PROPERTIES OF INTUITIONISTIC FUZZY BOUNDARY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

A. Manimaran, P. Thangaraj & K. Arun Prakash

Abstract

In this paper, we have introduced the concept of intuitionistic fuzzy boundary in intuitionistic fuzzy topological spaces. We have studied some characteristics of intuitionistic fuzzy boundary and obtained intuitionistic fuzzy boundary in product related spaces.

AMS classification: 54A40, 54D20.


1. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [8], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [2] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [5] introduced the notion of intuitionistic fuzzy topological spaces. For the past few years, many researchers were going on in intuitionistic fuzzy topological spaces and many concepts in fuzzy topology were extended to intuitionistic fuzzy topology.

Fuzzy boundary and fuzzy semi boundary were introduced M. Athar and B. Ahmad in [1] in 2008. In this paper, we are extending the above concept to intuitionistic fuzzy topological space. We study some of the basic properties of intuitionistic fuzzy boundary with examples. Properties of intuitionistic fuzzy semi-interior, intuitionistic fuzzy semi-closure and intuitionistic boundary have been obtained in product related spaces. We give necessary conditions for fuzzy continuous function.

2. PRELIMINARIES

Before entering to our work, we recall the following notations, definitions and intuitionistic fuzzy sets as given by Atanassov [2], Coker [5] and Hanafy [7].
Throughout this paper, \((X, \tau), (Y, \sigma)\) and \((Z, \eta)\) always means an intuitionistic fuzzy topological spaces in which no separation axioms are assumed unless otherwise mentioned.

**Definition 2.1:** [2] Let \(X\) be a non-empty fixed set. An intuitionistic fuzzy set (IFS, for short), \(A\) is an object having the form
\[
A = \{ < x, \mu_A(x), \gamma_A(x) > : x \in X \},
\]
where the function \(\mu_A : X \to I\) and \(\gamma_A : X \to I\) denotes respectively the degree of membership (namely \(\mu_A(x)\)) and the non-membership (namely \(\gamma_A(x)\)) of each element \(x \in X\) to a set \(A\), and \(0 \leq \mu_A(x) + \gamma_A(x) \leq 1\) for each \(x \in X\).

Obviously, every set \(A\) on a non-empty set \(X\) is an IFS having the form
\[
A = \{ < x, \mu_A(x), \gamma_A(x) > : x \in X \}.
\]

**Definition 2.2:** [2] Let \(X\) be a non-empty set and let the IFS's \(A\) and \(B\) in the form
\[
A = \{ < x, \mu_A(x), \gamma_A(x) > : x \in X \}; \quad B = \{ < x, \mu_B(x), \gamma_B(x) > : x \in X \}
\]
Let \(\{ A_j : j \in J \}\) be an arbitrary family of IFS's in \((X, \tau)\). Then,

(i) \(A \leq B\) if and only if \(\forall x \in X, \mu_A(x) \leq \mu_B(x)\) and \(\gamma_A(x) \geq \gamma_B(x)\)

(ii) \(\overline{A} = \{ < x, \mu_A(x), \gamma_A(x) > : x \in X \}\)

(iii) \(\cap A_j = \{ < x, \wedge \mu_{A_j}(x), \vee \gamma_{A_j}(x) > : x \in X \}\)

(iv) \(\cup A_j = \{ < x, \vee \mu_{A_j}(x), \wedge \gamma_{A_j}(x) > : x \in X \}\)

(v) \(\hat{1} = \{ < x, 1, 0 > : x \in X \}\) and \(\hat{0} = \{ < x, 0, 1 > : x \in X \}\)

(vi) \(\overline{\overline{A}} = A, \overline{\overline{\hat{1}}} = 0, \overline{\overline{\hat{0}}} = 1\).

**Definition 2.3:** [2] Let \(X\) and \(Y\) be two non-empty sets and \(f : (X, \xi) \to (Y, \sigma)\) be a function. If \(B = A = \{ < y, \mu_B(y), \gamma_B(y) > : y \in Y \}\) is an IFS in \(Y\), then the pre image of \(B\) under \(f\) is denoted and defined by \(f^{-1}(B) = \{ < x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) > : x \in X \}\), since, \(\mu_B, \gamma_B\) are fuzzy sets, we explain that
\[
f^{-1}(\mu_B)(x) = \mu_B(f(x)).
\]

**Definition 2.4.** [5] An intuitionistic fuzzy topology (IFT, for short) on a non-empty set \(X\) is a family of IFS's in \(X\) satisfying the following axioms:

(i) \(1, 0 \in \tau\)

(ii) \(A_j \cap A_2 \in \tau\) for some \(A_j, A_2 \in \tau\)

(iii) \(\cup A_j \in \tau\) for any \(\{ A_j : j \in J \}\).
In this case, the ordered pair \((X, \tau)\) is called intuitionistic fuzzy topological space (IFS, for short) and each IFS in \(\tau\) is known as an intuitionistic fuzzy open set (IFOS, for short) in \(X\). The complement of an intuitionistic fuzzy open set is called intuitionistic fuzzy closed set (IFCS, for short).

**Definition 2.5:** [5] Let \((X, \tau)\) be an IFTS and let \(A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}\) be an IFS in \(X\). Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of \(A\) are defined by

\[
\text{int}(A) = \{G/G \text{ is an IFOS in } X \text{ and } G \subseteq A\},
\]

\[
\text{cl}(A) = \{K/K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.
\]

**Remark 2.6:** [5] For any IFS \(A\) in \((X, \tau)\), we have \(\text{cl}(A) = \text{int}(\text{cl}(A))\) and \(\text{int}(A) = \text{cl}(\text{int}(A))\).

**Definition 2.7:** [6] A function \(f : (X, \tau) \to (Y, \delta)\) is said to be intuitionistic fuzzy continuous if \(f^{-1}(B)\) is an IFOS of \(X\) for each IFOS \(B\) of \(Y\).

**Definition 2.8:** [7] Let \(X, Y\) be non-empty sets and \(A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}\), \(B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X\}\) be IFS's of \(X\) and \(Y\) respectively. The \(A \times B\) is an IFS of \(X \times Y\) is defined by

\[
(A \times B)(x, y) = \langle(x, y), \min(\mu_A(x), \mu_B(y)), \max(\gamma_A(x), \gamma_B(y))\rangle : x \in X, y \in Y
\]

\[
1 - (A \times B)(x, y) = \langle(x, y), \max(\gamma_A(x), \gamma_B(y)), \min(\mu_A(x), \mu_B(y))\rangle : x \in X, y \in Y
\]

**Theorem 2.10:** [7] Let \((X, \Psi)\) and \((Y, \Phi)\) be product-related IFTS's. Then, for an IFS \(A\) of \(X\) and a IFS \(B\) of \(Y\), one has

1. \(\text{cl}(A \times B) = \text{cl}(A) \times \text{cl}(B)\)
2. \(\text{int}(A \times B) = \text{int}(A) \times \text{int}(B)\)

**Definition 2.11:** [6] Let \(A\) be a fuzzy set in an IFTS \((X, \tau)\). Then, \(A\) is called a fuzzy semiopen set of \(X\) if there exists a \(B \in \tau\) such that \(B \subseteq A \subseteq \text{cl}(B)\).

**Definition 2.12:** [6] Let \(A\) be a fuzzy set in an IFTS \((X, \tau)\). Then, semiclosure (briefly \(\text{scl}\)) and semi-interior (briefly \(\text{sint}\)) are given as

\[
\text{scl}(A) = \cap\{B/A \subseteq B, B \text{ is fuzzy semiclosed}\},
\]

\[
\text{sint}(A) = \cap\{B/B \subseteq A, B \text{ is fuzzy semiopen}\}.
\]

3. **INTUITIONISTIC FUZZY BOUNDARY**

**Definition 3.1:** Let \(A\) be an IFS in an IFTS \(X\). Then, the intuitionistic fuzzy boundary of \(A\) (IBd \(A\), for short) is defined as \(\text{IBd} A = \text{cl} A \wedge \text{cl} A'\).

**Remark 3.2:** IBd \(A\) is an IFCS.
Remark 3.3: In classical topology, for an arbitrary set $A$ of a topological space $X$, we have $A \cup \text{Bd} A = \text{cl} A$, but in IFT we have $A \vee \text{IBd} A \leq \text{cl} A$, for an IFS in $A$, the converse of which is not true.

Example 3.4: Let $X = \{a, b\}$. Let $A = \langle x, (\frac{a}{0.3}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.4}) \rangle$. Then $\tau = \{\emptyset, \bar{A}, A\}$ be an IFTS in $X$. Let $B = \langle x, (\frac{a}{0.6}, \frac{b}{0.7}), (\frac{a}{0.4}, \frac{b}{0.3}) \rangle$ be an IFS in $X$.

Now, $\text{cl} (B) = \bar{A}$. $\text{IBd} (B) = \text{cl} (B) \land \text{cl} (B)^c = \bar{A} \land A^c = A^c$.

Then, $B \vee \text{IBd} (B) = \langle x, (\frac{a}{0.6}, \frac{b}{0.7}), (\frac{a}{0.3}, \frac{b}{0.3}) \rangle$.

Clearly, $\text{cl} (B) \subseteq B \land \text{IBd} (B)$.

Proposition 3.5: For IFS’s $A$ and $B$ in an IFTS $X$, the following conditions hold.

1. $\text{IBd} A = \text{IBd} A^c$.

2. If $A$ is an IFCS, then $\text{IBd} A \leq A$.

3. If $A$ is an IFOS, then $\text{IBd} A \leq A^c$.

4. Let $A \leq B$ and $B \in \text{IFC} (X)$ (resp., $B \in \text{IFO} (X)$). Then, $\text{IBd} A \leq B$ (resp., $\text{IBd} B \leq A$).

5. $(\text{IBd} A)^c = \text{int} A \lor \text{int} A^c$.

Proof: (1) $\text{IBd} A = \text{cl} A \land \text{cl} A^c = \text{cl} A^c \land \text{cl} A = \text{cl} A^c \land \text{cl} (A^c)^c = \text{IBd} A^c$.

(2) $\text{IBd} A = \text{cl} A \land \text{cl} A^c \leq \text{cl} A = A$, hence $\text{IBd} A \leq A$.

(3) $A$ is IFOS implies $A^c$ is IFCS. By (2), $\text{IBd} A^c \leq A^c$ and by (1), we get $\text{IBd} A \leq A^c$.

(4) Since $A \leq B$ implies $\text{cl} (A) \leq \text{cl} (B)$, we have $\text{IBd} A = \text{cl} A \land \text{cl} A^c \leq \text{cl} B \land \text{cl} A^c \leq \text{cl} B = B$, since $B \in \text{IFC} (X)$.

(5) $(\text{IBd} A)^c = (\text{cl} A \land \text{cl} A^c)^c = (\text{cl} A)^c \lor (\text{cl} A^c)^c = \text{int} A^c \lor \text{int} A$.

The converse of (2) and (3) of Proposition 3.5 is not true in general, as seen from the following example.

Example 3.6: Let $X = \{a, b, c\}$. Let $A = \langle x, (\frac{a}{0.4}, \frac{b}{0.7}, \frac{c}{0.2}), (\frac{a}{0.2}, \frac{b}{0.1}, \frac{c}{0.1}) \rangle$ and $B = \langle x, (\frac{a}{0.6}, \frac{b}{0.9}, \frac{c}{0.3}), (\frac{a}{0.3}, \frac{b}{0.1}, \frac{c}{0.1}) \rangle$. Then $\tau = \{0, A, B, A \cup B, A \cap B, 1\}$ be an IFTS in $X$.

Let $G = \langle x, (\frac{a}{0.5}, \frac{b}{0.7}, \frac{c}{0.9}), (\frac{a}{0.2}, \frac{b}{0.1}, \frac{c}{0.1}) \rangle$ be any IFS in $X$.

Now, $\text{cl} (G) = \bar{A}$, $\text{cl} (G)^c = A^c \land (A \cap B)^c = A^c$.

$\text{IBd} (G) = \text{cl} (G) \land \text{cl} (G)^c = \bar{A} \land A^c = A^c$. Clearly $\text{IBd} (G) \leq G$ But $G$ is not an IFS in $X$.
Let \( F = (x, (\frac{a}{0.4}, \frac{b}{0.7}, \frac{c}{0.1}), (\frac{a}{0.3} \cdot \frac{b}{0.1} \cdot \frac{c}{0.4})) \) be any IFS in \( X \). Now, \( cl(F) = \tilde{1} \), \( cl(F') = A' \). \( IBd(F) = cl(F) \cap cl(F') = 1 \land (A \cap B)' = (A \cap B)' \). Clearly \( IBd(F) \leq F' \), but \( F \) is not an IFOS in \( X \).

**Proposition 3.7:** Let \( A \) be an IFS in an IFTS \( X \). Then,

1. \( IBd A = cl A \cap \text{int} A \); 
2. \( IBd \text{int} A \leq IBd A \); 
3. \( IBd cl A \leq IBd A \); 
4. \( \text{int} A \leq A - IBd A \).

**Proof:** (1) Since \( (cl A)' = \text{int} A \), therefore we have \( IBd A = cl A \cap cl A' = cl A - (cl A)' = cl A - \text{int} A \). Thus \( IBd A = cl A - \text{int} A \).

(2) \( IBd \text{int} A = cl \text{int} A - \text{int} \text{int} A = cl \text{int} A - \text{int} A \leq cl A - \text{int} A = IBd A \),

(3) \( IBd cl A = cl cl A - \text{int} cl A = cl A - \text{int} A \leq cl A - \text{int} A = IBd A \),

(4) \( A - IBd A = A \land (IBd A)' = A \land (cl A \land cl A')' = A \land (\text{int} A' \lor \text{int} A) = (A \land \text{int} A') \lor (A \land \text{int} A) = (A \land \text{int} A') \lor A \geq A \).

The following examples shows that the equalities (2), (3) & (4) of the proposition 3.7 does not hold in general.

**Example 3.8:** In example 3.6, Consider an IFS \( F = (x, (\frac{a}{0.1}, \frac{b}{0.7}, \frac{c}{0.1}),(\frac{a}{0.7} \cdot \frac{b}{0.5} \cdot \frac{c}{0.4})) \).

Now, \( \text{int}(F) = 0 \), \( IBd(\text{int}(F)) = IBd(0) = cl(0) \cap cl(0)' = 0 \land \tilde{1} = \tilde{0} \).

Also, \( IBd(F) = cl(F) \cap cl(F') = (A \cap B)' \land \tilde{1} = (A \cap B)' \).

Therefore, \( IBd(\text{int}(F)) \neq IBd(F) \).

**Example 3.9:** In example 3.6, Consider an IFS \( C = (x, (\frac{a}{0.5}, \frac{b}{0.7}, \frac{c}{0.1}),(\frac{a}{0.2} \cdot \frac{b}{0.1} \cdot \frac{c}{0.4})) \).

Now, \( cl(C) = \tilde{1} \), \( IBd(cl(C)) = IBd(\tilde{1}) = cl(0) \cap cl(0)' = 0 \land \tilde{0} = 0 \).

Also, \( IBd(C) = cl(C) \cap cl(C') = \tilde{1} \land A' = A' \).

Therefore, \( IBd(cl(C)) \neq IBd(C) \).

**Example 3.10:** In example 3.6, Consider an IFS \( C = (x, (\frac{a}{0.6}, \frac{b}{0.7}, \frac{c}{0.8}),(\frac{a}{0.2} \cdot \frac{b}{0.1} \cdot \frac{c}{0.4})) \).

Now, \( cl(C) = \tilde{1} \), \( cl(C') = A' \land (A \cap B)' = A' \), then \( IBd(C) = cl(C) \cap cl(C') = \tilde{1} \land A' = A' \).

We have, \( C - IBd(C) = C \land (IBd(C))' = C \land A = A \). \( \text{int}(C) = A \).

Therefore, \( \text{int}(C) \neq C - IBd(C) \).

**Theorem 3.11:** Let \( A \) and \( B \) be IFS’s in an IFTS \((X, \tau)\). Then, \( IBd(A \lor B) \leq IBdA \lor IBdB \).
Proof: \[IBd (A \vee B) = cl (A \vee B) \land cl (A \vee B)^c\]
\[\leq (cl A \land cl B) \land (cl A^c \land cl B^c)\]
\[= (cl A \land (cl A^c \land cl B^c)) \lor (cl B^c \land (cl A^c \land cl B^c))\]
\[= (IBd A \land cl B^c) \lor (IBd B \land cl A^c)\]
\[\leq IBd A \lor IBd B.\]

In Theorem 3.11, the equality does not hold as shown by the following example.

**Example 3.12:** In example 3.6, Choose IFS's \(F = (\langle x, (\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.1}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.9}) \rangle, G = (\langle x, (\frac{a}{0.7}, \frac{b}{0.2}, \frac{c}{0.1}) \rangle, \langle (\frac{a}{0.7}, \frac{b}{0.2}, \frac{c}{0.1}) \rangle, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.9}) \rangle, \langle (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.9}) \rangle) \rangle.\n
Then, \(F \lor G = (\langle x, (\frac{a}{0.7}, \frac{b}{0.2}, \frac{c}{0.1}) \rangle, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.9}) \rangle)\), \(cl (F \lor G) = \tilde{1}\), \(cl (F \lor G) = A^c \land (A \lor B)^c = A^c\). \(IBd (F \lor G) = cl (F \lor G) \land cl (F \lor G)^c = 1\). \(IBd (F \lor G) = cl (F \lor G) \lor IBd (G) = \tilde{1} \lor A^c = \tilde{1}\).

Therefore, \(IBd (F \lor G) \neq IBd (F) \lor IBd (G)\).

**Theorem 3.13:** For any IFSs \(A\) and \(B\) in an IFTS \(X\), \(IBd (A \land B) \leq IBd A \lor IBd B\)

Proof: \[IBd (A \land B) = cl (A \land B) \land cl (A \land B)^c\]
\[\leq (cl A \land cl B) \land (cl A^c \land cl B^c)\]
\[= (cl A \land cl B) \land (cl A^c \land cl B^c)\]
\[= (IBd A \land cl B^c) \lor (IBd B \land cl A^c)\]
\[\leq IBd A \lor IBd B.\]

Therefore, \(IBd (A \land B) = IBd A \lor IBd B\).

In general, the equality of the above theorem does not hold as seen from the following example.

**Example 3.14:** In example 3.6, Choose IFS's \(F = (\langle x, (\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.1}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.9}) \rangle, G = (\langle x, (\frac{a}{0.7}, \frac{b}{0.2}, \frac{c}{0.1}) \rangle, \langle (\frac{a}{0.7}, \frac{b}{0.2}, \frac{c}{0.1}) \rangle, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.9}) \rangle, \langle (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.9}) \rangle) \rangle.\n
Then, \(F \land G = (\langle x, (\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.1}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.9}) \rangle, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.9}) \rangle)\), \(cl (F \land G) = \tilde{1}\), \(IBd (F \land G) = cl (F \land G) \land cl (F \land G)^c = 1\). \(IBd (F \land G) \lor IBd (G) = \tilde{1} \lor A^c = \tilde{1}\).

Therefore, \(IBd (F \land G) \neq IBd (F) \lor IBd (G)\).

**Remark 3.15:** In general topology, it is known that \(Bd Bd A = Bd Bd A\), for any subset \(A\) of a space \(X\). However, in IFT, we have the following proposition.

**Proposition 3.16:** For any IFS in an IFTS \(X\),

1. \(IBd IBd A \leq IBd A\);
2. \(IBd IBd IBd A \leq IBd IBd A\).
Properties of Intuitionistic Fuzzy Boundary in Intuitionistic Fuzzy...

Proof: (1) \( IBd IBd A = \text{cl}(IBd A) \cap \text{cl}(IBd A)^c \leq \text{cl} IBd A = IBd A; \)

(2) \( IBd IBd IBd A = \text{cl}(IBd IBd A) \cap \text{cl}(IBd IBd A)^c \)

\[ = IBd IBd A \cap \text{cl}(IBd IBd A)^c \leq IBd IBd A. \]

Remark 3.17: The equality in (1) of proposition 3.16, in general, does not hold as shown by the following example.

Example 3.18: In example 3.6, choose \( F = (x, (a_{0.6}, b_{0.6}, c_{0.7}), (a_{0.2}, b_{0.3}, c_{0.1})). \)

Now,

\[ IBd (F) = \text{cl}(F) \cap \text{cl}(F^c) = \tilde{1} \]

\[ IBd IBd (F) = IBd (\tilde{1}) = \text{cl}(\tilde{1}) \cap \text{cl}(\tilde{1})^c = 1 \land 0 = 0. \]

Therefore, \( IBd IBd (F) \neq IBd (F). \)

Lemma 3.19: For IFS’s \( A, B, C, \) and \( D \) in a set \( X, \) we have

\[ (A \land B) \times (C \land D) = (A \times D) \land (B \times C). \]

Proof:

\[
(A \land B) \times (C \land D)(x, y) = \begin{cases}
(x, y), & \text{min} (\mu_{(A \land B)}(x), \mu_{(C \land D)}(y)), \\
& \text{max} (\gamma_{(A \land B)}(x), \gamma_{(C \land D)}(y))
\end{cases}
\]

\[
= \begin{cases}
(x, y), & \text{min} (\mu_A(x), \mu_B(x)), \\
& \text{min} (\mu_C(y), \mu_D(y)), \\
& \text{max} (\gamma_A(x), \gamma_B(x)), \\
& \text{max} (\gamma_C(y), \gamma_D(y))
\end{cases}
\]

\[
= \begin{cases}
(x, y), & \text{min} (\mu_A(x), \mu_D(x)), \\
& \text{min} (\mu_B(y), \mu_C(y)), \\
& \text{max} (\gamma_A(x), \gamma_D(x)), \\
& \text{max} (\gamma_B(y), \gamma_C(y))
\end{cases}
\]

\[
= \begin{cases}
(x, y), & \text{min} (\mu_{(A \times D)}(x, y), \mu_{(B \times C)}(x, y)), \\
& \text{max} (\gamma_{(A \times D)}(x, y), \gamma_{(B \times C)}(x, y))
\end{cases}
\]

\[ = (A \times D) \cap (C \times B)(x, y). \]

Therefore, \( (A \land B) \times (C \land D) = (A \times D) \land (B \times C). \)

Theorem 3.20: Let \( X, \) \( i = 1, 2, ..., n \) be a family of product related IFTS’s. If each \( A_i \) is an IFS in \( X, \) then
\[ IBd \prod_{i=1}^{n} A_i = (IBd A_1 \times cl A_2 \times \cdots \times cl A_n) \vee (cl A_1 \times IBd A_2 \times \cdots \times cl A_n) \]
\[ \vee \cdots \vee (cl A_1 \times \cdots \times cl A_n) \]

**Proof:** For \( n = 2 \). Consider,
\[ IBd (A_1 \times A_2) = cl (A_1 \times A_2) - \text{int} (A_1 \times A_2) \]
\[ = (cl A_1 \times cl A_2) - (\text{int} A_1 \times \text{int} A_2) \]
\[ = (cl A_1 \times cl A_2) - ((\text{int} A_1 \times cl A_2) \times (\text{int} A_2 \times cl A_1)) \]
\[ = (cl A_1 - \text{int} A_1) \times cl A_2 \vee (cl A_1 \times (cl A_2 - \text{int} A_2)) \]
\[ = (IBd A_1 \times cl A_2 ) \vee (cl A_1 \times IBd A_2) \]

**Theorem 3.21:** Let \( f : X \to Y \) be an intuitionistic fuzzy continuous mapping. Then, \( IBdf^{-1}(A) \leq f^{-1}(IBd A) \), for any IFS in \( Y \).

**Proof:** Let \( f \) be an intuitionistic fuzzy continuous mapping \( A \) be an IFS in \( Y \). Then,
\[ IBdf^{-1}(A) = clf f^{-1}(A) \land cl (f^{-1}(A)^c) \leq (clf^{-1}(cl A)) \land clf^{-1}(cl A^c) \]
\[ = f^{-1}(cl A) \land f^{-1}(cl A^c) \]
\[ = f^{-1}(cl A \land cl A^c) \]
\[ = f^{-1}(IBd A) \]
Therefore, \( IBdf^{-1}(A) \leq f^{-1}(IBd A) \).

**REFERENCES**


**A. Manimaran**  
Department of Mathematics, Kongu Engineering College,  
Perundurai-638 052, Erode, Tamilnadu, India.  
*E-mail: manimaranthangaraj@gmail.com*

**P. Thangaraj**  
Department of Computer Science and Engineering,  
Bannariamman Institute of Technology,  
Sathyamangalam, Erode, Tamilnadu, India.

**K. Arun Prakash**  
Department of Mathematics, Kongu Engineering College,  
Perundurai-638 052, Erode, Tamilnadu, India.