EOQ MODEL WITH PERMISSIBLE DELAY IN PAYMENTS UNDER PROGRESSIVE INTEREST SCHEME

Dinesh Prasad & Ashutosh Kansal

ABSTRACT: The main purpose of this paper is to investigate the retailer’s optimal replenishment policy under permissible delay in payments within the economic order quantity (EOQ) framework. Previously published articles dealing with optimal order quantity with permissible delay in payments assumed that the supplier only offers the retailer single trade credit period. In fact, most suppliers frequently offer retailers (or customers) a two trade credit period to attract new retailers (customers) from their competitors. In this paper, the retailer faces a progressive interest charge from the supplier. If the retailer pays the outstanding balance by the grace period (say, $M$), the supplier does not charge any interest to the retailer but if the outstanding amount is paid after $M$, but by $N$ (with $N > M$), then the supplier charges the retailer the lower interest rate of $I_c_1$ on the unpaid balance. If the retailer pays the outstanding amount after $N$, the supplier charges the regular interest rate of $I_c_2$ (with $I_c_2 > I_c_1$).

We first establish an appropriate EOQ model for a retailer when the supplier offers a progressive interest charge, and then provide an easy solution procedure to the problem. We also provide numerical example to illustrate the proposed model and sensitivity analysis is also carried out.

Keywords: EOQ, Deterioration, Permissible delay.

1. INTRODUCTION

The traditional economic order quantity (EOQ) model assumes that the buyer must pay off as soon as the items are received. In practice, the supplier hopes to stimulate his products and so he offers the retailer a delay period, namely, the trade credit period. In fact, allowing customers to delay payment for goods already delivered is a very common business practice. Suppliers often offer credit as a marketing strategy to increase sales and reduce on-hand stock level. The permissible delay in payments produces two benefits to the supplier: (1) it not only encourages customers to order more, but also attract new customers, and (2) it may be applied as an alternative to price discount because it does not provoke competitors to reduce their prices and thus introduce lasting price reductions. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. However, if the payment is not paid in full by the end of the permissible delay period, then interest is charged on the outstanding amount. Therefore it makes economic sense for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the supplier. In fact, buyers, especially small business which tend to have a limited number of financing opportunities, rely on trade credit as a source of short-term funds.

Goyal [1] first developed an economic order quantity (EOQ) model under the condition of permissible delay in payments. Aggarwal and Jaggi [2] extended Goyal’s model to the case of deterioration. Jamal et al., [3] analyzed Aggarwal and Jaggi’s model to allow for shortages. Teng [4] amended Goyal’s model by considering the difference between unit price and unit cost. Chung and Huang [5] proposed an economic production quantity (EPQ) inventory model for a retailer when the supplier offers a permissible delay in payments by assuming that the selling price is the same as the purchase cost. Huang [6] extended Goyal’s model to develop an EOQ model in which supplier offers the retailer, the permissible delay period $M$ (i.e. the supplier trade credit), and the retailer in turn provides the trade credit period $N$ (with $N < M$) to its customers (i.e. the retailer trade credit). Huang [7] incorporated both Chung and Huang [5] and Huang [6], to investigate the optimal retailer’s replenishment decisions with two levels of trade credit policy in the EPQ framework. A numerous studies dealing with the trade credit problem have been presented by Shinn and Hwang [7], Chung and Liao [(8), (9)], Chung et al., [10] and etc.
Huang [11] proposed the optimal replenishment decisions in the EPQ model under two levels of trade credit policy. Goyal et al., [12] analyzed an inventory model with progressive interest rate. Teng [13] established an EOQ model for a retailer who receives a full trade credit by its supplier and offers its bad credit customers a partial trade credit and full trade credit to its good credit customers. Min et al., [14] presented an inventory model with stock-dependent demand rate in which retailer enjoys a fixed credit period offered by the supplier and in turn also offers a credit period to its customers. Recently an EOQ model is developed for a deteriorating item by Khara et al., [15] with demand rate taken to be a quadratic function of time under two different circumstances viz. (i) the credit period is less than or equal (ii) the credit period is greater than the cycle time for settling the account.

In real life situation, the effect of decay and deterioration play an important role in many inventory systems. As a matter of fact, most of the physical goods (e.g., fruits, vegetables, pharmaceuticals, volatile liquids etc) deteriorate continuously due to spoilage, obsolescence, evaporation etc). Ghare and Schrader [16] first derived an EOQ model by assuming exponential decay. Several related articles were published by a number of researchers (Maity et al., [17], Rong et al., [18], Lee and Hsu [19]). Later on, Kansal and Prasad [20] investigated an inventory model with time proportional deterioration, in which demand is linearly time varying over a finite planning horizon. Recently Prasad et al., [21] presented a partial backlogging EOQ model with Weibull deterioration under inflation.

In this paper we developed an EOQ model for deteriorating items when supplier offers a progressive interest rate to its retailer. If the retailer pays the outstanding balance by the grace period (say $M$), the supplier does not charge any interest but if the outstanding amount is paid after $M$, but by $N$, (with $N > M$), the supplier charges the interest rate $Ic_1$ from the retailer on the unpaid balance. If the retailer pays the outstanding amount after $N$, the supplier charges the interest rate of $Ic_2$ ($Ic_2 > Ic_1$). We provide an easy solution procedure to the developed model.

2. ASSUMPTIONS AND NOTATION

The following assumptions and notations are used throughout the paper.

1. The demand is constant with time.
2. Shortages are not allowed.
3. Replenishment is instantaneous.
4. Time horizon is infinite
5. The supplier provides the retailer trade credits as follows:
   - If the retailer pays by $M$, the supplier does not charge any interest.
   - If the retailer pays after $M$ but before $N$, the supplier charges the retailer an interest rate of $Ic_1$.
   - If the retailer pays after $N$, he will be charged an interest rate of $Ic_2$ ($Ic_2 > Ic_1$). $M$ and $N$ are taken in fraction of year.

In addition we use the following notations throughout the paper:

- $D$: the demand rate per year
- $c_p$: the holding cost per year excluding interest charges
- $\theta$: the deterioration rate, $0 \leq \theta < 1$
- $c$: the unit purchase cost
- $p$: the selling price per unit, with $c < p$
- $M$: the first period of the permissible delay in settling account without any charges.
- $N$: the second period of permissible delay in settling account with an interest charge of $Ic_1$ and $N > M$. 


The interest charged per $ in stock per year by the supplier when the retailer pays after \( M \) but before \( N \).

\( Ic_2 \) the interest charged per $ in stock per year by the supplier when the retailer pays after \( N \).

\( Ie \) the interest earned per $ per year

\( A \) the ordering cost per order

\( Q \) the ordering quantity

\( T \) the replenishment cycle time in years

\( I(t) \) the level of inventory at time \( t, 0 \leq t \leq T \)

\( TC(T) \) the total annual relevant cost

3. MATHEMATICAL FORMULATION

The inventory level gradually decreases due to combined effect of demand and deterioration. The differential equation governing the instantaneous state of \( I(t) \) is given by:

\[
\frac{dI(t)}{dt} + \theta I(t) = -D, \quad 0 \leq t \leq T
\]

(1)

With the boundary conditions: \( I(0) = Q, \) \( I(T) = 0 \),

The solution of equation (1) is given by

\[
I(t) = \frac{D}{\theta} \left[ e^{\theta(T-t)} - 1 \right].
\]

(2)

And the ordering quantity \( Q \) is given by

\[
Q = I(0) = \frac{D}{\theta} \left[ e^{\theta T} - 1 \right].
\]

(3)

Annual ordering cost = \( \frac{A}{T} \)

Annual stock holding cost excluding interest charge

\[
= \frac{c_h D}{T \theta} \left[ e^{\theta T} - \theta T - 1 \right].
\]

(4)

Annual deterioration cost

\[
= \frac{cD}{T \theta} \left[ e^{\theta T} - \theta T - 1 \right].
\]

(5)

Regarding interest charged and earned, based on the length of the replenishment cycle \( T \), we have three possible cases:

\[
\begin{align*}
(1) \quad & T \leq M \\
(2) \quad & M < T < N \\
(3) \quad & T \geq N
\end{align*}
\]

Case (1): \( T \leq M \)

In this case, the retailer sells \( Q \) units during \([0, T]\) and has \( cQ \) dollars to pay the supplier in full at time \( M \); consequently there is no interest payable, so interest charges are zero. However during \([0, T]\) period, the retailer sells \( Q \) units, and deposits the revenue in to an account that earns \( Ie/\$\) per year. During the period \([T, M]\) the retailer keep on depositing the total revenue in to the account to earn interest.
Therefore, the interest earned per year is

$$\frac{pI_e}{T} \left[ \int_0^T D\, dt + DT(m - T) \right] = pI_e \left( M - \frac{T}{2} \right).$$  \hspace{1cm} (6)$$

The total relevant cost per year is

$$TC_1(T) = \frac{A}{T} + \frac{D(c_h + c\theta)}{T\theta^2} \left[ e^{\theta T} - \theta T - 1 - pI_eD \left( M - \frac{T}{2} \right) \right].$$  \hspace{1cm} (7)$$

**Case (2):** \( M < T < N \)

During the period \([0, M]\), the retailer sells products and deposits the revenue in to an account that earns an interest of \(I_e/\$\) year. Therefore, the interest earned during the period \([0, M]\) is

$$pI_e \int_0^M D\, dt = \frac{1}{2} pI_eDM^2.$$  \hspace{1cm} (8)$$

The retailer buys \(Q\) units at time \(t = 0\) and owes \(cQ\) dollars to the supplier. At time \(M\), the retailer sells \(DM\) units in total and has \(pDM\) dollars plus interest earned \(\frac{1}{2} pI_eDM^2\) dollars to pay the supplier. From the difference

**Figure 1:** Graphical Representation of Case 1

**Figure 2:** Graphical Representation of Case 2
between the total purchase cost $cQ$ and the total amount of money in the account $pDM + \frac{1}{2}pl_eDM^2$. There are two possible sub-cases:

**Sub-Case (2.1):** $pDM + \frac{1}{2}pl_eDM^2 \geq cQ$

In this sub-case, the retailer has enough money in his/her account to pay off the total purchase cost at time $M$. Hence the total purchase cost is paid at $M$, and there is no interest charge. The interest earned per year is $\frac{pl_e}{T} \int_0^M Dtdt = \frac{pl_eDM^2}{2T}$. Therefore, the total relevant cost per year

$$TC_{2.1}(T) = \frac{A}{T} + \frac{D(c_h + c\theta)}{T\theta^2} [e^{\theta T} - \theta T - 1] - \frac{pl_eDM^2}{2T}. \tag{9}$$

**Sub-Case (2.2)** $pDM + \frac{1}{2}pl_eDM^2 < cQ$

In this sub-case the supplier starts to charge the retailer for unpaid balance with interest rate $Ic_1$ at time $M$. Interest payable per unit time is

$$= \frac{cIc_1}{T} \int_0^T \frac{D}{\theta} [e^{\theta (T-t)} - 1] dt = \frac{cIc_1D}{\theta^2T} [e^{\theta (T-M)} - \theta(T-M) - 1].$$

And interest earned per year is $\frac{pl_eDM^2}{2T}$

The total relevant cost per year is

$$TC_{2.2}(T) = \frac{A}{T} + \frac{D(c_h + c\theta)}{T\theta^2} [e^{\theta T} - \theta T - 1] - \frac{pl_eDM^2}{2T}$$

$$+ \frac{cIc_1D}{\theta^2T} [e^{\theta (T-M)} - \theta(T-M) - 1]. \tag{10}$$

**Case (3):** $T \geq N$

Based on the total purchase cost $cQ$, the total amount of money in account at $M$ is $pDM + \frac{1}{2}pl_eDM^2$ and the total amount of money in the account at $N$ is $pDM + \frac{1}{2}pl_eDM^2$, there are three possible sub-cases:

![Figure 3: Graphical Representation of Case 3](image-url)
Sub-case (3.1): This sub-case is the same as sub-case (2.1). Hence the retailer will pay the total purchase cost at $M$ and there is no interest charged. The total relevant cost per year is

$$TC_{3.1}(T) = \frac{A}{T} + \frac{D(c_h + c_0)}{T^2} [e^{\theta T} - \theta T - 1] - \frac{plDM^2}{2T}. \quad (11)$$

Sub-Case (3.2): $pDM + \frac{1}{2} pl_eDM^2 < cQ$

But

$$pD(N - M) + \frac{pl_eD(N - M)^2}{2} \geq cQ - pDM - \frac{pl_eDM^2}{2}.$$

In this sub-case the retailer has not enough money in his/her account to pay off the total purchase cost at time $M$, but he can pay off the total purchase cost before or on $N$. Hence the retailer only pays $pDM + \frac{1}{2} pl_eDM^2$ at $M$, and supplier starts to charge the retailer the unpaid balance with interest rate $Ic_1$ at time $M$. Therefore the sub-case is same as sub-case (2.2), and therefore the total relevant cost per year is

$$TC_{3.2}(T) = \frac{A}{T} + \frac{D(c_h + c_0)}{T^2} [e^{\theta T} - \theta T - 1] - \frac{pl_eDM^2}{2T}$$

$$+ \frac{clc_1D}{\theta^2T} [e^{\theta(T - M)} - \theta(T - M) - 1]. \quad (12)$$

Sub-Case (3.3): $pDM + \frac{1}{2} pl_eDM^2 < cQ$

But

$$pD(N - M) + \frac{pl_eD(N - M)^2}{2} < cQ - pDM - \frac{pl_eDM^2}{2}.$$

Since the retailer has not enough money in his/her account to pay off the total purchase cost at time $M$, he only pay $[pDM + \frac{1}{2} pl_eDM^2]$ at $M$ and $[pD(N - M) + \frac{1}{2} pl_eD(N - M)^2]$ at $N$. Hence the supplier charge the retailer an interest rate of $Ic_1$ during $[M, N]$ and an interest rate of $Ic_2$ after $N$. Therefore the interest payable per year is

$$= \frac{clc_1D}{\theta^2T} [e^{\theta(T - M)} - e^{\theta(T - N)} - \theta(N - M)] + \frac{clc_2D}{\theta^2T} [e^{\theta(T - N)} - \theta(T - N) - 1].$$

The total relevant cost per year is

$$TC_{3.3}(T) = \frac{A}{T} + \frac{D(c_h + c_0)}{T^2} [e^{\theta T} - \theta T - 1] - \frac{pl_eDM^2}{2T}$$

$$+ \frac{cD}{\theta^2T} [lc_1(e^{\theta(T - M)} - e^{\theta(T - N)} - \theta(N - M)) + [lc_2(e^{\theta(T - N)} - \theta(T - N) - 1)]. \quad (13)$$

4. THEORETICAL RESULTS

Now, we shall determine the optimal replenishment cycle time that minimizes the total relevant cost per year.
**Case 1:** The first order condition for $TC_1(T)$ to be minimized is $\frac{dTC_1(T)}{dT} = 0$ which implies that

\begin{equation}
-A - \frac{D(c_h + c\theta)}{\theta^2} [e^{\theta T} - \theta Te^{\theta T} - 1] + \frac{p I D T^2}{2} = 0. \tag{14}
\end{equation}

To show there exists a unique value of $T$ in $[0, M]$ at which $TC_1(T)$ is minimized, we let

\begin{equation}
\Delta_1 = -A - \frac{D(c_h + c\theta)}{\theta^2} [e^{\theta M} - \theta Me^{\theta M} - 1] + \frac{p I D M^2}{2} = 0. \tag{15}
\end{equation}

Now we have the following lemma:

**Lemma 1:**

i. If $\Delta_1 > 0$, the total relevant cost per year $TC_1(T)$ has a unique minimum value at the point $T = T_1$, where $T_1 \in [0, M]$ and satisfies equation (14).

ii. If $\Delta_1 = 0$, $TC_1(T)$ has a minimum value at the boundary point $T = M$.

iii. If $\Delta_1 < 0$, the value of $T \in [0, M]$, which minimizes $TC_1(T)$ does not exist.

**Proof:** See the appendix A.

**Case 2:** The first order necessary condition for $TC_{2.1}(T)$ to be minimum is $\frac{dTC_{2.1}(T)}{dT} = 0$ which implies:

\begin{equation}
-A - \frac{D(c_h + c\theta)}{\theta^2} [e^{\theta T} - \theta Te^{\theta T} - 1] + \frac{p I D M^2}{2} = 0. \tag{16}
\end{equation}

Also we let

\begin{equation}
\Delta_2 = -A - \frac{D(c_h + c\theta)}{\theta^2} [e^{\theta N} - \theta Ne^{\theta N} - 1] + \frac{p I D M^2}{2} = 0. \tag{17}
\end{equation}

It is obvious $\Delta_1 \leq \Delta_2$, when $M \leq N$. Now we have the following lemma:

**Lemma 2.1:**

i. If $\Delta_1 \geq 0 \leq \Delta_2$, the total relevant cost per year $TC_{2.1}(T)$ has the unique minimum value at the point $T = T_{2.1}$, where $T_{2.1} \in (M, N)$, and satisfies equation (16).

ii. If $\Delta_1 > 0$, the total relevant cost per year $TC_{2.1}(T)$ has a unique minimum value at the lower boundary $T = M$.

iii. If $\Delta_1 < 0$, the total relevant cost per year $TC_{2.1}(T)$ has a unique minimum value at the upper boundary $T = N$.

**Proof:** The proof is similar to that in Lemma 1. Hence we omit it.

Similarly the first order condition for $TC_{2.2}(T)$ to be minimized is $\frac{dTC_{2.2}(T)}{dT} = 0$, which leads to

\begin{equation}
-A - \frac{D(c_h + c\theta)}{\theta^2} [e^{\theta T} - \theta Te^{\theta T} - 1] + \frac{p I D M^2}{2} - \frac{c I D}{\theta^2} [e^{\theta(T - M)} - \theta Te^{\theta(T - M)} + \theta M - 1] = 0. \tag{18}
\end{equation}
To show that there exist a value of $T$ in the interval $(M, N)$, we let

\[
\Delta_3 = -A - \frac{D(c_h + c \theta)}{\theta^2} [e^{\theta N} - \theta Ne^{\theta N} - 1] + \frac{pLDM^2}{2} - \frac{clcD}{\theta^2} [e^{\theta(N-M)} - \theta Ne^{\theta(N-M)} + \theta M - 1] = 0.
\]  

(19)

It is obvious that $\Delta_1 \leq \Delta_3$ for $M \leq N$, then we have the following lemma:

**Lemma 2.2:**

i. If $\Delta_1 \leq 0 \leq \Delta_3$, the total relevant cost per year $TC_{2.2}(T)$ has the unique minimum value at the point $T = T_{2.2}$, where $T_{2.2} \in (M, N)$, and satisfies equation (18).

ii. If $\Delta_1 > 0$, the total relevant cost per year $TC_{2.2}(T)$ has a unique minimum value at the lower boundary $T = M$.

iii. If $\Delta_1 < 0$, the total relevant cost per year $TC_{2.2}(T)$ has a unique minimum value at the upper boundary $T = N$.

**Proof:** The proof is similar to that in Lemma 1. For simplicity, we omit it.

**Case 3:** Similar to the approach used in previous cases, the necessary condition for $TC_{3.1}(T)$ to be minimized is $\frac{dTC_{3.1}(T)}{dT} = 0$, which is same as equation (16), we have the following lemma:

**Lemma 3.1:**

i. If $\Delta_1 \leq 0$, the total relevant cost per year $TC_{3.1}(T)$ has a unique minimum value at the point $T = T_{3.1}$, where $T_{3.1} \in [N, \infty)$.

ii. If $\Delta_1 > 0$, the total relevant cost per year $TC_{3.1}(T)$ has a unique minimum value at the lower boundary $T = N$.

**Proof:** This proof is also similar to that of Lemma 1, so we omit it.

Also the first order necessary condition for $TC_{3.2}(T)$ to be minimum is $\frac{dTC_{3.2}(T)}{dT} = 0$, which is same as equation (18), we have the following lemma:

**Lemma 3.2:**

i. If $\Delta_1 \leq 0$, the total relevant cost per year $TC_{3.2}(T)$ has a unique minimum value at the point $T = T_{3.2}$, where $T_{3.2} \in [N, \infty)$.

ii. If $\Delta_1 > 0$, the total relevant cost per year $TC_{3.2}(T)$ has a unique minimum value at the lower boundary $T = N$.

**Proof:** This proof is also similar to that of Lemma 1, so we omit it.

Likewise, the first-order necessary condition for $TC_{3.3}(T)$ to be minimum is $\frac{dTC_{3.3}(T)}{dT} = 0$, which leads to

\[
-A - \frac{D(c_h + c \theta)}{\theta^2} [e^{\theta T} - \theta Te^{\theta T} - 1] + \frac{pLDM^2}{2}
- \frac{clcD}{\theta^2} [e^{\theta(T-M)} - \theta Te^{\theta(T-M)} - e^{\theta(T-N)} + \theta Te^{\theta(T-N)} - \theta(N-M)]
- \frac{clcD}{\theta^2} [e^{\theta(T-N)} - \theta Te^{\theta(T-N)} + \theta N - 1] = 0.
\]

(20)
Equation (20) becomes $\Delta_j$ when $T = N$.

We have following lemma:

**Lemma 3.3:**

i. If $\Delta_1 \leq 0$, the total relevant cost per year $TC_{3,3}(T)$ has a unique minimum value at the point $T = T_{3,3}$, where $T_{3,3} \in [N, \infty)$.

ii. If $\Delta_1 > 0$, the total relevant cost per year $TC_{3,3}(T)$ has a unique minimum value at the lower boundary $T = N$.

**Proof:** This proof is also similar to that of Lemma 1.

Combining lemma 1 to lemma 3.3 we have the following result:

**Theorem 1:**

<table>
<thead>
<tr>
<th>Situation</th>
<th>$TC^<em>(T^</em>)$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_1 &lt; 0, \Delta_2 \geq 0, \text{and } \Delta_3 \geq 0$</td>
<td>$\min {TC_{2,1}(T_{2,1}), TC_{2,2}(T_{2,2}), TC_{3,1}(T_{3,1})}$</td>
<td>$T_{2,1}$ or $T_{2,2}$ or $T_{3,1}$</td>
</tr>
<tr>
<td>$\Delta_1 \geq 0$</td>
<td>$\min {TC_1(T), TC_2(M), TC_2(M)}$</td>
<td>$T_1$ or $M$</td>
</tr>
<tr>
<td>$\Delta_2 &lt; 0, \text{and } \Delta_3 \geq 0$</td>
<td>$\min {TC_{2,1}(N), TC_{3,1}(N), TC_{3,2}(N), TC_{3,3}(N)}$</td>
<td>$T_{3,1}$ or $N$</td>
</tr>
<tr>
<td>$\Delta_1 &lt; 0$</td>
<td>$\min {TC_{2,2}(N), TC_{3,2}(T_{3,2}), TC_{3,3}(T_{3,3})}$</td>
<td>$N$ or $T_{3,2}$ or $T_{3,3}$</td>
</tr>
</tbody>
</table>

5. **NUMERICAL EXAMPLE**

To illustrate the proposed model in this paper the following examples have been taken:

**Example 1:** $A = 150$ $$/\text{order}, D = 2000$ $\text{units}, c_4 = 2$ $$/\text{unit/} \text{year}, \theta = 0.01, c = 5$ $$/\text{unit}, p = 10$ $$/\text{unit}, I = 0.08$$$/\text{year}, M = 0.2$ $\text{year}, N = 0.4$ $\text{year}, Ic_1 = 0.1$$$/\text{year}, Ic_2 = 0.15$$$/\text{year}.

**Example 2:** Taking all the parameters same as in Example 1 except $D = 2500$ $\text{units}$ and $M = 0.3$ $\text{year}$.

Solving Eq. 16, Eq. 18 and Eq. 20 by Newton Raphson method and using Lemma 1 – Lemma 3.3 and Theorem 1, the result is summarized in the Table 1.

<table>
<thead>
<tr>
<th>Example 1</th>
<th>Example 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_1 &lt; 0$</td>
<td>$\Delta_1 &gt; 0$</td>
</tr>
<tr>
<td>$T = T_1$ does not exist</td>
<td>$T = T_1 = 0.2051$</td>
</tr>
<tr>
<td>$TC^<em>(T^</em>) = \min {TC_{2,1}(T_{2,1}), TC_{2,2}(T_{2,2}), TC_{3,1}(T_{3,1})}$</td>
<td>$TC^<em>(T^</em>) = \min {TC_1(T), TC_{2,1}(M), TC_{2,2}(M)}$</td>
</tr>
<tr>
<td>$\Delta_1 \leq 0 \Delta_2$</td>
<td>$\Delta_1 &gt; 0$</td>
</tr>
<tr>
<td>$T = T_{2,1} = 0.2397$</td>
<td>$T = T_{2,1} = M$</td>
</tr>
<tr>
<td>$\min {984.06, 986.79, 1116.09}$</td>
<td>$\min {862.38, 1008.95, 1025.62}$</td>
</tr>
<tr>
<td>$\Delta_1 \leq 0 \Delta_3$</td>
<td>$\Delta_1 &gt; 0$</td>
</tr>
<tr>
<td>$T = T_{2,2} = 0.2325$</td>
<td>$T = T_{2,2} = M$</td>
</tr>
<tr>
<td>$\min {984.06, 986.79, 1116.09}$</td>
<td>$\min {862.38, 1008.95, 1025.62}$</td>
</tr>
<tr>
<td>$\Delta_2 &gt; 0$</td>
<td>$\Delta_2 &gt; 0$</td>
</tr>
<tr>
<td>$T = T_{3,1} = N$</td>
<td>$T = T_{3,1} = N$</td>
</tr>
<tr>
<td>$984.06 = TC_{2,1}(T_{2,1})$</td>
<td>$862.38 = TC_1(T_{1})$</td>
</tr>
<tr>
<td>$\Delta_1 &gt; 0$</td>
<td>$\Delta_1 &gt; 0$</td>
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<td>$T = T_{3,2} = N$</td>
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</tr>
<tr>
<td>$\Delta_3 &gt; 0$</td>
<td>$\Delta_3 &gt; 0$</td>
</tr>
<tr>
<td>$T = T_{3,3} = N$</td>
<td>$T = T_{3,3} = N$</td>
</tr>
<tr>
<td>$T^* = T_{2,1} = 0.2397$</td>
<td>$T^* = T_1 = 0.2051$</td>
</tr>
</tbody>
</table>

6. **SENSITIVITY ANALYSIS**

Now we study the effects of the changes to the parameters $A, D, c_4, \theta$, and $M$. For this we change one parameter at a time and keep remaining parameters unchanged, also we take the values of the parameters same as in Example 1.
It is clear from the Table 2 that when each of the parameters increases, both the optimal replenishment cycle time \( T^* \) and the optimal cost \( TC^* \) are increasing simultaneously. It means that the retailer may order more quantity to reduce the average total cost.

(2) It is clear from the Table 2 that when each of the parameters \( D, c_h \) and \( \theta \) increases, optimal replenishment cycle time \( T^* \) decreases while optimal cost \( TC^* \) increases.

(3) When \( M \) is increasing both the replenishment cycle time \( T^* \) and optimal cost \( TC^* \) are decreasing.

### 7. CONCLUSION

In this paper we have developed an EOQ model for deteriorating items when supplier offers a progressive interest scheme to its retailer. This paper is fairly in general as the supplier may offer a grace period to pay for the purchased items without any interest to entice new retailers. It may be applied as an alternative to price discount because it does not provoke competitors to reduce their prices.
Several possible extensions of the present model could constitute future research endeavors in this field. One immediate extension could be to investigate the effect of inflation. It might be interesting to consider partial backlogging and two-warehouses. In order to show the uncertainties, the present model could be extended applying stochastic demand.

APPENDIX - A

To prove Lemma 1, we let:

\[ F_1(T) = -A - \frac{D(c_h + c\theta)}{\theta^2} \left[ e^{\theta T} - \theta T e^{\theta T} - 1 \right] + \frac{pI_e DT^2}{2}, \quad T \in [0, M]. \]  

(A1)

Taking first order derivative of \( F_1(T) \) with respect to \( T \), we get:

\[ \frac{dF_1(T)}{dT} = D(c_h + c\theta) T e^{\theta T} + pI_e DT > 0, \quad \text{for} \quad T \in [0, M]. \]  

(A2)

Thus \( F_1(T) \) is a strictly increasing function of \( T \) in the interval \([0, M]\), moreover from (A1), we know that \( F_1(0) = -A \), and \( F_1(M) = \Delta_1 \). Therefore if \( F_1(M) = \Delta_1 \geq 0 \), by applying intermediate value theorem, there exists a unique \( T_1 \in [0, M] \), such that \( F_1(T_1) = 0 \).

Furthermore taking the second-order derivative of \( TC_1(T) \) with respect to \( T \) at the point \( T_1 \) we have

\[ \left. \frac{d^2 TC_1(T)}{dT^2} \right|_{T=T_1} = \frac{D(c_h + c\theta)}{T_1} e^{\theta T_1} > 0. \]

Thus \( T_1 \) is the unique minimum solution of \( TC_1(T) \). On the other hand if \( F_1(M) = \Delta_1 < 0 \), then \( F_1(T) < 0 \) for all \( T \in (0, M) \). Consequently we have \( \frac{dTC_1(T)}{dT} = \frac{F_1(T)}{T} < 0 \) for all \( T \in (0, M) \). Thus \( TC_1(T) \) is a strictly decreasing function of \( T \) in the open interval \((0, M)\). Therefore we cannot find a value of \( T \) in the interval \((0, M)\) that minimizes \( TC_1(T) \). This completes the proof.

REFERENCES


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