THE FORCING EDGE-TO-VERTEX DETOUR NUMBER OF A GRAPH

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Abstract: For two vertices \( u \) and \( v \) in a graph \( G = (V, E) \), the detour distance \( D(u, v) \) is the length of a longest \( u - v \) path in \( G \). A \( u - v \) path of length \( D(u, v) \) is called a \( u - v \) detour. For subsets \( A \) and \( B \) of \( V \), the detour distance \( D(A, B) \) is defined as \( D(A, B) = \min \{ D(x, y) : x \in A, y \in B \} \). A \( u - v \) path of length \( D(A, B) \) is called an \( A - B \) detour if \( x \) is a vertex of an \( A - B \) detour. A set \( S \subseteq E \) is called an edge-to-vertex detour set if every vertex of \( G \) is incident with an edge of \( S \) or lies on a detour joining a pair of edges of \( S \). The edge-to-vertex detour number \( dn^2(G) \) of \( G \) is the minimum order of its edge-to-vertex detour sets and any edge-to-vertex detour set of order \( dn^2(G) \) is an edge-to-vertex detour basis of \( G \). A subset \( T \) of an edge-to-vertex detour basis \( S \) is called a forcing subset for \( S \) if \( S \) is the minimum forcing subset of \( S \). The forcing edge-to-vertex detour number of \( S \), denoted by \( fdn^2(S) \), is the cardinality of a minimum forcing subset for \( S \). The forcing edge-to-vertex detour number of \( G \), denoted by \( fdn^2(G) \), is \( fdn^2(G) = \min \{ fdn^2(S) \} \), where the minimum is taken over all edge-to-vertex detour bases \( S \) in \( G \). The forcing edge-to-vertex detour numbers of certain standard graphs are obtained. It is shown that for every pair \( a, b \) of integers with \( 0 \leq a \leq b \) and \( b \geq 2 \) there exists a connected graph \( G \) with \( fdn^2(G) = a \) and \( dn^2(G) = b \).

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1. INTRODUCTION

By a graph \( G = (V, E) \) we mean a finite undirected graph without loops or multiple edges. The order and size of \( G \) are denoted by \( p \) and \( q \) respectively. We consider connected graphs with at least two vertices. For basic definitions and terminologies we refer to [1, 5]. For vertices \( u \) and \( v \) in a connected graph \( G \), the detour distance \( D(u, v) \) is the length of a longest \( u - v \) path in \( G \). A \( u - v \) path of length \( D(u, v) \) is called a \( u - v \) detour. It is known that the detour distance is a metric on the vertex set \( V \). The detour distance was studied in [2, 4].

A vertex \( x \) is said to lie on a \( u - v \) detour \( P \) if \( x \) is a vertex of \( P \) including the vertices \( u \) and \( v \). A set \( S \subseteq V \) is called a detour set if every vertex \( v \) in \( G \) lies on a detour joining a pair of vertices of \( S \). The detour number \( dn(G) \) of \( G \) is the minimum order of a detour set and any detour set of order \( dn(G) \) is called a detour basis of \( G \). These concepts were studied in [3]. The detour concepts and colorings are widely
Definition 1.1: ([6]) Let $G = (V, E)$ be a connected graph with at least two vertices. A set $S \subseteq V$ is called a weak edge detour set of $G$ if every edge in $G$ has both its ends in $S$ or it lies on a detour joining a pair of vertices of $S$. The weak edge detour number $dn_w(G)$ of $G$ is the minimum order of its weak edge detour sets and any weak edge detour set of order $dn_w(G)$ is called a weak edge detour basis of $G$.

Example 1.2: For the graph $G$ given in Fig. 1.1, it is clear that the set $S = \{v_1, v_2\}$ is a weak edge detour basis of $G$ so that $dn_w(G) = 2$. For the graph $G$ given in Fig. 1.2, it is clear that no two element subset of $V$ is a weak edge detour set of $G$. The set $S = \{v_1, v_2, v_3\}$ is a weak edge detour basis of $G$ so that $dn_w(G) = 3$. The set $S_1 = \{v_1, v_4, v_5\}$ is another weak edge detour basis of $G$.
**Definition 1.3:** ([7]) Let $G = (V, E)$ be a connected graph with at least two vertices. A set $S \subseteq V$ is called an edge detour set of $G$ if every edge in $G$ lies on a detour joining a pair of vertices of $S$. The edge detour number $dn_1(G)$ of $G$ is the minimum order of its edge detour sets and any edge detour set of order $dn_1(G)$ is called an edge detour basis of $G$. A graph $G$ is called an edge detour graph if it has an edge detour set.

**Example 1.4:** For the graph $G$ given in Fig. 1.2, it is clear that no two element subset of $V$ is an edge detour set of $G$. The set $S = \{v_1, v_4, v_5\}$ is an edge detour basis of $G$ so that $dn_1(G) = 3$ and hence it is an edge detour graph. But the graph $G$ given in Fig. 1.1 is not an edge detour graph.

**Definition 1.5:** [10] Let $G = (V, E)$ be a connected graph with at least three vertices. For subsets $A$ and $B$ of $V$, the detour distance $D(A, B)$ is defined as $D(A, B) = \min\{D(x, y): x \in A, y \in B\}$. A $u - v$ path of length $D(A, B)$ is called an $A - B$ detour joining the sets $A$ and $B$, where $u \in A$ and $v \in B$. A vertex $x$ is said to lie on an $A - B$ detour if $x$ is a vertex of an $A - B$ detour. For $A = \{u, v\}$ and $B = \{z, w\}$ with $uv$ and $zw$ edges, we write an $A - B$ detour as $uv - zw$ detour and $D(A, B)$ as $D(uv, zw)$.

**Example 1.6:** For the graph $G$ given in Fig. 1.3, with $A = \{v_1, v_2\}$ and $B = \{v_4, v_5, v_6\}$, $v_1, v_2, v_3, v_4$ and $v_1, v_6, v_2, v_4$ are the $v_1 - v_4$ detours, $v_1, v_2, v_3, v_4, v_6$ is the $v_1 - v_5$ detour, $v_1, v_2, v_3, v_4, v_5, v_6$ is the $v_1 - v_6$ detour, $v_2, v_1, v_6, v_5, v_4$ is the $v_2 - v_4$ detour, $v_2, v_1, v_6, v_4, v_5$ and $v_2, v_3, v_4, v_5, v_6$ are the $v_2 - v_5$ detours and $v_2, v_3, v_4, v_5, v_6$ is the $v_2 - v_6$ detour. Hence $D(A, B) = 3$ and an $A - B$ detour is a $v_1 - v_4$ detour so that $v_1, v_2, v_3, v_4$ and $v_1, v_6, v_5, v_4$ are the only two $A - B$ detours.

**Figure 1.3:** $G$

**Definition 1.7:** [10] Let $G = (V, E)$ be a connected graph with at least three vertices. A set $S \subseteq V$ is called an edge-to-vertex detour set of $G$ if every vertex of $G$ is incident with an edge of $S$ or lies on a detour joining a pair of edges of $S$. The edge-to-vertex detour number $dn_2(G)$ of $G$ is the minimum cardinality of its edge-
to-vertex detour sets and any edge-to-vertex detour set of cardinality $dn_2(G)$ is an edge-to-vertex detour basis of $G$.

**Example 1.8:** For the graph $G$ given in Fig. 1.4, the two $v_1v_2 - v_4v_5$ detours are $P : v_2, v_1, v_0, v_5$ and $Q : v_2, v_3, v_4, v_5$, each of length 3 so that $D(v_1v_2, v_4v_5) = 3$. Since the vertices $v_6$ and $v_3$ lie on the $v_1v_2 - v_4v_5$ detours $P$ and $Q$ respectively, $S_1 = \{v_1v_2, v_4v_5\}$ is an edge-to-vertex detour basis of $G$ so that $dn_2(G) = 2$. Also $S_2 = \{v_1v_6, v_3v_4\}$ is another edge-to-vertex detour basis of $G$. Thus there can be more than one edge-to-vertex detour basis for a graph.

![Figure 1.4: $G$](image)

The edge-to-vertex detour number of a graph was introduced and studied in [10]. The following theorem is used in the sequel.

**Theorem 1.9:** [10] Every end-edge of a connected graph $G$ belongs to every edge-to-vertex detour set of $G$. Also if the set $S$ of all end-edges of $G$ is an edge-to-vertex detour set, then $S$ is the unique edge-to-vertex detour basis for $G$.

Throughout this paper $G$ denotes a connected graph with at least three vertices.

### 2. THE FORCING EDGE-TO-VERTEX DETOUR NUMBER OF A GRAPH

**Definition 2.1:** Let $G$ be a connected graph and $S$ an edge-to-vertex detour basis of $G$. A subset $T \subseteq S$ is called a forcing subset for $S$ if $S$ is the unique edge-to-vertex detour basis containing $T$. A forcing subset for $S$ of minimum cardinality is a minimum forcing subset of $S$. The forcing edge-to-vertex detour number of $S$, denoted by $fdn_2(S)$, is the cardinality of a minimum forcing subset for $S$. The forcing edge-to-vertex detour number of $G$, denoted by $fdn_2(G)$, is $fdn_2(G) = \min \{fdn_2(S)\}$, where the minimum is taken over all edge-to-vertex detour bases $S$ in $G$.

**Example 2.2:** For the graph $G$ given in Fig. 2.1(a), it is easily verified that no two element subset of $E$ is an edge-to-vertex detour set of $G$. Also, it is clear that the set $S = \{ux, yz, wv\}$ is the unique edge-to-vertex detour basis of $G$ so that $dn_2(G) = 3$.
and \( fdn_2(G) = 0 \). For the graph \( G \) given in Fig. 2.1(b), any set \( S = \{ uv, e \} \) of two edges, where \( e \in E - \{ xu, uv, vy \} \) is an edge-to-vertex detour set of \( G \) so that \( dn_2(G) = 2 \). It is easily seen that no two element subset of \( E - \{ uv \} \) is an edge-to-vertex detour set of \( G \) so that every edge-to-vertex detour basis of \( G \) contains the edge \( uv \). Hence it follows that \( fdn_2(G) = 1 \). Also for the complete graph \( G = K_3 \), it is easily seen that \( fdn_2(G) = 2 \).

![Figure 2.1: G](image)

The next theorem follows immediately from the definitions of edge-to-vertex detour number and forcing edge-to-vertex detour number of a connected graph \( G \).

**Theorem 2.3:** For every connected graph \( G \), \( 0 \leq fdn_2(G) \leq dn_2(G) \).

**Remark 2.4:** The bounds in Theorem 2.3 are sharp. For any path \( P_n (n \geq 3) \), it is clear that the set of two end-edges is the unique edge-to-vertex detour basis so that \( fdn_2(P_n) = 0 \). For the cycle \( C_3 \), any set of two edges is an edge-to-vertex detour basis so that \( dn_2(C_3) = fdn_2(C_3) = 2 \). Also, the inequality in Theorem 2.3 can be strict. For the cycle \( C_4 \), it is clear that any set of two independent edges is an edge-to-vertex detour basis and so \( dn_2(C_4) = 2 \) and \( fdn_2(C_4) = 1 \). Thus \( 0 < fdn_2(G) < dn_2(G) \).

The following theorem is an easy consequence of the definition of forcing edge-to-vertex detour number of a graph.

**Theorem 2.5:** Let \( G \) be a connected graph. Then

(i) \( fdn_2(G) = 0 \) if and only if \( G \) has a unique edge-to-vertex detour basis,

(ii) \( fdn_2(G) = 1 \) if and only if \( G \) has at least two edge-to-vertex detour bases, one of which is a unique edge-to-vertex detour basis containing one of its elements, and

(iii) \( fdn_2(G) = dn_2(G) \) if and only if no edge-to-vertex detour basis of \( G \) is the unique edge-to-vertex detour basis containing any of its proper subsets.

An edge that belongs to each edge-to-vertex detour basis is called an **edge-to-vertex detour edge**.
Theorem 2.6: Let $G$ be a connected graph and $W$ be the set of all edge-to-vertex detour edges of $G$. Then $fdn_2(G) \leq dn_2(G) - |W|$. 

Proof: Let $S$ be an edge-to-vertex detour basis of $G$. Then $dn_2(G) = |S|$, $W \subseteq S$ and $S$ is the unique edge-to-vertex detour basis containing $S - W$. Hence $fdn_2(S) \leq |S - W| = |S| - |W| = dn_2(G) - |W|$ and the result follows.

Remark 2.7: The bound in Theorem 2.6 is sharp. For the graph $G$ given in Figure 2.1(b), $dn_2(G) = 2$, $|W| = 1$ and $fdn_2(G) = 1$ as in Example 2.2. Also, the inequality in Theorem 2.6 can be strict. As in Remark 2.4, for the cycle $C_4$, $dn_2(C_4) = 2$ and $fdn_2(G) = 1$. Further, there is no edge-to-vertex detour edge so that $|W| = 0$. Thus $fdn_2(G) < dn_2(G) - |W|$.

Theorem 2.8: (i) For the complete graph $K_p$ ($p \geq 4$), a set $S$ of edges is an edge-to-vertex detour basis if and only if $S$ consists of two independent edges of $K_p$.

(ii) For the complete bipartite graph $K_{m,n}$ ($2 \leq m \leq n$), a set $S$ of edges is an edge-to-vertex detour basis if and only if $S$ consists of two independent edges of $K_{m,n}$.

Proof: (i) Let $S = \{e, f\}$ be any set of two independent edges of $K_p$. Then it is clear that $D(e,f) = p - 1$ and hence it follows that $S$ is an edge-to-vertex detour set of $K_p$. Now, let $S$ be an edge-to-vertex detour basis of $K_p$. Let $S'$ be any set consisting of two independent edges. Then as in the first part of this theorem $S'$ is an edge-to-vertex detour basis of $K_p$. Hence $|S| = |S'| = 2$. Let $S = \{e, f\}$. If $e$ and $f$ are not independent, then $D(e,f) = 0$ and since $p \geq 4$, $S$ can not be an edge-to-vertex detour set of $G$, which is a contradiction. Thus $S$ consists of two independent edges.

(ii) Let $X$ and $Y$ be the bipartite sets of $K_{m,n}$ ($2 \leq m \leq n$) with $|X| = m$ and $|Y| = n$ and let $S = \{uv, zw\}$ be a set of any two independent edges of $K_{m,n}$ such that $u, z \in X$ and $v, w \in Y$. We show that $S$ is an edge-to-vertex detour basis of $K_{m,n}$.

Case 1: $m = n = 2$. Then $K_{m,n} = C_4$ and it is clear that every vertex of $K_{m,n}$ is incident with an edge of $S$ so that $S$ is an edge-to-vertex detour basis of $K_{m,n}$.

Case 2: $2 \leq m \leq n$ and $n \neq 2$. We consider two subcases:

Subcase 1: $m < n$. It is clear that $D(u, z) = 2 (m - 1)$, $D(u, w) = D(v, z) = 2m - 1$, $D(v, w) = 2m$ and so $D(uv, zw) = 2 (m - 1)$. Let $y \in Y$ be any vertex different from $v$ and $w$. If $m > 2$, consider any set of $m - 2$ vertices $x_1, y, x_2, \ldots, y_{m-2}$ from $Y - \{v, w\}$. Then the vertex $y$ lies on the $uv - wz$ detour $P : u = x_1, y, x_2, y_1, x_3, y_2, \ldots, x_{m-1}, y_{m-2}$, $x_m = z$, where $x_1, x_2, \ldots, x_m \in X$. If $m = 2$, then $y$ lies on the $uv - wz$ detour $Q : u, y, z$. 

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Since every vertex of $X$ also lies on the same detour $P$ and $Q$ in respective cases, it follows that $S$ is an edge-to-vertex detour basis of $K_{m,n}$ and hence $dn_2(K_{m,n}) = 2$.

**Subcase 2: $m = n$.** It is clear that $D(u, z) = D(v, w) = 2(m - 1)$, $D(u, w) = D(v, z) = 2m - 1$ and so $D(uv, zw) = 2(m - 1)$. Also $P : u, v, x_1, y_1, x_2, y_2, ..., x_m, y_m, z$, where $u, x_1, x_2, ..., x_m, z \in X$ and $v, y_1, y_2, ..., y_m \in Y$ with $w \neq v_i (1 \leq i \leq m - 2)$ is a $uv - zw$ detour containing all vertices of $K_{m,n}$ other than the vertex $w$. Since $w$ is incident with the edge $zw$, it follows that $S$ is an edge-to-vertex detour basis of $K_{m,n}$.

The proof of the converse is similar to that of Theorem 2.8(i).

**Corollary 2.9:** (i) If $G$ is the complete graph $K_p (p \geq 3)$, then $dn_2(G) = 2$.

(ii) If $G$ is the complete bipartite graph $K_{m,n}$ $(2 \leq m \leq n)$, then $dn_2(G) = 2$.

**Theorem 2.10:** For any cycle $G = C_p (p \geq 3)$, $dn_2(G) = 2$.

**Proof:** For $p = 3$, the result follows from the Corollary 2.9(i). For $p \geq 4$, let $C_p : v_1, v_2, ..., v_{p-1}, v_p, v_1$ be the cycle of length $p \geq 4$. Let $S = \{v_1v_2, v_{p-1}v_p\}$. Then $S$ is an edge-to-vertex-detour basis of $C_p$ and so $dn_2(G) = 2$.

**Theorem 2.11:** For the complete graph $K_p (p \geq 3)$,

(i) $dn_2(K_p) = fdn_2(K_p) = 2$ for $p = 3$ or $p \geq 5$.

(ii) $dn_2(K_p) = 2$ and $fdn_2(K_p) = 1$ for $p = 4$.

**Proof:** (i) For $p = 3$, this follows from Corollary 2.9(i). Let $p \geq 5$. Then by Theorem 2.8(i), a set $S$ of two edges is an edge-to-vertex detour basis of $K_p$ if and only if $S$ consists of two independent edges of $K_p$. For each edge $e$ in $K_p (p \geq 5)$ there are at least two edges independent with $e$. Thus the edge $e$ belongs to more than one edge-to-vertex detour basis of $K_p$. Hence it follows that no set consisting of a single edge is a forcing subset for any edge-to-vertex detour basis of $K_p$. Thus $fdn_2(K_p) = 2$. Also, by Corollary 2.9(i), $dn_2(K_p) = 2$ and the result follows.

(ii) By Corollary 2.9(i), $dn_2(K_p) = 2$ when $p = 4$. Let $v_1, v_2, v_3, v_4$ be the vertices of $K_4$. Then it is clear that the edge-to-vertex detour bases are $S_1 = \{v_1v_2, v_3v_4\}$, $S_2 = \{v_1v_4, v_2v_3\}$ and $S_3 = \{v_1v_3, v_2v_4\}$ and hence it follows that $fdn_2(K_4) = 1$.

**Theorem 2.12:** For the cycle $C_p (p \geq 4)$,

(i) $dn_2(C_p) = 2$ and $fdn_2(C_p) = 1$ for $p = 4$.

(ii) $dn_2(C_p) = fdn_2(C_p) = 2$ for $p \geq 5$.

**Proof:** (i) By Theorem 2.10, $dn_2(C_4) = 2$. Let $v_1, v_2, v_3, v_4$ be the vertices of $C_4$. Then it is clear that the edge-to-vertex detour bases are $S_1 = \{v_1v_2, v_3v_4\}$ and $S_2 = \{v_1v_4, v_2v_3\}$ and hence it follows that $fdn_2(C_4) = 1$. 

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(ii) Let $C_p$ be the cycle $v_1, v_2, ..., v_{p-1}, v_p, v_1$. For any edge, say $v_i v_{i+1}$ of $C_p$, it is clear that the sets $S_1 = \{v_1 v_2, v_1 v_3\}$, $S_2 = \{v_1 v_2, v_{p-1} v_p\}$ of edges $C_p$ are edge-to-vertex detour bases of $C_p$. Thus an edge $e$ belongs to more than one edge-to-vertex detour basis of $C_p$. So it follows that no single edge is a forcing subset for any edge-to-vertex detour basis of $C_p$ and hence $fdn_2(C_p) = 2$. By Theorem 2.10, $dn_2(C_p) = 2$ and the result follows.

**Theorem 2.13:** For the complete bipartite graph $K_{m,n} \neq K_{2,2}$ ($2 \leq m \leq n$), $dn_2(K_{m,n}) = fdn_2(K_{m,n}) = 2$.

**Proof:** By Theorem 2.8(ii), any set of two edges $e$ and $f$ in $K_{m,n}$ ($2 \leq m \leq n$) is an edge-to-vertex detour basis of $K_{m,n}$ if and only if $e$ and $f$ are independent. It is clear that for each edge $e$ in $K_{m,n}$ there are at least two edges independent with $e$. Thus the edge $e$ belongs to more than one edge-to-vertex detour basis of $K_{m,n}$. Hence it follows that no set consisting of a single edge is a forcing subset for any edge-to-vertex detour basis of $K_{m,n}$. Thus $fdn_2(K_{m,n}) = 2$. Also, by Corollary 2.9(ii), $dn_2(K_{m,n}) = 2$ and the result follows.

**Theorem 2.14:** If $G$ is a tree of order $p \geq 3$ with $k$ end-vertices, then $dn_2(G) = k$ and $fdn_2(G) = 0$.

**Proof:** The set of all end-edges of a tree is the unique edge-to-vertex detour basis and so the result follows from Theorem 1.9 and Theorem 2.5(i).

In view of Theorem 2.3, we have the following realization result.

**Theorem 2.15:** For each pair $a, b$ of integers with $0 \leq a \leq b$ and $b \geq 2$, there is a connected graph $G$ with $fdn_2(G) = a$ and $dn_2(G) = b$.

**Proof. Case 1:** $a = 0$. For each $b \geq 2$, let $G$ be a tree with $b$ end-vertices. Then $fdn_2(G) = 0$ and $dn_2(G) = b$ by Theorem 2.14.

**Case 2:** $a \geq 1$. For each $i \ (1 \leq i \leq a)$, let $F_i : u_i, v_i, w_i$ be a path of order 3 and let $H = K_{1, b-a}$ be the star at $v$ whose set of end-vertices is $\{z_1, z_2, ..., z_{b-a}\}$. Let $G$ be the graph $G = G_1 \cup G_2 \cup H$ such that $G_1$ is a tree with $a$ end-vertices, $G_2$ is a tree with $b-a$ end-vertices, and $H$ is a star with $b$ end-vertices.
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graph obtained by joining the central vertex ν of H to both vertices u_i, w_i of each F_i (1 ≤ i ≤ a). The graph G is connected and is shown in Fig. 2.2.

Let W = \{v_{z1}, v_{z2}, ..., v_{zb-a}\} be the set of all (b – a) end-edges of G. First, we show that \(dn_2(G) = b\). By Theorem 1.9, every edge-to-vertex detour basis contains W. Also, it is clear that every edge-to-vertex detour basis must contain at least one edge from each F_i (1 ≤ i ≤ a). Thus \(dn_2(G) ≥ (b – a) + a = b\). Let S = W \cup \{e_1, e_2, ..., e_a\}, where \(e_i ∈ \{u_i, v_i, v_i, w_i\}\ (1 ≤ i ≤ a)\). Then it is clear that S is an edge-to-vertex detour set of G so that \(dn_2(G) ≤ |S| = b\). Therefore, \(dn_2(G) = b\).

Next we show that \(fdn_2(G) = a\). It is clear that W is the set of all edge-to-vertex detour edges of G. Hence it follows from Theorem 2.6 that \(fdn_2(G) ≤ dn_2(G) – |W| = b – (b – a) = a\). Now, since \(dn_2(G) = b\), it is easily seen that a set S is an edge-to-vertex detour basis of G if and only if S is of the form S = W \cup \{e_1, e_2, ..., e_a\}, where \(e_i ∈ \{u_i, v_i, v_i, w_i\}\ (1 ≤ i ≤ a)\). Let T be a subset of S with |T| < a. Then there is an edge \(e_j (1 ≤ j ≤ a)\) such that \(e_j ∈ T\). Let \(f_j\) be an edge of \(F_j\) distinct from \(e_j\). Then \(S' = (S – \{e_j\}) \cup \{f_j\}\) is an edge-to-vertex detour basis that contains T. Thus S is not the unique edge-to-vertex detour basis containing T. Thus \(fdn_2(S) ≥ a\). Since this is true for all edge-to-vertex detour basis of G, it follows that \(fdn_2(G) ≥ a\) and so \(fdn_2(G) = a\).

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