Design of a PSO-based Sliding Mode Controller for Controlling Chaotic Brushless DC Motors with Model Uncertainty and External Disturbances

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ABSTRACT
In this paper, the problem of robust stabilization and chaos suppression of nonlinear chaotic brushless dc motors (BLDCMs) is investigated. We propose a PSO-based robust sliding mode controller for stabilization and chaos suppression of the BLDCM in the presence of both unknown model uncertainties and external disturbances. First, based on the Lyapunov stability theory, a robust SMC is designed to ensure the occurrence of sliding motion. Then, a PSO algorithm is applied to optimize the parameters of the proposed controller. An illustrative example is presented to verify the robustness and efficiency of the proposed method and to validate the theoretical results of the paper.

Keywords: chaos suppression, BLDCM, sliding mode control, uncertainty.

1. INTRODUCTION
Electrical machines have wide applications in industrial machinery, electrical locomotives and electrical submersibles thruster drives. In recent years, investigation of the chaos in electric motors has become an interesting research topic, especially since Kuroe and Hayashi [1] have addressed the occurrence of chaos in electric motors in the late 1980s.

Moreover, adaptive dynamic surface control [2], adaptive backstepping control approach [3], feedback linearization control [4], method of time delay feedback control [5] have been proposed to suppress the chaos and to control the PMSM. However, the controller in [5] is not suitable when the desired target is other than the equilibrium or an unstable periodic orbit of the system. Conversely, the chaotification of the PMSM from an orderly mode has been proposed by Ren and Han [6].

Furthermore, chaos in BLDCM and its control are other active areas in the field of nonlinear electric motors’ control, due to its interesting features of high efficiency, high power density and low maintenance cost and its applications in robotics and aerospace, computers, CNCs, etc [7].

Recently, nonlinear control [8], optimal control [9], variable structure control [10], PID control [11], adaptive control [12], backstepping approach [13] have been proposed for a linear BLDCM model that does not exhibit chaotic dynamics. Chaotic behavior in BLDCM mainly appears as intermittent ripples of torque, and/or low-frequency oscillations of rotational speed. This can extremely destroy the stabilization and performance of the motor. Thus, it is indispensable to study the method of controlling and suppressing chaos in BLDCMs. One drawback of the controller in [13] is that the equilibrium points

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of the uncontrolled system cannot be used as the control target. It is also practically impossible to reach the points near these equilibrium points; because the feedback gain may become either too small, which is not robust to parametric perturbations and measurement noise, or too large, which cannot be realized in practice. Using nonlinear control methods in [14], regulation of the system around the origin is achieved; but after the controller is turned on, a noticeable chattering phenomenon is observed. In the feedback linearization and sliding mode controller developed in [15], only the stator current state equation has been perturbed by uncertainties. Besides, no control parameter selection procedure is used in the above mentioned methods. Recently, chaotic anticontrol and chaos synchronization of BLDCM systems have been studied [16, 17] to synchronize two BLDCM chaotic motors, but not to control and suppress the system chaos.

Practically, the BLDCM is subjected to model uncertainties and load disturbances, which degrade the performance of the system. Therefore, the main aim of this paper is to find a proper and applicable method for controlling chaos in BLDCM with both unknown model uncertainties and external disturbances.

Sliding mode controller (SMC) [18] is a robust powerful control methodology which has several useful performances such as fast response, low sensitivity to external disturbances, robustness to the plant uncertainties and easy realization. On the other hand, particle swarm optimization (PSO) [19] is an intelligent numerical optimization technique that has found many successful applications in engineering problems. In this paper, a PSO-based SMC is introduced to control an uncertain chaotic BLDCM and suppress its chaotic motions. Bounded unknown model uncertainties and external disturbances are considered in all state equations of the BLDCM. The stability and robustness of the designed sliding mode controller is proved using Lyapunov stability theory. The parameters of the proposed sliding mode controller have been optimally chosen by PSO method to minimize a cost function, including state errors and control the used energy. Some numerical simulations are presented to validate the robustness and applicability of the proposed scheme.

2. DESCRIPTION OF THE BLDCM SYSTEM AND PROBLEM FORMULATION

BLDCM is an electromechanical system. Using time scaling and an affine state transformation, Hemati [8] has derived a non-dimensionalized model for the BLDCM with smooth air gap. He has shown that the unforced transformed system is equivalent to the Lorenz system, which is known to exhibit chaotic behavior. The model is given by

\[
\begin{align*}
\dot{I}_q &= -I_q - I_q \omega + \rho \omega + v_q \\
\dot{I}_d &= -I_d - I_d \omega - v_d \\
\dot{\omega} &= \sigma (I_q - \omega) - T_L
\end{align*}
\]

where \( \rho \) and \( \sigma \) are system parameters, \( I_q, I_d, v_q, v_d \), and \( \omega \) are the transformed direct and quadrature axis for stator current and voltage, external load torque including friction and angular velocity, respectively.

When \( v_q = v_d = T_L = 0, \rho = 20 \) and \( \sigma = 5.5 \) the BLDCM system (1) exhibits chaotic motion. It can be shown that the system has three equilibrium points: \((0, 0, 0)\), \((\sqrt{19}, 19, \sqrt{19})\) and \((-\sqrt{19}, 19, -\sqrt{19})\), which are all unstable [15]. The phase portrait of the BLDCM system (1) is illustrated in Fig. 1.

The BLDCM system (1) with control inputs can be rewritten in the following form:
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where $x = [x_1, x_2, x_3]^T$ denotes $I_q$, $I_d$ and $\omega$, respectively; $u = [u_1, u_2, u_3]^T$ are control inputs, $\Delta f_i(x, t)$ and $d_i(t)$, $(i = 1, 2, 3)$ are unknown model uncertainties and external disturbances, respectively. $f_i(x)$, $(i = 1, 2, 3)$ are nonlinear terms of equations.

**Assumption 1.** We assume the uncertainties $\Delta f_i(x, t)$ are bounded by
\[
|\Delta f_i(x, t)| \leq \alpha_i \|x\| + \beta_i, i = 1, 2, 3
\]
where $\alpha_i$ and $\beta_i$ are given positive constants.

**Assumption 2.** In general it is assumed that external disturbances are norm-bounded in $C^1$, i.e.
\[
|d_i| \leq \gamma_i, i = 1, 2, 3
\]
where $\gamma_i$ are known positive constants.

3. **DESIGN PROCEDURE OF PSO-BASED SLIDING MODE CONTROLLER FOR BLDCM**

3.1. **Design of a Robust Sliding Mode Controller for Stabilization of BLDCM System**

The design procedure of a sliding mode controller is a two-stage process. The first phase is selecting a switching surface for the desired dynamics and the second phase is establishing a discontinuous control law such that the system trajectories reach the surface and remain on it evermore.
As the first step of design procedure, in this paper, a PI type sliding surface is defined as

$$s_i = \lambda_i x_i(t) + \int_0^t x_i(\tau)d\tau, \quad i = 1, 2, 3$$

(5)

where $s_i \in R$ and the sliding surface parameters $\lambda_i$ are positive constants to be chosen later by PSO. For the existence of the sliding mode it is necessary and sufficient that

$$s_i = \lambda_i x_i(t) + \int_0^t x_i(\tau)d\tau \quad \text{and} \quad s_i(t) = \lambda_i x_i(t) + x_i(t), \quad i = 1, 2, 3$$

(6)

Hence, the following sliding mode dynamics can be achieved as

$$\dot{x}_i(t) = -\frac{1}{\lambda_i} x_i(t), \quad i = 1, 2, 3$$

(7)

It is apparent that the sliding mode dynamics (7) is asymptotically stable.

The second step is to determine an input signal $u(t)$ to guarantee that the trajectories go on to sliding surface $s(t)=0$ (i.e. to satisfy the reaching condition $s(t)\dot{s}(t) \leq 0$) and remain on it, for the subsequent time interval. Therefore, to ensure the existence of the sliding motion, a discontinuous control can be designed as follows:

$$u_i(t) = -(f_i(x) + \frac{1}{\lambda_i} x_i(t) + \xi_i s_i + (\alpha_i \|x\| + \beta_i + \gamma_i + k_i) s_i) \text{sgn}(s_i), \quad i = 1, 2, 3$$

(8)

where $\xi_i$ and $k_i$ are positive constants to be chosen later by PSO and $\text{sgn}(\cdot)$ is the sign function.

**Theorem 1.** Consider the chaotic BLDCM system (2). If this system is controlled by $u(t)$ in (8), then the system trajectories converge to the sliding surface $s(t) = 0$.

**Proof.** Choosing Lyapunov function candidate in the form of $V(t) = \frac{1}{2} \sum_{i=1}^3 s_i^2$ and taking its time derivative, one has

$$\dot{V}(t) = \sum_{i=1}^3 \xi_i s_i$$

(9)

Knowing $s_i(t) = \lambda_i x_i(t) + x_i(t)$ and replacing $\dot{x}_i(t)$ from Eq. (2), it yields

$$\dot{V}(t) = \sum_{i=1}^3 \lambda_i s_i (f_i(x) + \Delta f_i(x,t) + d_i(t) + u_i(t)) + x_i(t))$$

(10)

$$\dot{V}(t) \leq \sum_{i=1}^3 \lambda_i s_i (|\Delta f_i(x,t)| + |d_i(t)| + s_i (\lambda_i (f_i(x) + u_i(t)) + x_i(t)))$$

(11)

Using Assumptions 1 and 2, we have

$$\dot{V}(t) \leq \sum_{i=1}^3 \lambda_i s_i (|\alpha_i \|x\| + \beta_i + \gamma_i + k_i) + s_i (\lambda_i (f_i(x) + u_i(t)) + x_i(t))$$

(12)

Replacing $u_i(t)$ from Eq. (9) and using $s_i \text{sgn}(s_i) = |s_i|$, one can obtain

$$\dot{V}(t) \leq -\sum_{i=1}^3 \left(\xi_i |s_i| + \frac{\xi_i \lambda_i s_i^2}{\eta_i}\right) = -\sum_{i=1}^3 \left(\eta_{i1} |s_i| + \eta_{i2} s_i^2\right) \leq 0$$

(13)
Therefore \( \dot{V}(t) \) becomes

\[
\dot{V}(t) \leq - \sum_{i=1}^{3} (\eta_{li} |s_i| + \eta_{12i} s_i^2) = -\omega(t) \leq 0
\]

(14)

where \( \omega(t) = \sum_{i=1}^{3} (\eta_{li} |s_i| + \eta_{12i} s_i^2) \geq 0 \). Integrating Eq. (14) from 0 to \( t \) yields

\[
V(0) \geq V(t) + \int_{0}^{t} \omega(\tau)d\tau
\]

(15)

Since \( \dot{V}(t) \leq 0, V(0) - V(t) \geq 0 \) is positive and finite and that results in: \( \lim_{t \to \infty} \int_{0}^{t} \omega(\tau)d\tau \) to exists and to be finite (i.e. \( \lim_{t \to \infty} \int_{0}^{t} \omega(\tau)d\tau = V(0) - V(t) \geq 0 \)). Thus, according to the Barbalat lemma, it can be obtained that \( \lim_{t \to \infty} \omega(t) = \lim_{t \to \infty} \sum_{i=1}^{3} (\eta_{li} |s_i| + \eta_{12i} s_i^2) = 0 \). Since \( \eta_{li}, \eta_{12i} > 0 \), it can be concluded that \( s(t) = 0 \). Hence the proof is achieved completely.

**Remark 1.** Since the control law (8) contains the sign function as a hard switcher, the undesirable chattering phenomenon occurs. In order to chattering reduction, the function \( \tanh(\varepsilon s_i) \) is replaced by the sign function. Therefore, the final control input becomes

\[
u_i(t) = -(f_i(x) + \frac{1}{\lambda} x_i(t) + \xi_i s_i + (\alpha_i \|x\| + \beta_i + \gamma_i + k_i) \tanh(\varepsilon s_i)), i = 1, 2, 3
\]

(16)

where \( \varepsilon > 0 \) is a constant to be chosen later by PSO.

### 3.2. Optimization of SMC Parameters by PSO

After designing a robust SMC and proving its robustness and stability, the next step is to use a PSO technique to optimal selection of the controller parameters. The core operation of PSO is the updating formulae of the particles, i.e. the velocity updating equation and position updating equation. The global optimizing model proposed by Shi and Eberhart is as follows [21]:

\[
v_{i+1} = w \times v_i + RAND \times c_1 \times (P_{best} - x_i) + rand \times c_2 \times (G_{best} - x_i)
\]

(17)

\[
x_{i+1} = x_i + v_{i+1}
\]

(18)

where \( w \) is the inertia weight factor, \( v_i \) is the velocity of particle \( i \), \( x_i \) is the particle position, \( c_1 \) and \( c_2 \) are two positive constant parameters called acceleration coefficients. \( RAND \) and \( rand \) are the random numbers in the range \([0, 1]\), \( P_{best} \) is the best position of the \( i^{th} \) particle and \( G_{best} \) is the best position among all particles in the swarm.

In the proposed sliding mode control law (16), there are 10 parameters \( (\lambda, \xi_i, k_i, i = 1, 2, 3 \text{ and } \varepsilon) \) left to be selected by PSO. In order to design the PSO-based SMC, the performance criterion or objective function should be defined first. In general, three kinds of performance criteria, the integrated absolute error (IAE), the integral of squared-error (ISE), and the integrated of time-weighted-squared-error (ITSE) are usually considered in controller design under step testing input, as they can be evaluated analytically in the frequency domain. It is worthy to notice that using different performance indices probably makes different solutions for PSO-based SMC. The three integral performance criteria in the frequency domain have their own
advantages and disadvantages. For example, a disadvantage of the IAE and ISE criteria is that their minimization can result in a response with relatively small overshoot but a long settling time. Although the ITSE performance criterion can overcome this, the derivation processes of the analytical formula are complex and time-consuming. However, in this paper, another performance criterion as cost function is defined by

$$J(K) = \int_0^\infty w_1 x^2(\tau) + w_2 u^2(\tau) d\tau$$

where \( K = [\lambda_1, \lambda_2, \xi_1, \xi_2, k_1, k_2, k_3, \varepsilon] \) is the parameter vector and \( w_1 \) and \( w_2 \) are weights. One can see that the optimum value of the cost function corresponds to the minimum state error and minimum used control energy.

4. NUMERICAL SIMULATIONS

In this section, numerical simulations are presented to verify the efficiency and effectiveness of the proposed PSO-based SMC. Numerical simulations are carried out using the MATLAB software. In both examples, weights of the cost function (i.e. \( w_1 \) and \( w_2 \)) are set equal to 1. Here, the following uncertainties and external disturbances are added to the BLDCM system.

$$\Delta f_i(x, t) = 0.5 \sin(x_i) - 0.1 \cos(2t),$$
$$\Delta f_2(x, t) = 0.3 \cos(3x_2) + 0.1 \cos(5t),$$
$$\Delta f_3(x, t) = -0.4 \sin(2x_3) + 0.2 \sin(3t),$$
$$d_1(t) = 0.3 \cos(5t),$$
$$d_2(t) = 0.2 \sin(3t),$$
$$d_3(t) = -0.25 \cos(2t)$$

The initial conditions of the BLDCM system is selected as \( x(0) = [1, -1, 2] \). The lower and upper bounds of the controller parameters are specified as \( 0 < c_i, \xi_i, k_i < 10 \) and \( 0 < \varepsilon < 10 \). The population size of the swarm is considered to be 10. The maximum number of iterations is selected equal to 1000. Inertial coefficient, \( w \), is set as a gradually decreasing function of iterations. Both parameters \( c_1 \) and \( c_2 \) are chosen equal to 2.

After running the PSO-based SMC, the optimal value of the vector parameter is found as \( K = [5.22, 4.65, 4.34, 1.1, 0.95, 0.4, 0.54, 0.83, 1.2, 93] \). Results of controlling the chaotic BLDCM with optimal parameters found by PSO are illustrated in Fig. 2. It can be seen that the closed loop system is not chaotic anymore and therefore, there is no strange attractor after the controller is applied. The applied control inputs are depicted in Fig. 3. It can be seen that the control inputs are bounded and feasible in practice. Also, it is obvious that not only the system is stabilized asymptotically around the origin, but also the closed loop system is robust against model uncertainties and external disturbances.

5. CONCLUSIONS

In this paper, a PSO-based robust sliding mode controller is introduced to stabilize chaotic nonlinear brushless dc motors (BLDCMs). It is assumed that all states of BLDCM are perturbed by unknown model uncertainties and external disturbances. Then, a robust sliding mode controller is designed to guarantee the existence of the sliding motion. Robustness and stability of the proposed controller is proved using Lyapunov stability theory. PSO algorithm is used to find the parameters of the proposed controller in an optimal manner. Numerical simulations show that the proposed controller works well for stabilization and chaos suppression.
of the BLDCM system, even in the presence of both unknown model uncertainties and external disturbances in the system dynamics.

REFERENCES


