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STRUCTURES ON INTUITIONISTIC Q-FUZZY QUOTIENT SUBLATTICES INTERMS OF FUZZY LATTICE

ABSTRACT: In this paper, we introduce the notion of intuitionistic fuzzy quotient lattice in a fuzzy lattice and then some basic properties are investigated. Characterizations of intuitionistic fuzzy quotient lattices are given. Using a collection of lattices, an intuitionistic fuzzy quotient lattice is established. The notion of fuzzy quotient lattice relation on the family of all intuitionistic fuzzy sub lattices of \( L \) are discussed upper and lower level sets of fuzzy quotient lattices are studied.

Keywords: Q-fuzzy set, Fuzzy lattice, Fuzzy quotient Lattice, level cut, intuitionistic fuzzy quotient sub lattice, Homomorphism

1. INTRODUCTION

The theory of fuzzy sets proposed by L.A. Zadeh [25] in 1965, has achieved a great success in various fields. After that time, some author’s [12, 16] applied this concepts to groups and rings theory. With the research of fuzzy sets, in 1986, K. Atanassov [1] presented intuitionistic fuzzy sets which are very effective to deal with vagueness. The concept of the intuitionistic fuzzy sets is a generalization of one of the fuzzy sets. Recently Coker and his colleagues [8, 9] and Lee [15] introduce the concept of intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets and investigated some of its properties. In 1989, Biswas [6] introduced the concept of intuitionistic fuzzy subgroups and studied some of its properties. In 2003 Banerjee and Basnet [5] investigated intuitionistic fuzzy subgroups and intuitionistic fuzzy ideals using intuitionistic fuzzy sets. Also Hur and his colleagues [11, 12, 13, 14] studied various properties of intuitionistic fuzzy sub groupoids, intuitionistic fuzzy sub rings and intuitionistic fuzzy topological groups. In particular Bustince and Burillo [7] introduce the concept of intuitionistic fuzzy relations and investigated some of its properties and Yon and Kim [24] introduced the notion of intuitionistic fuzzy sub lattices, filters and ideals. In a series of papers [2, 3, 4] various sub lattices of the lattice \( L \) of all fuzzy groups of the group \( G \) are constructed and examined. In papers [21,22] studied the rough sets corresponding to an ideals of a lattice and introduced rough sub lattice and intuitionistic fuzzy sub lattices. N. Ajmal and K.V. Thomas initiated such types of study in the year 1994. It was latter independently established by N. Ajmal that the set of all fuzzy normal sub groups of a group constitute a sub lattice of the lattice of all subgroups and is modular. Nanda. S [17] proposed the notice of fuzzy lattice using the concept of fuzzy partial ordering. More recently in the notion of set product is discussed in details and in the lattice theoretical aspects of fuzzy sub groups and fuzzy normal sub groups are explored. G.S.V. Satya Saibaba [20] initiated the study of L-fuzzy lattice ordered groups and introducing the notion L fuzzy sub -1 groups. J.A. Goguen [10] replaced the valuation set \([0, 1]\) by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. A.Solairaju and R.Nagarajan investigated the idea of
Constructions of $Q$-fuzzy groups [18,19]. In this paper, we investigate intuitionistic fuzzy quotient lattice, upper and lower level sets and characterization of intuitionistic fuzzy quotient lattices. The notion of fuzzy quotient lattice relation on the family of all intuitionistic fuzzy sub lattices of $L$ are discussed.

2. PRELIMINARIES

Definition 2.1: A mapping $\mu : X \times Q \rightarrow [0, 1]$, where $X$ is an arbitrary non-empty set and is called $Q$-fuzzy set in $X$.

Definition 2.2: Let $X$ be a non-empty set. An intuitionistic $Q$-fuzzy set (IFS) $A$ of $X$ is an object of the following form $A = \{(x, \mu_A(x, q), \gamma_A(x, q)) / x \in X\}$. Where $\mu_A : X \times Q \rightarrow [0, 1]$ and $\gamma_A : X \times Q \rightarrow [0, 1]$ defined the degree of membership and the degree of non-membership of the element $x \in X$, $0 \leq \mu_A(x, q) + \gamma_A(x, q) \leq 1$

Definition 2.3: A fuzzy lattice $L$ under $\mu$ is called equipotent $Q$-fuzzy quotient lattice if

(i) $\mu(x + y, q) \geq T\{\mu(x, q), \mu(y, q)\}$

(ii) $\mu(\neg x, q) \geq \mu(x, q)$

(iii) $\mu(x \vee y, q) \geq T\{\mu(x, q), \mu(y, q)\}$, for all $x, y \in L$

Definition 2.4: Let $I : X \rightarrow L$ is called intuitionistic $Q$-fuzzy quotient lattice over $L$ if

(i) $I(x + y, q) \geq T\{I(x, q), I(y, q)\}$

(ii) $I(\neg x, q) \geq I(x, q)$

(iii) $I(x \vee y, q) \geq T\{I(x, q), I(y, q)\}$

(iv) $I(x \wedge y, q) \geq T\{I(x, q), I(y, q)\}$ Where $I = (\mu, \gamma)$. For all $x, y \in L$

Definition 2.5: An intuitionistic fuzzy set $A$ in $L$ is called intuitionistic $Q$-fuzzy quotient sublattice of $L$ if, the following conditions are satisfied.

(i) $I_A(x + y, q) \geq T\{I_A(x, q), I_A(y, q)\}$

(ii) $I_A(\neg x, q) \geq I_A(x, q)$

(iii) $I_A(x \vee y, q) \geq T\{I_A(x, q), I_A(y, q)\}$

Here $*$ represents the combination of meet and joint operations.

Definition 2.6: Let $\mu$ be a $Q$-fuzzy subset of a set $L$ and $t \in [0, 1]$. Then the set $\mu_t = \{x \in L / \mu(x, q) \geq t\}$ is called level sub set of $\mu$.

Definition 2.7: An intuitionistic Q-fuzzy quotient lattice $A$ is said to be self-distributive intuitionistic $Q$-fuzzy set in $L$ and $I_A$ is fuzzy quotient lattice then $I_A^a(x, q) = a'^aI(x, q)$, for any $a, b \in L$.

Definition 2.8: Let $f : L \rightarrow L'$ be a lattice homomorphism $f$ is fuzzy lattice homomorphism if $f(x + y) = f(x) + f(y)$, for all $x, y \in L$. 
**Definition 2.9:** A fuzzy set \( S \) dominates \( S^* \) if \( S^* \supseteq S \). (ie) \( S^* \) dominates \( S \).

**Definition 2.10:** Let \( \mu \) and \( \gamma \) be two fuzzy quotient lattice. Then fuzzy quotient class is defined as \[ \mu \cdot \gamma \]_S \((x, q) = \max \{ \mu \cdot \gamma \} \] \( S^* \) dominates \( S \) for all \( x, y \in L \).

### 3. Properties of Intuitionistic Q-Fuzzy Quotient Sublattices

**Proposition 3.1:** If an intuitionistic \( Q \)-fuzzy set \( A \) in \( L \) is an intuitionistic \( Q \)-fuzzy Quotient sublattice of \( L \) then so, is \( A = \{(x, \mu_A(x, q), 1 - \mu_A(x, q)) / x \in L \} \)

**Proof:** Suppose \( A \) is an intuitionistic \( Q \)-fuzzy quotient lattice of \( L \). Then for any \( x, y \in A \), \( x + y \in A \) and \( x^* y \in A \).

\[ \mu_A(x, q) = \mu_A(y, q) = 1 \quad \text{and} \quad 1 - \gamma_A(x, q) = 1 - \gamma_A(y, q) = 0 \]

Then \( \mu_A(x + y, q) = T \{ \mu_A(x, q), \mu_A(y, q) \} \mu_A(x^* y)_q = \max \{ \min \{ \mu_A(x, q), \mu_A(y, q) \} \} \)

\[ (1 - \gamma_A) (x + y, q) = S \{(1 - \gamma_A) (x, q), (1 - \gamma_A) (y, q) \} \]

Suppose \( x, y \in L \) and atleast one of them say \( y \notin A \), then

\[ \mu_A(y, q) = 0, (1 - \gamma_A) (y, q) = 1, \gamma_A(y, q) = 0 \]

\[ (1 - \gamma_A) (x, q) V (1 - \gamma_A) (y, q) = 1 \]

Thus \( \mu_A, 1 - \gamma_A \) satisfies the properties of intuitionistic \( Q \)-fuzzy quotient lattice.

Conversely, Suppose \( \mu_A, 1 - \gamma_A \) is \( Q \)-fuzzy quotient lattice of \( L \). Let \( x, y \in A \). \( \mu_A(x, q) = \mu_A(y, q) = 1, T \{ \mu_A(x, q), \mu_A(y, q) \} = 1 \). But, both \( \mu_A(x + y, q) \) and \( \mu_A(x^* y)_q \geq T \{ \mu_A(x, q), \mu_A(y, q) \} \). Thus, \( A \) is intuitionistic \( Q \)-fuzzy quotient lattice of \( L \).

**Proposition 3.2:** If an intuitionistic \( Q \)-fuzzy set \( A \) in \( L \) is intuitionistic \( Q \)-fuzzy quotient sublattice of \( L \) if \( \mu_A \) and \( \gamma_A^C \) are \( Q \)-fuzzy lattice of \( L \).

**Proof:** Let \( I_A = (\mu_A, \gamma_A) \) be an intuitionistic \( Q \)-fuzzy quotient lattice of \( L \). Then obviously \( \mu_A \) is \( Q \)-fuzzy quotient lattice of \( L \). Let \( x, y \in L \). Then,

\[ (IFEPL1) \quad \gamma_A^C (x + y, q) = 1 - \gamma_A (x + y, q) \geq 1 - \max \{ \gamma_A(x, q), \gamma_A(y, q) \} \]

\[ \geq T \{ \gamma_A^C (x, q), \gamma_A^C (y, q) \} \]

\[ (IFEPL2) \quad \gamma_A^C (-x, q) = 1 - \gamma_A (-x, q) \geq 1 - \gamma_A (x, q) = \gamma_A^C (x, q) \]

\[ \gamma_A^C (x^* y)_q = 1 - \gamma_A (x^* y)_q \leq 1 - \min \{ \max \{ \gamma_A (x, q), \gamma_A (y, q) \} \} \]

\[ \left( \min \{ \gamma_A (x, q), \gamma_A (y, q) \} \right) = \min \{ \gamma_A^C (x, q), \gamma_A^C (y, q) \} \]
Conversely, suppose that \( \mu_A \) and \( \gamma_A^C \) are intuitionistic \( Q \)-fuzzy quotient lattice of \( L \). Let \( x, y \in L \).

Then, \[ 1 - \gamma_A(x + y, q) = \gamma_A^C(x + y, q) \geq T \{ \gamma_A^C(x, q), \gamma_A^C(y, q) \} \]
\[ \geq T \{ 1 - \gamma_A(x, q), 1 - \gamma_A(y, q) \} \geq 1 - \max \{ \gamma_A(x, q), \gamma_A(y, q) \} \]
\[ 1 - \gamma_A(-x, q) = \gamma_A^C(-x, q) \geq \gamma_A(x, q). \]

Which simply \( \gamma_A(x + y, q) \leq S(\gamma_A(x, q), \gamma_A(y, q)) \)
\[ \gamma_A(x, q) \leq \gamma_A(y, q) \]

Finally, for any \( x, y \in L \),
\[ 1 - \gamma_A(x * y, q) = \gamma_A^C(x * y, q) \geq \max \{ \min \{ \gamma_A^C(x, q), \gamma_A^C(y, q) \} \} \geq \min \{ 1 - \gamma_A(x, q), 1 - \gamma(y, q) \}. \]

**Proposition 3.3:** For any \( t \in [0, 1] \), the maps \( U_t \) and \( V_t \) are surjective from \( F(L) \) to \( I(L) \) \( U \{ 0, 1 \} \). Moreover the quotient sets \( F(L) / \sim \mu \) and \( F(L) / \sim \gamma \) are equipotent to \( I(L) \) \( U \{ 0, 1 \} \).

**Proof:** Let \( t \in [0, 1] \). Note that \( 0 \sim = [0, 1] \) proof is in \( F(L) \). Where 0 and 1 are fuzzy sets in \( L \) defined by \( 0(x) = 0 \) and \( 1(x) = 1 \) for all \( x \in L \) obviously.
\[ f_t(0 \sim) = U(0; t) = \phi = L(0; t) = g_t(0 \sim) \]

Let \( J (\# \phi) \in I(L) \). For \( J \sim = (X_{J, \bar{X}_J}) \in F(L) \), we have \( f_t(J \sim) = U(X_J; t) = J = L \)
\[ \bar{X}_J; t) = g_t(J \sim). \] Hence \( f_t \) and \( g_t \) are surjective.

Let \( f_t^* \) be a map from \( F(L) / \sim \mu \) to \( I(L) \) \( U \{ \phi \} \) defined by \( f_t^*([I_A]_\mu) = f_t(I_A) \).

Assume that \( U(\mu_A; t) = U(\mu_B; t) \) and \( L(\gamma_A; t) = L(\gamma_B; t) \) for \( A, B \) is \( F(L) \).

Then \( A \sim \mu_B \) and \( A \sim \gamma_B \), and hence \( [I_A]_\mu = [I_B]_\mu \), \( [I_A]_\gamma = [I_B]_\gamma \).

Therefore the maps \( f_t^* \) and \( g_t^* \) are injective.

Now let \( J (\# \phi) \in I(L) \).

For \( J \sim = (X_{J, \bar{X}_J}) \in F(L) \), we have \( f_t^*([J \sim]_\mu) = f_t(J \sim) = g_t([J \sim]_\mu). \)

Finally, for \( 0 \sim = [0, 1] \in F(L) \), we get \( f_t^*([0 \sim]_\mu) = f_t(0 \sim) = U(0; t) = \phi = L(0; t) = g_t(0 \sim) = g_t([0 \sim]_\mu) \). This shows that \( f_t^* \) and \( g_t^* \) are surjective.

**Proposition 3.4:** Let \( \{ M_t / t \in A \subseteq [0, 1] \} \) be a collection of quotient lattices of \( L \) such that

(i) \( J = U M_t, t \in A \)

(ii) For any \( s, t \in A \), \( S > t \) if and only if \( M_S \subseteq M_t \).

**Proof:** Let \( \{ M_t / t \in A \subseteq [0, 1] \} \) be a collection of fuzzy quotient lattices of \( L \).
We consider the following two cases.

(i) \( S = \text{sup} \{ t \in \Lambda | t < s \} \) and (ii) \( S \neq \text{sup} \{ t \in \Lambda | t < s \} \)

Case (i) implies that

\[ x \in I_S \iff x \in M_t \text{ for all } t < S. \iff x \in \cap M_t \text{ when } t < p \]

\( I_S = \cap M_t \) which is a lattice of \( L, \) \( t < p \)

For the case (ii), there exists \( \epsilon > 0 \) such that \( (S - \epsilon, S) \cap \Lambda = \emptyset. \)

We claim that \( I_S = \bigcup_{t \in S} M_t \), then \( x \in M_t \) for some \( t \geq S. \)

It follows that \( I_A (x) \geq t \geq S \) so that \( x \in I_S. \)

Conversely if \( x \notin \bigcup_{t \in S} M_t, \) then \( x \notin M_t \) for all \( t \geq S. \)

Which implies that \( x \notin M_t \) for all \( t > S - \epsilon, \) that is if \( x \in M_t \) then \( t \in S - \epsilon. \)

Thus \( I_A(x) \leq S - \epsilon \) and so \( x \notin I_S. \)
Consequently \( I_S = \bigcup_{t \in S} M_t \) which is fuzzy quotient lattice of \( L. \) This completes the proof.

**Proposition 3.5:** Let \( A \) be an IFS in \( L \) such that the non-empty upper and lower level sets \( U(\mu_A; t) \) and \( L(\gamma_A; t) \) of \( A \) are quotient lattices of \( L \) for every \( t \in [0, 1]. \) Then \( A \) is an intuitionistic \( Q \)-fuzzy quotient sub lattice of \( L. \)

**Proof:** Let \( A \) be an IFS in a lattice \( L. \) For any \( x, y \in U(\mu_A; t) \). We have \( \mu_A(x, q) \geq t \) and \( \mu_A(y, q) \geq t \) and for \( x, y \in L(\gamma_A; t) \), we have \( \gamma_A(x, q) \leq t, \gamma_A(y, q) \leq t. \) Now (IFEPL1) \( \mu_A(x + y, q) = \text{sup} \{ \mu_A(x, q), \mu_A(y, q) \} \geq \text{sup} \{ t, t \} = t, \) thus \( x + y \in U(\mu_A; t) \)

\( \gamma_A(x + y, q) = S \{ \gamma_A(x, q), \gamma_A(y, q) \} \leq \text{sup} \{ t, t \} \text{ and thus } x + y \leq L(\gamma_A; t) \)

(IFEPL2) \( \mu_A(-x, q) \geq \mu_A(x, q) \geq t, \) therefore \( -x \in U(\mu_A; t) \) and \( \gamma_A(-x, q) \leq \gamma_A(x, q) \leq t, \) therefore \( -x \in L(\gamma_A; t) \)

(IFEPL3) \( \mu_A(x \ast y, q) = \text{max} \{ \min \{ I_A(x, q), I_A(y, q) \} \geq \text{max} \{ \min \{ t, t \} \geq t, \) thus \( x \ast y \in U(\mu_A; t) \)

\( \gamma_A(x \ast y, q) = \text{min} \{ \max \{ I_A(x, q), I_A(y, q) \} \leq \text{min} \{ \max \{ t, t \} \leq t, \) thus \( x \ast y \in L(\gamma_A; t) \)

Hence \( A \) is an intuitionistic \( Q \)-fuzzy quotient lattice in \( L. \)

**Proposition 3.6:** If IFS \( A \) in \( L \) is an intuitionistic \( Q \)-fuzzy quotient sub lattice then the non-empty upper and lower level sets \( U(\mu_A; t) \) and \( L(\mu_A; t) \) of \( A \) are lattices of \( L \) for every \( t \in [0, 1]. \)

**Proof:** Let \( \mu \) be a \( Q \)-fuzzy quotient lattice of \( L \) and \( t \in [0, 1]. \). For any \( x, y \in I_A, \) we have

\( I_A(x + y, q) \geq T \{ I_A(x, q), I_A(y, q) \} \geq t \text{ and so } x + y \in I_A. \)

Let \( -x \in L \) and \( x \in I_A. \) Then \( I_A(-x, q) \geq I_A(-x, q) \geq t. \) Let \( x, y \in L \) we have \( I_A(x \ast y, q) \geq \text{max} \{ \min \{ I_A(x, q), I_A(y, q) \} \geq \text{max} \{ \min \{ t, t \} \geq t, \) which shows that \( x \ast y \in I_A. \) Conversely, assume that \( I_A \) is a quotient lattice of \( L \) for every \( t \in [0, 1]. \).
If \( I_A(x_0 + y_0, q) < T \{ I_A(x_0, q), I_A(y_0, q) \} \) for some \( x_0, y_0 \in L \).

Then by taking

\[
t_0 = \frac{1}{2} \{ I_A(x_0 + y_0, q) + T \{ I_A(x_0, q), I_A(y_0, q) \} \}
\]

We have \( I_A(x_0 + y_0, q) < t_0, I_A(x_0, q) > t_0 \) and \( I_A(y_0, q) > t_0 \). Hence \( x_0 + y_0 \notin I_{A_0} \).

\( x_0 \in I_{A_0} \) and \( y_0 \in I_{A_0} \). This is a contradiction, and so \( I_A(x + y, q) \geq T \{ I_A(x, q), I_A(y, q) \} \), for all \( x, y \in L \). Assume that \( I_A(-x_0, q) < I_A(x_0, q) \) for some \( x_0, y_0 \in L \).

Putting \( M_0 = \frac{1}{2} \{ I_A(-x_0, q) + I_A(y_0, q) \} \), then \( I_A(-x_0, q) < M_0 < I_A(x_0, q) \). It follows that \( x_0 \in I_{A_0} \) and \( -x_0 \notin I_{A_0} \), which is impossible. Hence \( I_A(-x, q) \geq I_A(x, q) \) for all \( x, y \in L \). If the condition \((ii)\) of the definition \(2.4\) is not true, then for fixed \( P_0 = \frac{1}{2} \{ I_A(x^* y, q) + I_A(x, q) \} \).

Then \( I_A(x + y, q) \notin I_{R_1} \) and \( x \in I_{R_1} \) and \( y \in I_{R_1} \). This is a contradiction. Similarly we can show lower level set are lattice in \( L \) for every \( t \in \{0, 1\} \).

**Proposition 3.7:** If \( A \) is an intuitionistic Q-fuzzy quotient lattice of \( L \), then the sets \( L \mu_A = \{ x \in L / \mu_A(x, q) = \mu_A(0, q) \} \) and \( L \gamma_A = \{ x \in L / \gamma_A(x, q) = \gamma_A(0, q) \} \) are lattices of \( L \).

**Proof:** A be an intuitionistic Q-fuzzy quotient lattice and let \( x, y \in L \mu_A \).

Then \( I_A(x + y, q) \geq T \{ I_A(x, q), I_A(y, q) \} = I_A(0, q) \) and so, \( I_A(x + y, q) = I_A(0, q) \) or \( x + y \in L \mu_A \). For every \( x \in L \) and \( x \in L \mu_A \), we have

\( I_A(-x, q) \geq I_A(x, q) = I_A(0, q) \). Hence \(-x \in L \mu_A \), which shows that \( L \mu_A \) is a negative of \( L \).

Let \( x, y \in L \mu_A \) and hence \( I_A(x * y, q) \geq \max \{ \min \{ I_A(x, q), I_A(y, q) \} \} \) \( \max \{ \min \{ I_A(0, q), I_A(0, q) \} \} = I_A(0, q) \).

Therefore \( L \mu_A \) is a lattice of \( L \). Similarly, we can show the complement of \( \mu_A \).

**Proposition 3.8:** Let \( A \) be a self-distributive intuitionistic Q-fuzzy sets in \( L \). Then the IFS \( I^b_a \) in \( L \) is intuitionistic Q-fuzzy quotient sub lattice of \( L \) for all \( a, b \in L \).

**Proof:** Since \( A \) be self-distributive intuitionistic Q-fuzzy sets in \( L \) and \( I_A \) is Q-fuzzy quotient lattice (ie) \( I^a_a \) \((x, q) = a^b I(x, q)\), for any \( a, b \in L \).

(IFEPL 1) \( I^b_a (x + y, q) = a^b I(x + y, q) \geq T \{ a^b I(x, q), a^b I(y, q) \} \geq T \{ I^a_a (x, q), I^a_a (y, q) \}\)

(IFEPL 2) \( I^b_a (-x, q) = a^b I(-x, q) \geq a^b I(x, q) \geq I^a_a (x, q)\)

(IFEPL 3) \( I^b_a (x * y, q) = a^b I(x * y, q) \geq a^b \max \{ \min \{ I(x, q), I(y, q) \} \}\)

\( \geq \max \{ a^b I(x, q), a^b I(y, q) \} \geq \min \{ I^a_a (x, q), I^a_a (y, q) \}\)

Hence \( I^b_a \) is intuitionistic Q-fuzzy quotient lattice in \( L \).
Proposition 3.9: Let $L$ be a lattice and $I$ be a sub set of $L$. If $I$ is an intuitionistic quotient Q-fuzzy sub lattice of $L$, then the characteristic function $\psi_I : L \to [0, 1]$. $I$ is intuitionistic Q-fuzzy quotient sub lattices with respect to $S$.

Proof: Since $L$ be a lattice and $I \subseteq L$. The characteristic function of $I$ is $I : L \to [0, 1]$. $I$ is intuitionistic Q-fuzzy quotient sub lattice of $L$.

Claim: $I$ is intuitionistic Q-fuzzy quotient sub lattice of $L$ for any $x, y \in L$.

(IFEPL 1) $\psi_I (x + y, q) = \psi_I (y) = \max \{ \psi(x, q), \psi(y, q) \}$

(IFEPL 2) $\psi_I (x, q) = \psi_I (y, q) = \max \{ \psi(x, q), \psi(y, q) \}$

(IFEPL 3) $\psi_I (x \cdot y, q) = \psi_I (x, y, q) = \max \{ \psi(x, q), \psi(y, q) \}$

Proposition 3.10: Let $S$ be a $S$ norm and $\mu, \gamma$ be a two intuitionistic quotient Q- Fuzzy sub lattice of $L$ with respect to $S$. If $S^*$ dominates $S$, then $S^*\text{-}product$, $[\mu \cdot \gamma]_{S^*}$ of $\mu$ and $\gamma$ is intuitionistic Q-fuzzy quotient sub lattice of $L$.

Proof: $\mu$ and $\gamma$ be two intuitionistic Q-fuzzy quotient sub lattice of $L$ with respect to $S\text{-}norms$. Since $S^*$ dominates the norm $S$.

Claim: $S^*$ - product forms intuitionistic Q-fuzzy quotient sub lattice of $L$.

(IFEPL1) $[\mu \cdot \gamma]_{S^*} (x + y, q) = \max \{ \mu_{S^*} (x + y, q), \gamma_{S^*} (x + y, q) \}$

$\gamma(y, q) \geq \max \{ T \{ S^* \{ \mu(x, q), \gamma(x, q) \}, S^* \{ \mu(y, q), \gamma(y, q) \} \} \}$

$\geq T \{ \mu_{S^*} (x, q), \gamma_{S^*} (x, q), \mu_{S^*} (y, q), \gamma_{S^*} (y, q) \} \}$

$\geq T \{ \mu_{S^*} (x, q), \gamma_{S^*} (x, q), \max \{ \mu_{S^*} (y, q), \gamma_{S^*} (y, q) \} \}$

$\geq T \{ \mu_{S^*} (x, q), \gamma_{S^*} (x, q) \}$

(IFEPL 2) $[\mu \cdot \gamma]_{S^*} (x, q) = \max \{ \mu_{S^*} (x, q), \gamma_{S^*} (x, q) \}$

$\mu(x, q) \geq \max \{ S^* \{ \mu(x, q), S^* \gamma(x, q) \} \} \geq \max \{ \mu_{S^*} (x, q), \gamma_{S^*} (x, q) \}$

$\geq \max \{ \mu_{S^*} (x, q), \gamma_{S^*} (x, q) \}$

(IFEPL 3) $[\mu \cdot \gamma]_{S^*} (x \cdot y, q) = \max \{ \mu_{S^*} (x \cdot y, q), \gamma_{S^*} (x \cdot y, q) \}$

$\mu(x \cdot y, q) \geq \max \{ S^* \{ \min \{ \mu(x, q), \mu(y, q) \} \}, S^* \{ \max \{ \gamma(x, q), \gamma(y, q) \} \} \} \geq \max \{ \mu_{S^*} (x, q), \gamma_{S^*} (x, q) \}$

$\geq \max \{ \min \{ \mu_{S^*} (x, q), \gamma_{S^*} (x, q) \}, \max \{ \mu_{S^*} (y, q), \gamma_{S^*} (y, q) \} \}$
Proposition 3.11: Let \( f: R \to R' \) be a homomorphism of \( R \) and \( R' \). If \( \mu \) and \( \gamma \) are two intuitionistic Q-fuzzy quotient sub lattice of \( L' \) with respect to \( S \) then \( f^{-1}(\gamma_{S'}) \) is intuitionistic Q-fuzzy quotient sub lattice of \( L \) with respect to \( S \).

Proof: A mapping \( f: R \to R' \) be a homomorphism \( \mu \) and \( \gamma \) are intuitionistic Q-fuzzy quotient sub lattice of \( L' \) with respect to \( S \). Since \( S^* \) dominates \( S \). We have \( S^* \subseteq S \).

Claim: \( f^{-1}(\gamma_{S'}) \) is intuitionistic Q-fuzzy quotient sub lattice of \( L \) with respect to \( S \).

(IFEPL 1): \( f^{-1}(\gamma_{S'}) (x + y, q) = \gamma_{S'} f(x, q) \)

\[ \geq \max \{ \mu \gamma_{S'} (f(x, q) + f(y, q)), \gamma_{S'} (f(x, q) + f(y, q)) \} \]

\[ = \max \{ \mu \gamma_{S'} (f(x, q) + f(y, q)), \gamma_{S'} (f(x, q) + f(y, q)) \} \]

\[ \geq T \{ \max \{ \mu \gamma_{S'} (f(x, q), f(y, q)), \gamma_{S'} (f(x, q), f(y, q)) \} \}

\[ \geq T \{ \max(f^{-1}\mu_{S'} (x, q), f^{-1}\gamma_{S'} (x, q)), \max (f^{-1}\mu_{S'} (y, q), f^{-1}\gamma_{S'} (y, q)) \}

\[ \geq T(f^{-1}(\mu \gamma_{S'} (x, q), f^{-1}(\mu \gamma_{S'} (y, q))) \}

(IFEPL 2): \( f^{-1}(\mu \gamma_{S'}) (x, y) = \mu \gamma_{S'} f(x, y) \)

\[ \geq \max \{ \mu \gamma_{S'} f(x, y), \gamma_{S'} f(x, y) \} \]

\[ = \max \{ \mu \gamma_{S'} f(x, y), \gamma_{S'} f(x, y) \} \]

\[ \geq \max \{ \mu \gamma_{S'} f(x, y), \gamma_{S'} f(x, y) \} \]

\[ \geq \max \{ \mu \gamma_{S'} f(x, y), \gamma_{S'} f(x, y) \} \]

\[ \geq \max \{ \mu \gamma_{S'} f(x, y), \gamma_{S'} f(x, y) \} \]

\[ = \max \{ \mu \gamma_{S'} f(x, y), \gamma_{S'} f(x, y) \} \]

\[ \geq \max \{ \mu \gamma_{S'} f(x, y), \gamma_{S'} f(x, y) \} \]

\[ \geq \max \{ \mu \gamma_{S'} f(x, y), \gamma_{S'} f(x, y) \} \]

\[ \geq \max \{ \mu \gamma_{S'} f(x, y), \gamma_{S'} f(x, y) \} \]

Therefore \( f^{-1}(\mu \gamma_{S'}) \) is intuitionistic Q-fuzzy quotient sub lattice of \( L \) under the domination of \( S^* \).

Proposition 3.12: A lattice homomorphic image of intuitionistic Q-fuzzy quotient sub lattice of \( L \) with sup property is intuitionistic Q-fuzzy quotient sub lattice.
**Proof:** Let \( f: L \rightarrow L' \) be a lattice homomorphism of \( L \) and let \( I_A \) be \( Q \)-fuzzy quotient lattice of \( L \) with sup property. Given \( x, y \in L \). We let \( x_0 \in f^{-1}(x') \) and \( y_0 \in f^{-1}(y') \) be such that \( I_A(x_0) = \text{Sup} I_A(h), I_A(y_0) = \text{Sup} I_A(h) \), respectively.

\[
h \in f^{-1}(x') \quad h \in f^{-1}(y')
\]

Then we can deduce that

(IFEPL 1): 
\[
I_A'(x' + y', q) = \text{sup} I_A(z, q),
\]

\[
z \in f^{-1}(x' + y') \geq T\{I_A(x_0, q), I_A(y_0, q)\}
\]

\[
\geq T\{\text{sup} I_A(h, q), \text{sup} I_A(h, q)\}
\]

\[
h \in f^{-1}(x') h \in f^{-1}(y')
\]

\[
\geq T\{I_A'(x', q), I_A'(y', q)\}
\]

(IFEPL 2): 
\[
I_A'(-x', q) = \text{sup} I_A(z, q),
\]

\[
z \in f^{-1}(-x')
\]

\[
\geq I_A(x_0, q)
\]

\[
\geq \text{sup} I_A(x_0, q)
\]

\[
h \in f^{-1}(x')
\]

\[
\geq I_A'(x, q)
\]

(IFEPL 3): 
\[
I_A'(x' \ast y', q)q = \text{sup} I_A(z, q),
\]

\[
z \in f^{-1}(x' \ast y')
\]

\[
\geq \max\{\min\{I_A(x_0, q), I_A(y_0, q)\}\}
\]

\[
\geq \max\{\min\{\text{sup} I_A(h, q), \text{sup} I_A(h, q)\}\}
\]

\[
h \in f^{-1}(x') h \in f^{-1}(y')
\]

\[
\geq \max\{\min\{I_A'(x, q), I_A'(y, q)\}\}, \text{ for all } x, y \in L.
\]

Hence, lattice homomorphic image of \( Q \)-fuzzy quotient lattice with sup-property forms intuitionistic \( Q \)-fuzzy quotient sub lattice on \( L \).

**Applications:** Lattice structure has been found to be extremely important in the areas of quantum logic Ergodic theory. Reynold’s operations. Soft Computing, Communication system, Information analysis system, artificial intelligences and physical science.
4. CONCLUSION

Y.H. Yon and K.H. Kim introduced the concept of intuitionistic fuzzy filters and ideals of lattices and A. Solairaju and R. Nagarajan investigated the new idea of Structure and Constructions of Q- fuzzy groups. In this paper, the concept of Intuitionstic Q- fuzzy sub lattices of fuzzy lattices are analyzed.

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