigma}_{56i} = \rho \cdot B_{wi} R_{1i}, \quad \tilde{\Sigma}_{58i} = \varepsilon_i \cdot \rho \cdot B_{wi} R_{1i}, \]

\[ \tilde{\Sigma}_{66i} = \Pi_{22} + \Pi_{12} D_{wi} + D_{wi} \Pi_{12}, \quad \tilde{\Sigma}_{68i} = \varepsilon_i \cdot \Pi_{12} D_{wi} R_{1i}, \quad \tilde{\Sigma}_{77i} = \Pi_{11}^{-1}, \quad \tilde{\Sigma}_{78i} = \varepsilon_i \cdot D_{wi} R_{1i}, \]

\[ \Lambda_i = \begin{bmatrix} I & -I & 0 & 0 \\ -I & I & 0 & 0 \end{bmatrix}, \quad \Lambda_2 = \begin{bmatrix} 0 & I & -I & 0 \end{bmatrix}, \quad \Lambda_3 = \begin{bmatrix} 0 & 0 & I & -I \end{bmatrix}. \]

Then the system (21) with (22) is asymptotically stabilizable via the reliable control in (23b) with \( K_i = \hat{K}_i \hat{Q}_0^{-1} \) with IQC performance.

**Proof.** Define the Lyapunov function as

\[
V(x_i) = x^T(t)Q_0 x(t) + \int_{t-h_u}^{t} x^T(s)Q_1 x(s)ds + \int_{t-h_u}^{t} x^T(s)Q_2 x(s)ds + \int_{t-h_u}^{t-h_M} x^T(s)Q_3 x(s)ds + \int_{t-h_M}^{t-h_u} x^T(s)Q_4 x(s)ds + \int_{t-h_u}^{t-h_M} (s - h_u) x^T(s)Q_5 x(s)ds + \int_{t-h_M}^{t} (s - (t - h_M)) x^T(s)Q_5 x(s)ds + (h_M - h_u) \int_{t-h_u}^{t} x^T(s)Q_5 x(s)ds,
\]

where \( Q_0 = Q_0^{-1} > 0, \quad Q_i = \hat{Q}_0^{-1} \hat{Q}_i \hat{Q}_0^{-1}, \quad i = 1, 2, \ldots, 5 \). The time derivative of \( V(x_i) \) in (26), along the trajectories of the system (21) with (22) and (23), is given by
\[ \dot{V}(x_i) = 2 \sum_{i=1}^{m} \lambda_i(t, \sigma) \cdot x_i^T(t)Q_0\left\{ A_{0i}x(t) + A_{1i}x(t-h(t)) - B_{mi}RK_i x(t) + B_{wi}w(t) \right\} \]
\[ + x^T(t)Q_1x(t) - x^T(t-h_m)(Q_1 - Q_2)x(t-h_m) \]
\[ - \left(1 - h(t) \right) \cdot x^T(t-h(t))(Q_2 - Q_3)x(t-h(t)) - x^T(t-h_M)Q_3x(t-h_M) \]
\[ + h_m \cdot \dot{x}^T(t)Q_4 \dot{x}(t) - \int_{t-h_m}^{t} \dot{x}^T(s)Q_4 \dot{x}(s)ds + (h_M - h_m) \cdot \dot{x}^T(t-h_m)Q_5 \dot{x}(t-h_m) - \int_{t-h_m}^{t} \dot{x}^T(s)Q_5 \dot{x}(s)ds \]
\[ + (h_M - h_m) \cdot \left[ \dot{x}^T(t)Q_3 \dot{x}(t) - \dot{x}^T(t-h_m)Q_3 \dot{x}(t-h_m) \right]. \] (27)

Define

\[ X(t) = \begin{bmatrix} x^T(t) & x^T(t-h_m) & x^T(t-h(t)) & x^T(t-h_M) \end{bmatrix}^T, \]
\[ Y(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) & w^T(t) \end{bmatrix}^T. \]

By Leibniz-Newton formula and LMIs (25a), we have

\[ \int_{t-h_m}^{t} \frac{X(t)}{\dot{x}(s)} ds = \int_{t-h_m}^{t} \frac{X(t)}{\dot{x}(s)} \begin{bmatrix} P_{111} & P_{112} & P_{121} & P_{122} \end{bmatrix} \frac{X(t)}{\dot{x}(s)} ds \]
\[ = h_m \cdot X^T(t)P_{111}X(t) + 2X^T(t)P_{112} \left[ x(t) - x(t-h_m) \right] + \int_{t-h_m}^{t} \dot{x}^T(s)P_{122} \dot{x}(s) ds \geq 0, \] (28a)

\[ \int_{t-h(t)}^{t} \frac{X(t)}{\dot{x}(s)} ds = \int_{t-h(t)}^{t} \frac{X(t)}{\dot{x}(s)} \begin{bmatrix} P_{211} & P_{212} & P_{221} & P_{222} \end{bmatrix} \frac{X(t)}{\dot{x}(s)} ds \]
\[ = [h(t) - h_m] \cdot X^T(t)P_{211}X(t) + 2X^T(t)P_{212} \left[ x(t-h_m) - x(t-h(t)) \right] + \int_{t-h(t)}^{t} \dot{x}^T(s)P_{222} \dot{x}(s) ds \geq 0, \] (28b)

\[ \int_{t-h_M}^{t} \frac{X(t)}{\dot{x}(s)} ds = \int_{t-h_M}^{t} \frac{X(t)}{\dot{x}(s)} \begin{bmatrix} P_{311} & P_{312} & P_{321} & P_{322} \end{bmatrix} \frac{X(t)}{\dot{x}(s)} ds \]
\[ = [h_M - h(t)] \cdot X^T(t)P_{311}X(t) + 2X^T(t)P_{312} \left[ x(t-h(t)) - x(t-h_M) \right] + \int_{t-h_M}^{t} \dot{x}^T(s)P_{322} \dot{x}(s) ds \geq 0, \] (28c)

where \( P_i = \hat{Q}_i \hat{Q}_i^T, \ i = 1, 2, 3, \) with \( \hat{Q} = \text{diag}\left[ \hat{Q}_0^1 \hat{Q}_0^1 \hat{Q}_0^1 \hat{Q}_0^1 \right] \).

From (23d) with \( \rho > 0 \), we have

\[ -2 \rho \cdot \dot{x}^T(t)Q_0 \dot{x}(t) + 2 \rho \cdot \dot{x}^T(t)Q_0 \sum_{i=1}^{m} \lambda_i(t, \sigma) \cdot \left\{ A_{0i}x(t) + A_{1i}x(t-h(t)) - B_{mi}RK_i x(t) + B_{wi}w(t) \right\} = 0. \] (29)

From the system (21) with (22), (23), and (28)-(29), we have

\[ \dot{V}(x_i) + \begin{bmatrix} z(t) \end{bmatrix}^{T} \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12} & \Pi_{22} \end{bmatrix} \begin{bmatrix} z(t) \\ w(t) \end{bmatrix} \]
\[ = \dot{V}(x_i) + 2 \sum_{i=1}^{m} \lambda_i(t, \sigma) \cdot w^T(t) \Pi_{12} \left\{ C_0x(t) + C_1x(t-h(t)) - D_{ni}RK_i x(t) + D_{wi}w(t) \right\} \]
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\[
+ \sum_{i=1}^{m} \lambda_i(t, \sigma) \cdot \{ C_{0i} x(t) + C_{1i} x(t - h(t)) - D_{wi} R_{K} x(t) + D_{wi} w(t) \}^T \Pi_{11} \cdot \{ C_{0i} x(t) + C_{1i} x(t - h(t)) - D_{wi} R_{K} x(t) + D_{wi} w(t) \} + \Pi_{12} w(t)
\]

\[
- 2 \rho \cdot \bar{x}^T(t) Q_0 \bar{x}(t) + 2 \rho \cdot \bar{x}^T(t) Q_0 \sum_{i=1}^{m} \lambda_i(t, \sigma) \cdot \{ A_{0i} x(t) + A_{1i} x(t - h(t)) - B_{wi} R_{K} x(t) + B_{wi} w(t) \}
\]

\[
\leq \sum_{i=1}^{m} \lambda_i(t, \sigma) Y_i(t) Y_i(t) - \int_{t-h_m}^{t} \bar{x}^T(s) Q_4 \bar{x}(s) ds - \int_{t-h_m}^{t-h(t)} \bar{x}^T(s) Q_5 \bar{x}(s) ds
\]

\[
- \int_{t-h_m}^{t} \bar{x}^T(s) (Q_5 - P_{222}) \bar{x}(s) ds \, ,
\]

(30a)

where

\[
\Sigma_i = \begin{bmatrix}
\Sigma_{11i} & 0 & \Sigma_{13i} & 0 & \Sigma_{15i} & \Sigma_{16i} \\
* & \Sigma_{22i} & 0 & 0 & 0 & 0 \\
* & * & \Sigma_{33i} & 0 & \Sigma_{35i} & \Sigma_{36i} \\
* & * & * & \Sigma_{44i} & 0 & 0 \\
* & * & * & * & \Sigma_{55i} & \Sigma_{56i} \\
* & * & * & * & * & \Sigma_{66i}
\end{bmatrix}
+ \begin{bmatrix}
\Omega_i & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
+ \begin{bmatrix}
\Pi_{11}^{-1} & 0 \\
0 & \Pi_{11}^{-1} \Sigma_i T \\
0 & 0
\end{bmatrix}
+ \begin{bmatrix}
C_{0i} - K_i^T R_{K} D_{wi}^T \\
0 & \Pi_{11}^{-1} \Sigma_i T \\
0 & 0
\end{bmatrix},
\]

(30b)

\[
\Sigma_{11i} = Q_0 A_{0i} + A_{0i}^T Q_0 - Q_0 B_{wi} R_{K} - K_i^T R_{K} B_{wi}^T Q_0 + Q_1 , \Sigma_{13i} = Q_0 A_{1i} , \Sigma_{15i} = \rho \cdot (A_{0i} - B_{wi} R_{K})^T Q_0 ,
\]

\[
\Sigma_{16i} = Q_0 B_{wi} + (C_{0i} - D_{wi} R_{K})^T \Pi_{12} , \Sigma_{22i} = -(Q_1 - Q_2) , \Sigma_{33i} = - (1 - h_D) (Q_2 - Q_3) , \Sigma_{35i} = \rho \cdot A_{1i}^T Q_0 ,
\]

\[
\Sigma_{36i} = C_{1i}^T \Pi_{12} + \Sigma_{44i} = - Q_3 , \Sigma_{55i} = h_D Q_4 + (h_D - h_m) Q_5 - 2 \rho \cdot Q_0 , \Sigma_{56i} = \rho \cdot Q_0 B_{wi} ,
\]

\[
\Sigma_{66i} = \Pi_{22} + \Pi_{12} D_{wi} + D_{wi}^T \Pi_{12} , \Omega_i = h_D \cdot P_{311} + P_{112} \Lambda_i + A_{1i}^T P_{112} + P_{212} \Lambda_2 + A_{2i}^T P_{212} + P_{312} \Lambda_3 + A_{3i}^T P_{312} ,
\]

\[
\Lambda_i = [I \ -I \ 0 \ 0] , \Lambda_2 = [0 \ I \ -I \ 0] , \Lambda_3 = [0 \ 0 \ I \ -I] .
\]

Premultiplying and postmultiplying the matrix \( \Sigma_i \) in (30b) by

\[
\text{diag}[Q_0^1, Q_0^1, Q_0^1, Q_0^1, I] > 0
\]

with \( \hat{Q}_0 = Q_0^{-1} \), we can obtain the following matrix with (30b), \( \hat{K}_i = K_i \hat{Q}_0 \), \( \hat{\Omega}_i = \hat{Q}_0 \hat{K}_i \hat{Q}_0 \), \( i = 1, \ldots, 5 \).

\[
\hat{\Sigma}_i = \begin{bmatrix}
\hat{\Sigma}_{11i} & 0 & \hat{\Sigma}_{13i} & 0 & \hat{\Sigma}_{15i} & \hat{\Sigma}_{16i} \\
* & \hat{\Sigma}_{22i} & 0 & 0 & 0 & 0 \\
* & * & \hat{\Sigma}_{33i} & 0 & \hat{\Sigma}_{35i} & \hat{\Sigma}_{36i} \\
* & * & * & \hat{\Sigma}_{44i} & 0 & 0 \\
* & * & * & * & \hat{\Sigma}_{55i} & \hat{\Sigma}_{56i} \\
* & * & * & * & * & \hat{\Sigma}_{66i}
\end{bmatrix}
+ \begin{bmatrix}
\hat{\Omega}_i & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
+ \begin{bmatrix}
\hat{Q}_0 C_{0i}^T - \hat{K}_i^T R_{K} D_{wi}^T \\
0 & \hat{Q}_0 C_{1i}^T \\
0 & 0
\end{bmatrix}
+ \begin{bmatrix}
\hat{Q}_0 C_{0i}^T - \hat{K}_i^T R_{K} D_{wi}^T \\
0 & \hat{Q}_0 C_{1i}^T \\
0 & 0
\end{bmatrix},
\]

(31)
\[ \dot{\Sigma}_{1i} = A_{0i} \dot{Q}_0 + \dot{Q}_0 A_{0i}^T - B_{ui} R \hat{K}_i - \hat{K}_i^T R^T B_{ui}^T + \dot{Q}_i, \quad \dot{\Sigma}_{13i} = \ddot{\Sigma}_{13i}, \quad \dot{\Sigma}_{15i} = \rho \cdot (A_{0i} \dot{Q}_0 - B_{ui} R \hat{K}_i)^T, \]
\[ \dot{\Sigma}_{16i} = B_{wi} + (C_{0i} \dot{Q}_0 - D_{wi} R \hat{K}_i) \Pi_{12}, \quad \dot{\Sigma}_{22i} = \ddot{\Sigma}_{22i}, \quad \dot{\Sigma}_{33i} = \ddot{\Sigma}_{33i}, \quad \dot{\Sigma}_{35i} = \ddot{\Sigma}_{35i}, \quad \dot{\Sigma}_{37i} = \ddot{\Sigma}_{37i}, \]
\[ \dot{\Sigma}_{55i} = \ddot{\Sigma}_{55i}, \quad \dot{\Sigma}_{56i} = \ddot{\Sigma}_{56i}, \quad \dot{\Sigma}_{66i} = \ddot{\Sigma}_{66i}, \quad \dot{\Omega}_i = \ddot{\Omega}_i. \]

Define the matrix
\[
\Sigma_i = \begin{bmatrix}
\dot{\Sigma}_{1i} & 0 & \dot{\Sigma}_{13i} & 0 & \dot{\Sigma}_{15i} & \dot{\Sigma}_{16i} & \dot{\Sigma}_{17i} \\
* & \dot{\Sigma}_{22i} & 0 & 0 & 0 & 0 & 0 \\
* & * & \dot{\Sigma}_{33i} & 0 & \dot{\Sigma}_{35i} & 0 & \dot{\Sigma}_{37i} \\
* & * & * & \dot{\Sigma}_{44i} & 0 & 0 & 0 \\
* & * & * & * & \dot{\Sigma}_{55i} & \dot{\Sigma}_{56i} & 0 \\
* & * & * & * & * & \dot{\Sigma}_{66i} & \dot{\Sigma}_{67i} \\
* & * & * & * & * & * & \dot{\Sigma}_{77i}
\end{bmatrix} + \begin{bmatrix}
\dot{\Omega}_i & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\ddot{\Sigma}_{1i} & 0 & \ddot{\Sigma}_{13i} & 0 & \ddot{\Sigma}_{15i} & \ddot{\Sigma}_{16i} & \ddot{\Sigma}_{17i} \\
* & \ddot{\Sigma}_{22i} & 0 & 0 & 0 & 0 & 0 \\
* & * & \ddot{\Sigma}_{33i} & 0 & \ddot{\Sigma}_{35i} & 0 & \ddot{\Sigma}_{37i} \\
* & * & * & \ddot{\Sigma}_{44i} & 0 & 0 & 0 \\
* & * & * & * & \ddot{\Sigma}_{55i} & \ddot{\Sigma}_{56i} & 0 \\
* & * & * & * & * & \ddot{\Sigma}_{66i} & \ddot{\Sigma}_{67i} \\
* & * & * & * & * & * & \ddot{\Sigma}_{77i}
\end{bmatrix} + \begin{bmatrix}
\ddot{\Omega}_i & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
B_{wi} R_{1i} & -\hat{K}_i^T & -\hat{K}_i^T & B_{wi} R_{1i} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\Delta J & 0 & 0 & \Delta J^T \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} + \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
B_{wi} R_{1i} & -\hat{K}_i^T & -\hat{K}_i^T & B_{wi} R_{1i} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\Delta J & 0 & 0 & \Delta J^T \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} + \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
B_{wi} R_{1i}^T \\
0 \\
0 \\
0 \\
0 \\
\rho \cdot B_{wi} R_{1i} \\
\Pi_{12} D_{wi} R_{1i} \\
D_{wi} R_{1i}
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
-\hat{K}_i^T \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
= \begin{bmatrix}
B_{wi} R_{1i}^T \\
0 \\
0 \\
0 \\
0 \\
\rho \cdot B_{wi} R_{1i} \\
\Pi_{12} D_{wi} R_{1i} \\
D_{wi} R_{1i}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
B_{wi} R_{1i}^T \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
\rho \cdot B_{wi} R_{1i} \\
\Pi_{12} D_{wi} R_{1i} \\
D_{wi} R_{1i}
\]

where
\[
\dot{\Sigma}_{17i} = \hat{Q}_0 C_{0i}^T - \hat{K}_i^T R_{1i}^T D_{wi}^T, \quad \dot{\Sigma}_{37i} = \hat{Q}_0 C_{1i}^T, \quad \dot{\Sigma}_{67i} = D_{wi}^T, \quad \dot{\Sigma}_{77i} = -\Pi_{11}^{-1}.
\]

By Lemmas 2.1-2.2, we get that the LMI condition \( \Sigma < 0 \) in (25b) is equivalent to \( \Sigma < 0 \) in (32). By Lemma 2.2, \( \Sigma < 0 \) is equivalent to \( \hat{\Sigma} < 0 \) in (31). Condition \( \hat{\Sigma} < 0 \) is also equivalent to \( \Sigma < 0 \) in (30b).

From (28), (29), (30a), and \( \Sigma < 0 \) with \( w(t) = 0 \), there exists a constant \( \lambda > 0 \) satisfying
\[ V(x(t))_{w(t)=0} + z^T(t)\Pi_{i1} z(t) \leq -\lambda \cdot \|x(t)\|^2. \]

With \( \Pi_{i1} > 0 \), we obtain the following condition:

\[ V(x(t))_{w(t)=0} \leq -\lambda \cdot \|x(t)\|^2. \]

Hence, the closed system (21)-(22) with (23b) and \( w(t) = 0 \) is asymptotically stable (see [16]-[17]).

Integrating the function in (30a) from 0 to \( \ell > 0 \) and from \( \Sigma < 0 \), we have

\[ V(x(\ell)) - V(x_0) + \int_0^\ell \left[ z(t)^T \Pi_{i1} z(t) \right] dt \leq 0. \]

With the zero initial condition \( (x_0 = 0) \), we have

\[ V(x_0) = 0, \quad V(x(\ell)) \geq 0, \]

and

\[ \int_0^\ell \left[ z(t)^T \Pi_{i1} z(t) \right] dt \leq 0, \quad \forall w \in L_2[0, \infty), \ w \neq 0. \]

By the Definition 2.1, the system (21)-(22) with (23b) satisfies IQC performance.

7. RELIABLE ROBUST CONTROL FOR UNCERTAIN SWITCHED SYSTEMS WITH IQC PERFORMANCE

Consider the following uncertain switched system with time-varying delay:

\begin{align*}
\dot{x}(t) &= A_{0i}(t)x(t) + A_{1i}(t)x(t-h(t)) + B_{ui}(t)u(t) + B_{wi}(t)w(t), \quad t \geq 0, \\
z(t) &= C_{0i}(t)x(t) + C_{1i}(t)x(t-h(t)) + D_{ui}(t)u(t) + D_{wi}(t)w(t), \quad t \geq 0, \\
x(t) &= \phi(t), \quad t \in [-h_M, 0],
\end{align*}

where

\[ A_{0i}(t) = A_{0i} + \Delta A_{0i}(t), \quad A_{1i}(t) = A_{1i} + \Delta A_{1i}(t), \quad B_{ui}(t) = B_{ui} + \Delta B_{ui}(t), \quad B_{wi}(t) = B_{wi} + \Delta B_{wi}(t), \]

\[ C_{0i}(t) = C_{0i} + \Delta C_{0i}(t), \quad C_{1i}(t) = C_{1i} + \Delta C_{1i}(t), \quad D_{ui}(t) = D_{ui} + \Delta D_{ui}(t), \quad D_{wi}(t) = D_{wi} + \Delta D_{wi}(t), \]

are some given constant matrices, \( \Delta A_{0i}(t), \Delta A_{1i}(t), \Delta B_{ui}(t), \Delta B_{wi}(t), \Delta C_{0i}(t), \Delta C_{1i}(t), \Delta D_{ui}(t), \Delta D_{wi}(t) \), are some time-varying functions satisfying

\[ \begin{bmatrix} \Delta A_{0i}(t) & \Delta A_{1i}(t) & \Delta B_{ui}(t) & \Delta B_{wi}(t) \\ \Delta C_{0i}(t) & \Delta C_{1i}(t) & \Delta D_{ui}(t) & \Delta D_{wi}(t) \end{bmatrix} = \begin{bmatrix} M_{xi} & M_{zi} \\ \end{bmatrix} \cdot \Delta F_i(t) \cdot \begin{bmatrix} N_{0i} & N_{1i} & N_{ui} & N_{wi} \end{bmatrix}, \]

\[ M_{xi}, M_{zi}, N_{0i}, N_{1i}, N_{ui}, N_{wi}, i = 1, 2, \cdots, N, \] are some given constant matrices, \( \Delta F_i(t), i = 1, 2, \cdots, N, \) are real time-varying functions with appropriate dimensions and bounded as follows:

\[ F_i^T(t) \cdot F_i(t) \leq I, \quad \forall t \geq 0. \]

With the result of Theorem 6.1 and by comparing system (21) and system (33), a result to design the robust reliable control (23b) with IQC performance for system (33) is presented.
**Theorem 7.1.** Suppose for a constant \( \rho > 0 \), there exist some \( n \times n \) positive-definite symmetric matrices \( \hat{Q}_i, i = 0, 1, 2, \cdots, 5 \), some \( 4 \times 4 \) positive-definite symmetric matrices \( \hat{P}_{j11}, j = 1, 2, 3 \), and some \( 4 \times n \) matrices \( \hat{P}_{j12}, j = 1, 2, 3 \), \( \hat{K}_i \in \mathbb{R}^{m_{\text{out}}} \), positive constants \( \varepsilon_i \) and \( \mu_i, l = 1, 2, \cdots, N \), such that the following LMI conditions are satisfied:

\[
\hat{P}_j = \begin{bmatrix}
\hat{P}_{j11} & \hat{P}_{j12} \\
* & \hat{P}_{j22}
\end{bmatrix} > 0, \quad j = 1, 2, 3,
\hat{P}_{211} > P_{111}, \quad \hat{P}_{311} > P_{211}, \quad \hat{Q}_4 > \hat{P}_{122}, \quad \hat{Q}_5 > \hat{P}_{222}, \quad \hat{Q}_5 > \hat{P}_{322},
\]

\[
\left[ \begin{array}{cc}
\bar{\Sigma}_i & \Lambda_{li} \\
* & \Lambda_{2i}
\end{array} \right] < 0, \quad i = 1, 2, \cdots, N,
\]

where \( \bar{\Sigma}_i \) is defined in (5b) and

\[
\Lambda_{li} = \begin{bmatrix}
M_{wi}^T & 0 & 0 & \rho \cdot M_{wi}^T & M_{wi}^T \Pi_{12} & M_{wi}^T R_{11} & M_{wi}^T R_{10} & 0
\end{bmatrix}^T,
\Lambda_{2i} = \begin{bmatrix}
-\mu_i \cdot I & 0 \\
0 & -\mu_i \cdot I
\end{bmatrix},
\]

Then the system (33) with (22) is asymptotically stabilizable via the reliable control in (23b) with \( K_i = \hat{K}_i \hat{Q}_i^{-1} \) with IQC performance.

**Proof:** This proof can be done by the similar derivation of Theorem 6.1.

**8. \textit{H}_\infty \textit{ CONTROL FOR UNCERTAIN SWITCHED SYSTEMS }**

In Remark 6.1 with \( \Pi_{11} = I, \Pi_{12} = 0, \Pi_{22} = -\gamma^2 I \), the reliable \( \textit{H}_\infty \) control can be obtained from Theorem 3.1.

**Theorem 8.1.** Suppose for a constant \( \rho > 0 \), the following LMI optimization problem:

\[
\text{Minimize } \bar{\gamma},
\]

Subject to

\[
\hat{P}_j = \begin{bmatrix}
\hat{P}_{j11} & \hat{P}_{j12} \\
* & \hat{P}_{j22}
\end{bmatrix} > 0, \quad j = 1, 2, 3,
\hat{P}_{211} > P_{111}, \quad \hat{P}_{311} > P_{211}, \quad \hat{Q}_4 > \hat{P}_{122}, \quad \hat{Q}_5 > \hat{P}_{222}, \quad \hat{Q}_5 > \hat{P}_{322},
\]

\[
\left[ \begin{array}{cc}
\bar{\Sigma}_i & \bar{\Lambda}_{li} \\
* & \bar{\Lambda}_{2i}
\end{array} \right] < 0, \quad i = 1, 2, \cdots, N,
\]

has a feasible solution for \( n \times n \) positive-definite symmetric matrices \( \hat{Q}_i, i = 0, 1, 2, \cdots, 5 \), \( \hat{P}_{j12}, j = 1, 2, 3 \), some \( 4 \times 4 \) positive-definite symmetric matrices \( \hat{P}_{j11}, j = 1, 2, 3 \), and \( 4 \times n \) matrices \( \hat{P}_{j12}, j = 1, 2, 3 \), \( \hat{K}_i \in \mathbb{R}^{m_{\text{out}}} \), \( l = 1, 2, \cdots, N \), positive constants \( \varepsilon_i \) and \( \mu_i \), where

\[
\bar{\Lambda}_{li} = \begin{bmatrix}
M_{wi}^T & 0 & 0 & \rho \cdot M_{wi}^T & M_{wi}^T \Pi_{12} & M_{wi}^T R_{11} & M_{wi}^T R_{10} & 0
\end{bmatrix}^T,
\bar{\Lambda}_{2i} = \begin{bmatrix}
-\mu_i \cdot I & 0 \\
0 & -\mu_i \cdot I
\end{bmatrix},
\]
where \( \tilde{\Sigma}_{kl} \), \( k, l = 1, 2, 3, \ldots, 9 \) are defined in (25b). Then the system (33) with (22) is asymptotically stabilizable via the reliable \( H_n \) control in (23b) with \( K = \hat{K} \hat{Q}^{-1} \) and the disturbance attenuation \( \gamma = \sqrt{\tilde{\gamma}} \).

9. RELIABLE GUARANTEED COST CONTROL FOR UNPERTURBED SWITCHED SYSTEMS

Consider the following switched system with interval time-varying delay:

\[
\dot{x}(t) = A_{0\sigma} x(t) + A_{1\sigma} x(t-h(t)) + B_{u\sigma} u_f(t), \quad t \geq 0, \tag{36a}
\]

\[
x(t) = \phi(t), \quad t \in [-h_M, 0], \tag{36b}
\]

where \( x \in \mathbb{R}^n \) is state at time \( t \) defined by \( x(t) := x(t+\theta), \) \( \forall \theta \in [-H, 0], u_f \in \mathbb{R}^m \) is the control input of actuator or sensor fault. Switching signal \( \sigma \) may depend on \( t \) or \( x \) and takes its values in the finite set \( N \).

Interval time-varying delay \( h(t) \) satisfies \( 0 \leq h_m \leq h(t) \leq h_M, \) \( h_M \) are given. \( A_{0i}, A_{1i} \in \mathbb{R}^{nxn}, B_{ui} \in \mathbb{R}^{nxm}, \) \( i \in N, \) are some given constant matrices. The initial vector \( \phi \) is a differentiable function on \( [-h_M, 0] \).

Define the cost function of system (36) with (22) as follows:

\[
J = \int_0^\ell \left[ x^T(s) \cdot S_1 \cdot x(s) + u_f^T(s) \cdot S_2 \cdot u_f(s) \right] ds, \tag{37}
\]

where constant \( \ell > 0, S_1 \in \mathbb{R}^{nxn} \) and \( S_2 \in \mathbb{R}^{nxn} \) are two given positive definite symmetric matrices. We wish to design a feedback control in (23b)-(23c) and find a positive constant \( J' \), such that the system (36) with (22) is asymptotically stable and \( J \leq J' \), where \( J' \) is the guaranteed cost for this feedback control in (23b)-(23c) of system (36) with (22).

The switched system in (36) can be rewritten as follows:

\[
\dot{x}(t) = \sum_{i=1}^N \lambda_i(t, \sigma) \cdot \{ A_{0i} x(t) + A_{1i} x(t-h(t)) - B_{ui} R K_i x(t) \}, \quad t \geq 0, \tag{38a}
\]

\[
x(t) = \phi(t), \quad t \in [-H, 0], \tag{38b}
\]
Now the reliable state feedback control (23b) will be designed from the following result.

**Theorem 9.1.** Suppose for a constant \( p > 0 \), there exist some \( n \times n \) positive-definite symmetric matrices \( \hat{Q}_i, i = 0, 1, 2, \ldots, 5, \hat{P}_{ij}, j = 1, 2, 3, \) some \( 4n \times 4n \) positive-definite symmetric matrices \( \hat{P}_{ij}, j = 1, 2, 3, \) and some \( 4n \times n \) matrices \( \hat{P}_{ij}, j = 1, 2, 3, \hat{K}_i \in \mathbb{R}^{m_m}, \) constants \( \varepsilon_i > 0, i = 1, 2, \ldots, N, \) such that following LMI conditions are satisfied:

\[
\hat{P}_j = \begin{bmatrix}
\hat{P}_{j11} & \hat{P}_{j12} \\
\ast & \hat{P}_{j22}
\end{bmatrix} > 0, \quad j = 1, 2, 3, \quad \hat{P}_{211} > P_{111}, \quad \hat{P}_{311} > P_{211}, \quad \hat{Q}_4 > \hat{P}_{122}, \quad \hat{Q}_5 > \hat{P}_{222}, \quad \hat{Q}_5 > \hat{P}_{322}, \quad (39a)
\]

\[
\hat{\Sigma}_j = \begin{bmatrix}
\hat{\Sigma}_{11i} & \hat{\Sigma}_{13i} & 0 & \hat{\Sigma}_{15i} & \hat{\Sigma}_{16i} & \hat{\Sigma}_{17i} & \hat{\Sigma}_{18i} & \hat{\Sigma}_{19i} \\
\ast & \hat{\Sigma}_{22i} & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \hat{\Sigma}_{33i} & 0 & \hat{\Sigma}_{35i} & 0 & 0 & 0 \\
\ast & \ast & \ast & \hat{\Sigma}_{44i} & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \hat{\Sigma}_{55i} & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \hat{\Sigma}_{66i} & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \hat{\Sigma}_{77i} & \hat{\Sigma}_{78i} & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\varepsilon_i \cdot I \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\varepsilon_i \cdot I
\end{bmatrix} + \begin{bmatrix}
\tilde{\Omega}_i \\
\ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \ast \a
with \( Q_0 = \hat{Q}_0^{-1} \), \( Q_i = \hat{Q}_0^{-1} \hat{Q}_i \hat{Q}_0^{-1} \).

**Proof.** Define the Lyapunov function as

\[
V(x_t) = x^T(t) Q_0 x(t) + \int_{t-h_0}^{t} x^T(s) Q_1 x(s) ds + \int_{t-h(t)}^{t-h_0} x^T(s) Q_2 x(s) ds + \int_{t-h(t)}^{t-h(t)} x^T(s) Q_3 x(s) ds
\]

\[
+ \int_{t-h_0}^{t} (s - (t - h_m) x^T(s) Q_4 x(s) ds + \int_{t-h_0}^{t} (s - (t - h_M)) x^T(s) Q_5 x(s) ds + (h_M - h_m) \int_{t-h_0}^{t} x^T(s) Q_6 x(s) ds \right),
\]

where \( Q_0 = Q_0^{-1} > 0 \), \( Q_i = \hat{Q}_0^{-1} \hat{Q}_i \hat{Q}_0^{-1} \), \( i = 1, 2, \cdots, 5 \). The time derivative of \( V(x_t) \) in (40), along the trajectories of the system (36) with (22) and (23b)-(23c), is given by

\[
\dot{V}(x_t) = 2 \sum_{i=1}^{m} \lambda_i(t, \sigma) \cdot x^T(t) Q_0 \{ A_{0i} x(t) + A_{1i} x(t - h(t)) - B_{ai} RK, \dot{x}(t) \}
\]

\[
+ x^T(t) Q_1 x(t) - x^T(t - h_m)(Q_1 - Q_2)x(t - h_m)
\]

\[
- (1 - \hat{h}(t)) x^T(t - h(t)) (Q_2 - Q_3)x(t - h(t)) - x^T(t - h_M) Q_3 x(t - h_M)
\]

\[
+ h_m \cdot x^T(t) Q_4 \dot{x}(t) - \int_{t-h_0}^{t} \dot{x}^T(s) Q_4 \dot{x}(s) ds + (h_M - h_m) \cdot x^T(t - h_M) Q_5 \dot{x}(t - h_M) - \int_{t-h_0}^{t} \dot{x}^T(s) Q_6 \dot{x}(s) ds
\]

\[
+ (h_M - h_m) \left[ x^T(t) Q_6 \dot{x}(t) - x^T(t - h_m) Q_5 \dot{x}(t - h_m) \right].
\]

Define

\[
X(t) = \begin{bmatrix} x^T(t) & x^T(t - h_m) & x^T(t - h(t)) & x^T(t - h_M) \end{bmatrix},
\]

\[
Y(t) = \begin{bmatrix} X^T(t) & \dot{x}^T(t) \end{bmatrix}.
\]

By Leibniz-Newton formula and LMIs (39a), we have

\[
\int_{t-h_0}^{t} \left[ X(t) \right]^T P_{1} \left[ X(t) \right] ds = \int_{t-h_0}^{t} \left[ \dot{x}(s) \right]^T \left[ \begin{array}{c} P_{111} \ P_{112} \\ P_{112}^T \ X(t) \end{array} \right] ds
\]

\[
= h_m \cdot X^T(t) P_{211} X(t) + 2 X^T(t) P_{121} \left[ x(t) - x(t - h_m) \right] + \int_{t-h_0}^{t} \dot{x}^T(s) P_{222} \dot{x}(s) ds \geq 0,
\]

(41a)

\[
\int_{t-h_0}^{t} \left[ X(t) \right]^T P_{2} \left[ X(t) \right] ds = \int_{t-h_0}^{t} \left[ \dot{x}(s) \right]^T \left[ \begin{array}{c} P_{21} \ P_{212} \\ P_{212}^T \ X(t) \end{array} \right] ds
\]

\[
= [h(t) - h_m] \cdot X^T(t) P_{211} X(t) + 2 X^T(t) P_{212} \left[ x(t - h_m) - x(t - h(t)) \right] + \int_{t-h_0}^{t} \dot{x}^T(s) P_{222} \dot{x}(s) ds \geq 0,
\]

(41b)

\[
\int_{t-h_0}^{t} \left[ X(t) \right]^T P_{3} \left[ X(t) \right] ds = \int_{t-h_0}^{t} \left[ \dot{x}(s) \right]^T \left[ \begin{array}{c} P_{31} \ P_{312} \\ P_{312}^T \ X(t) \end{array} \right] ds
\]

\[
= [h_M - h(t)] \cdot X^T(t) P_{311} X(t) + 2 X^T(t) P_{312} \left[ x(t - h(t)) - x(t - h_m) \right] + \int_{t-h_0}^{t} \dot{x}^T(s) P_{322} \dot{x}(s) ds \geq 0,
\]

(41c)

where \( P_i = \hat{Q}_0 \hat{P}_i \hat{Q}_0 \), \( i = 1, 2, 3 \), with \( \hat{Q} = \text{diag} \left[ \hat{Q}_0^{-1} \hat{Q}_0^{-1} \hat{Q}_0^{-1} \right] \). From (38a) with \( \rho > 0 \), we have

\[
-2 \rho \cdot \dot{x}^T(t) Q_0 \dot{x}(t) + 2 \rho \cdot \dot{x}^T(t) Q_0 \sum_{i=1}^{m} \lambda_i(t, \sigma) \left[ A_{0i} x(t) + A_{1i} x(t - h(t)) - B_{ai} RK, \dot{x}(t) \right] = 0.
\]

(41d)
From (23c) with $\rho > 0$, we have

$$u'_f(t) \cdot S_2 \cdot u_f(t) = \sum_{i=0}^{m} \sum_{j=0}^{m} \lambda_i(t, \sigma) \lambda_j(t, \sigma) \cdot x^T(t) K_i^T R^T S_2 R K_j x(t) = \sum_{i=1}^{m} \lambda_i(t, \sigma) \cdot x^T(t) K_i^T R^T S_2 R K_j x(t). \quad (41e)$$

From the system (21) with (22), (23), and (41), we have

$$\dot{V}(x_i) + x^T(t) \cdot S_i \cdot x(t) + u'_f(t) \cdot S_2 \cdot u_f(t)$$

$$\leq \sum_{i=1}^{m} \lambda_i(t, \sigma) Y^T(t) \Sigma_i Y(t) - \int_{t-h_m}^{t} x^T(s)(Q_4 - P_{122}) \dot{x}(s) ds - \int_{t-h_m}^{t-h(t)} x^T(s)(Q_5 - P_{222}) \dot{x}(s) ds$$

$$- \int_{t-h_m}^{t-h(t)} x^T(s)(Q_5 - P_{222}) \dot{x}(s) ds,$$  \quad (42a)

where

$$\Sigma_i = \begin{bmatrix} \Sigma_{1i} + S_1 + K_i^T R^T S_2 R K_i & 0 & \Sigma_{1i} & 0 & \Sigma_{1i} \\ 0 & \Sigma_{22i} & 0 & 0 & 0 \\ \Sigma_{33i} & 0 & \Sigma_{33i} & 0 & \Sigma_{33i} \\ 0 & 0 & 0 & \Sigma_{44i} & 0 \\ \Sigma_{55i} \end{bmatrix} + \begin{bmatrix} \Omega_i & 0 \\ 0 & 0 \end{bmatrix}, \quad (42b)$$

$$\Sigma_{1i} = Q_0 A_{0i} + A_{0i}^T Q_0 - Q_0 B_{mi} R K_i - K_i^T R^T B_{mi}^T Q_0 + Q_i, \quad \Sigma_{13i} = Q_0 A_{0i},$$

$$\Sigma_{15i} = \rho \cdot (A_{0i} - B_{mi} R K_i)^T Q_0, \quad \Sigma_{22i} = -(Q_4 - P_{122}), \quad \Sigma_{33i} = -(1 - h_D)(Q_4 - P_{122}), \quad \Sigma_{35i} = \rho \cdot A_{0i}^T Q_0, \quad \Sigma_{44i} = -Q_5,$$

$$\Sigma_{55i} = h_m Q_4 + (h_m - h_m) \cdot Q_5 - 2 \rho \cdot Q_0, \quad \Omega_i = h_m \cdot P_{311} + P_{112} L_1 + \Lambda_1^T P_{112} + P_{212} L_2 + \Lambda_2^T P_{212} + P_{312} L_3 + \Lambda_3^T P_{312},$$

$$\Lambda_1 = \begin{bmatrix} I & -I & 0 & 0 \end{bmatrix}, \quad \Lambda_2 = \begin{bmatrix} 0 & I & -I & 0 \end{bmatrix}, \quad \Lambda_3 = \begin{bmatrix} 0 & 0 & I & -I \end{bmatrix}.$$

Premultiplying and postmultiplying the matrix $\Sigma$ in (42b) by

$$\text{diag}\begin{bmatrix} Q_0 & Q_0 & Q_0 & Q_0 & Q_0 \end{bmatrix} > 0$$

with $\hat{Q}_0 = Q_0^{-1}$, we can obtain the following matrix with $\hat{K}_i = K_i \hat{Q}_0$, $\hat{Q}_0 = \hat{Q}_0 Q_0 \hat{Q}_0$, $i = 1, \ldots, 5$,

$$\hat{\Sigma}_i = \begin{bmatrix} \hat{\Sigma}_{11i} & 0 & \hat{\Sigma}_{13i} & 0 & \hat{\Sigma}_{15i} \\ 0 & \hat{\Sigma}_{22i} & 0 & 0 & 0 \\ \hat{\Sigma}_{33i} & 0 & \hat{\Sigma}_{33i} & 0 & \hat{\Sigma}_{33i} \\ 0 & 0 & 0 & \hat{\Sigma}_{44i} & 0 \\ \hat{\Sigma}_{55i} \end{bmatrix} + \begin{bmatrix} \hat{Q}_0 & 0 & \hat{Q}_0 & 0 & \hat{Q}_0 \end{bmatrix} + \begin{bmatrix} \hat{Q}_0 & 0 & \hat{Q}_0 & 0 & \hat{Q}_0 \end{bmatrix} + \begin{bmatrix} \hat{Q}_0 & 0 & \hat{Q}_0 & 0 & \hat{Q}_0 \end{bmatrix}$$

$$= \begin{bmatrix} A_{0i} \hat{Q}_0 + \hat{Q}_0 A_{0i}^T - B_{mi} \hat{K}_i - \hat{K}_i^T R^T B_{mi}^T + \hat{Q}_0, \hat{\Sigma}_{13i} = \hat{\Sigma}_{13i}, \hat{\Sigma}_{15i} = \rho \cdot (A_{0i} \hat{Q}_0 - B_{mi} R K_i), \hat{\Sigma}_{22i} = \hat{\Sigma}_{22i}, \hat{\Sigma}_{33i} = \hat{\Sigma}_{33i}, \hat{\Sigma}_{55i} = \hat{\Sigma}_{55i}, \hat{\Sigma}_{44i} = \hat{\Sigma}_{44i} \end{bmatrix}.$$

Define the matrix
\[
\tilde{\Sigma} = \begin{bmatrix}
\hat{\Sigma}_{1i} & 0 & \hat{\Sigma}_{13i} & 0 & \hat{\Sigma}_{15i} & \hat{\Sigma}_{16i} & \hat{\Sigma}_{17i} \\
* & \hat{\Sigma}_{22i} & 0 & 0 & 0 & 0 & 0 \\
* & * & \hat{\Sigma}_{33i} & 0 & \hat{\Sigma}_{35i} & 0 & 0 \\
* & * & * & \hat{\Sigma}_{44i} & 0 & 0 & 0 \\
* & * & * & * & \hat{\Sigma}_{55i} & 0 & 0 \\
* & * & * & * & * & \hat{\Sigma}_{66i} & 0 \\
* & * & * & * & * & * & \hat{\Sigma}_{77i}
\end{bmatrix}
\]

\[
\quad + \begin{bmatrix}
\hat{\omega} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
B_{ii}R_1 \\
0 \\
0 \\
\Delta J \\
\rho \cdot B_{ii}R_1 \\
0 \\
R_1
\end{bmatrix}
- \hat{K}_i^T \tilde{\Sigma} \hat{K}_i^T
- \hat{K}_i^T \\
\Delta J^T
\rho \cdot B_{ii}R_1 \\
0 \\
R_1
\end{bmatrix}
\]

\[
\text{where}\]

\[
\hat{\Sigma}_{16i} = \hat{\omega}_0, \quad \hat{\Sigma}_{17i} = -\hat{K}_i^T R_i, \quad \hat{\Sigma}_{66i} = -S_1^{-1}, \quad \hat{\Sigma}_{77i} = -S_2^{-1}.
\]

By Lemmas 2.1-2.2, we get that the LMI condition \( \hat{\Sigma} < 0 \) in (39b) is equivalent to \( \Sigma < 0 \) in (44). By Lemma 2.2, \( \Sigma < 0 \) is equivalent to \( \hat{\Sigma} < 0 \) in (43). Condition \( \hat{\Sigma} < 0 \) is also equivalent to \( \Sigma < 0 \) in (42b). From (39a), (42a), and \( \Sigma < 0 \), there exists a constant \( \lambda > 0 \) satisfying

\[
\dot{V}(x(t)) + x^T(t) \cdot S_1 \cdot x(t) + u_f^T(t) \cdot S_2 \cdot u_f(t) \leq -\lambda \cdot \|x(t)\|^2.
\]

With \( S_1, S_2 > 0 \), we obtain the following condition:

\[
\dot{V}(x(t)) \leq -\lambda \cdot \|x(t)\|^2.
\]
Hence, the closed system (36), (22) with (23b) is asymptotically stable (see [16]-[17]). Integrating the function in (30a) from 0 to $\ell > 0$ and from $\Sigma < 0$, we have

$$V(x(\ell)) - V(x_0) + \int_0^\ell x^T(t) \cdot S_1 \cdot x(t) + u_j^T(t) \cdot S_2 \cdot u_j(t) dt \leq 0,$$

where $J^*$ is defined in (39c).

10. RELIABLE GUARANTEED COST CONTROL FOR UNCERTAIN SWITCHED SYSTEMS

Consider the following uncertain switched system with interval time-varying delay:

$$\dot{x}(t) = A_{0\sigma}(t)x(t) + A_{1\sigma}(t)x(t - h(t)) + B_{u\sigma}(t)u_j(t), \quad t \geq 0,$$

where

$$A_{0j}(t) = A_{0i} + \Delta A_{0j}(t), \quad A_{1j}(t) = A_{1i} + \Delta A_{1j}(t), \quad B_{uj}(t) = B_{ui} + \Delta B_{uj}(t),$$

$A_{0i}, A_{1i}, B_{ui}$ are some given constant matrices, $\Delta A_{0j}(t), \Delta A_{1j}(t), \Delta B_{uj}(t)$ are some time-varying functions satisfying

$$[\Delta A_{0j}(t) \quad \Delta A_{1j}(t) \quad \Delta B_{uj}(t)] = M_i \cdot \Delta F_i(t) \cdot [N_{0i} \quad N_{1i} \quad N_{ui}],$$

$M_i, N_{0i}, N_{1i}, N_{ui}, i = 1, 2, ..., N,$ are some given constant matrices, $\Delta F_i(t), i = 1, 2, ..., N,$ are real time-varying functions with appropriate dimensions and bounded as follows:

$$F_i^T(t) \cdot F_i(t) \leq I, \quad \forall t \geq 0.$$

With the result of Theorem 9.1 and by comparing system (33) and system (45), a result to design the robust reliable guaranteed cost control (23b) for system (25) is presented.

**Theorem 10.1.** Suppose for a constant $\rho > 0$, there exist some $n \times n$ positive-definite symmetric matrices $\hat{Q}_i, i = 0, 1, 2, \cdots, 5,$ $\hat{P}_{j22}, j = 1, 2, 3,$ some $4n \times 4n$ positive-definite symmetric matrices $\hat{P}_{j11}, j = 1, 2, 3,$ and some $4n \times n$ matrices $\hat{P}_{j12}, j = 1, 2, 3, \hat{K}_i \in \mathbb{R}^{n \times n}$, constants, $\varepsilon_i > 0, \mu_i > 0, l = 1, 2, \cdots, N$, such that following LMI conditions are satisfied:

$$\hat{P}_j = \begin{bmatrix} \hat{P}_{j11} & \hat{P}_{j12} \\ \ast & \hat{P}_{j22} \end{bmatrix} > 0, \quad j = 1, 2, 3, \quad \hat{P}_{211} > P_{111}, \quad \hat{P}_{311} > P_{211}, \quad \hat{Q}_4 > \hat{P}_{122}, \quad \hat{Q}_5 > \hat{P}_{222}, \quad \hat{Q}_3 > \hat{P}_{322},$$

$$\begin{bmatrix} \hat{\Sigma}_i & \Lambda_{li} \\ \ast & \Lambda_{2i} \end{bmatrix} < 0, \quad i = 1, 2, \cdots, N,$$

where $\hat{\Sigma}_i$ is defined in (39b) and

$$\Lambda_{li} = \begin{bmatrix} M_i^T \\ N_{0i} \hat{Q}_0 - N_{ui} R_0 \hat{K}_i \\ 0 \\ N_{1i} \hat{Q}_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \Lambda_{2i} = \begin{bmatrix} -\mu_i \cdot I & 0 \\ 0 & -\mu_i \cdot I \end{bmatrix}.$$
Then the system (45) with (22) is asymptotically stabilizable via the reliable guaranteed cost control in (23b) with $K_i = \hat{K}_i \hat{Q}_0^{-1}$. The guaranteed cost is given in (39c) with $Q_0 = \hat{Q}_0^{-1}, \hat{Q}_i = \hat{Q}_0^{-1} \hat{Q}_i \hat{Q}_0^{-1}$.

The optimal guaranteed cost control for system (45) with (22) and cost function (37) is given in the following result.

**Theorem 10.2.** Suppose for a given constant $\rho > 0$, the following optimization problem:

Minimize $\alpha + \text{trace}(W_1^T \Phi_1 W_1 + W_2^T \Phi_2 W_2 + W_3^T \Phi_3 W_3 + W_4^T \Phi_4 W_4 + W_5^T \Phi_5 W_5 + W_6^T \Phi_6 W_6)$, \hfill (47a)

subject to

(i) \hfill (46a)-(46b),

(ii) $\begin{bmatrix} -\alpha & x(0)^T \\ x(0) & -\hat{Q}_0 \end{bmatrix} < 0, \begin{bmatrix} -2\hat{Q}_0 + \hat{Q}_1 & I \\ I & -\Phi_1 \end{bmatrix} < 0, \begin{bmatrix} -2\hat{Q}_0 + \hat{Q}_2 & I \\ I & -\Phi_2 \end{bmatrix} < 0, \begin{bmatrix} -2\hat{Q}_0 + \hat{Q}_3 & I \\ I & -\Phi_3 \end{bmatrix} < 0,

\begin{bmatrix} -2\hat{Q}_0 + \hat{Q}_4 & I \\ I & -\Phi_4 \end{bmatrix} < 0, \begin{bmatrix} -2\hat{Q}_0 + \hat{Q}_5 & I \\ I & -\Phi_5 \end{bmatrix} < 0, \begin{bmatrix} -2\hat{Q}_0 + \hat{Q}_5 & I \\ I & -\Phi_5 \end{bmatrix} < 0,$ \hfill (47b)

has a solution with constants $\alpha > 0$, $\varepsilon_i > 0$, $\mu_i > 0$, $l = 1, 2, \ldots, N$, $n \times n$ matrices $\hat{Q}_i > 0$, $i = 0, 1, 2, \ldots, 5$, $\hat{P}_{ij2}$, $j = 1, 2, 3$, $\Phi_k > 0$, $k = 1, 2, 3, 4, 5$, $\Phi_5 > 0$, $4n \times 4n$ matrices $\hat{P}_{j11} > 0$, $j = 1, 2, 3$, $4n \times 4n$ matrices $\hat{P}_{j12}$, $j = 1, 2, 3$, $\hat{K}_i \in \mathbb{R}^{m \times n}$, where

\[
\int_{-h_m}^{0} x(s) x^T(s) ds = W_1 W_1^T, \quad \int_{-h(0)}^{h_m} x(s) x^T(s) ds = W_2 W_2^T, \quad \int_{-h(0)}^{h_m} \dot{x}(s) \dot{x}^T(s) ds = W_3 W_3^T,
\]

\[
\int_{-h_m}^{0} (s + h_m) \dot{x}(s) \dot{x}^T(s) ds = W_4 W_4^T, \quad \int_{-h_m}^{0} (s + h_m) \dot{x}(s) \dot{x}^T(s) ds = W_5 W_5^T,
\]

\[
(h_m - h_m) \cdot \int_{-h_m}^{0} \dot{x}(s) \dot{x}^T(s) ds = \dot{W}_5 \dot{W}_5^T. \quad (47c)
\]

Then the control (23b) with the gain $K_i = \hat{K}_i \hat{Q}_0^{-1}$ is the guaranteed cost control of system (45) with the guaranteed cost in (39c) with $Q_0 = \hat{Q}_0^{-1}, \hat{Q}_i = \hat{Q}_0^{-1} \hat{Q}_i \hat{Q}_0^{-1}$.

**Proof.** By the Schur complement, LMIs (47b) are equivalent to

$x^T(0)Q_0x(0) < \alpha, -2\hat{Q}_0 + \hat{Q}_k + \Phi_k^{-1} < 0, k \in \{1, 2, 3, 4, 5\}, -2\hat{Q}_0 + \hat{Q}_5 + \Phi_5^{-1} < 0.$ \hfill (48)

Note that

\[
[\hat{Q}_0 - \Phi_k^{-1}] \Phi_k [\hat{Q}_0 - \Phi_k^{-1}] = \hat{Q}_0 \Phi_k \hat{Q}_0 - 2\hat{Q}_0 + \Phi_k^{-1} \geq 0, k \in \{1, 2, 3, 4, 5\},
\]

\[
[\hat{Q}_0 - \Phi_5^{-1}] \Phi_5 [\hat{Q}_0 - \Phi_5^{-1}] = \hat{Q}_0 \Phi_5 \hat{Q}_0 - 2\hat{Q}_0 + \Phi_5^{-1} \geq 0.
\]

The following results are obtained from condition (48):

\[-\hat{Q}_0 \Phi_k \hat{Q}_0 + \hat{Q}_k < 0, k \in \{1, 2, 3, 4, 5\}, -\hat{Q}_0 \Phi_5 \hat{Q}_0 + \hat{Q}_5 < 0. \quad (29)\]
Conditions in (49) are equivalent to
\[ Q_k = \hat{Q}_0^{-1} \hat{Q}_k \hat{Q}_0^{-1} < \Phi_k, \quad k \in \{1, 2, 3, 4, 5\}, \]
\[ Q_5 = \hat{Q}_0^{-1} \hat{Q}_5 \hat{Q}_0^{-1} < \bar{\Phi}. \]

Hence we have
\[
\int_{-h_0}^{0} x^T(s)Q_kx(s)ds = \text{trace}\left(\int_{-h_0}^{0} x^T(s)Q_kx(s)ds\right) = \text{trace}\left(\int_{-h_0}^{0} x(s)x^T(ds)\right)
\]
\[ = \text{trace}(W_1^TW_1^T) = \text{trace}(W_1^TW_1^T) \leq \text{trace}(W_1^T\Phi_1W_1^T), \]
\[ \int_{-h(0)}^{0} x^T(s)Q_2x(s)ds = \text{trace}(W_2^TW_2^T) \leq \text{trace}(W_2^T\Phi_2W_2^T), \]
\[ \int_{-h(0)}^{0} x^T(s)Q_3x(s)ds = \text{trace}(W_3^TW_3^T) \leq \text{trace}(W_3^T\Phi_3W_3^T), \]
\[ \int_{-h_m}^{0} x^T(s)Q_4x(s)ds = \text{trace}(W_4^TW_4^T) \leq \text{trace}(W_4^T\Phi_4W_4^T), \]
\[ \int_{-h_m}^{0} x^T(s)Q_5x(s)ds = \text{trace}(W_5^TW_5^T) \leq \text{trace}(W_5^T\Phi_5W_5^T). \]

This completes this proof.

**Remark 10.1.** All the proposed approaches in this paper can be used to design static output feedback and observer-based controls [18-20], but some Linear Matrix Equality (LME) conditions may be proposed. The respective LMI and LME conditions could be solved by some efficient computer optimization programs; such as Scilab (see http://scilabsoft.inria.fr/). Hence the standard LMI procedure can be directly employed to find a feasible solution of these conditions. The main purpose for equality constraints are used to convert the non-convex problem into convex one. For another type perturbations can also be considered; such as nonlinear perturbation in [21].

**11. CONCLUSIONS**

The robust reliable control with IQC performance for uncertain neutral systems with time-varying input delays has been investigated in the first part of this paper. The reliable controls (IQC performance, \( H_{\infty} \), guaranteed cost controls) for switched systems with interval time-varying delays have been investigated in the second part of this paper. An extensive understand for switched systems and a significant meaning for designing the switching signal to achieve the requirement of system performance have been provided.

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