NON-UNIFORM SLOT INJECTION (SUCTION) INTO MHD BOUNDARY LAYER FLOW OVER A CYLINDER

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Abstract: An analysis is performed to study the effect of non-uniform slot injection (suction) into a steady two-dimensional laminar boundary layer flow with an applied magnetic field. Non-similar solutions have been obtained from the starting point of the streamwise co-ordinate to the exact point of separation. The difficulties arising at the starting point of the streamwise co-ordinate, at the edges of the slot and at the point of separation have been overcome by applying an implicit finite difference scheme with the quasi-linearization technique and an appropriate selection of finer step size along the streamwise direction. Numerical results are obtained to display the effect of magnetic field and slot injection (suction) on skin friction and heat transfer coefficients at the wall. The results indicate that magnetic field increases both skin friction and heat transfer coefficients. And in the case of slot suction, the skin friction and heat transfer coefficients increase while in case of injection they decrease. Further boundary layer separation is delayed by non-uniform slot suction whereas slot injection does the opposite.

1. Introduction

Problems of contemporary practical interest that arise in boundary layer theory do not admit similarity transformations. A detailed analysis of the flow situation in the laminar boundary layer taking non-similarity into account is of prime importance, where the non-similarity may be due to the free stream velocity or due to the curvature of the body or due to the surface mass transfer or due to all these effects. A review of non-similarity solution methods along with citations of some relevant publications is given by Dewey and Gross [1]. Since then several researches [2-4] have attempted to study the behavior of non-similar boundary layer flows.

In many technological solutions, mass transfer through a wall slot (i.e mass transfer occurs in a small porous section of the body surface while there is no mass transfer in the remaining part of the body surface) into the boundary layer is of interest for the various potential applications including thermal protection, energizing for the inner portion of the boundary layer in adverse pressure gradient, and skin friction reduction on control surfaces. Moreover mass transfer through a slot strongly influences the development of a boundary layer along a surface and in particular can prevent or atleast delay separation of the viscous region. Several investigators [5-7]

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have studied the effect of slot injection (suction) into a laminar compressible boundary layer over a flat plate by considering the interaction between the boundary layer and oncoming stream. Uniform mass transfer in a slot causes finite discontinuities at the leading and the trailing edges of the slot. The discontinuities can be avoided by choosing a non-uniform mass transfer distribution along a streamwise slot as has been discussed in Minkowycz, et al., [8].

The purpose of the present investigation is to study the effect of non-uniform slot injection (suction) (i.e., mass transfer in a small porous section of the body surface and the remaining part of the body surface is solid) on the steady non-similar boundary layers flow over a two-dimensional body (cylinder). The non-similar solutions have been obtained starting from the origin of the streamwise coordinate to the point of separation (zero skin friction in the streamwise direction) using quasilinearization technique with an implicit finite difference scheme.

2. Analysis

Consider the steady, laminar non-similar boundary layer forced convection flow of an electrically conducting fluid over a two-dimensional body (cylinder) when the free stream velocity and non-uniform mass transfer (slot injection/suction) vary with the axial distance (x) along the surface. Let x and y be the curvilinear coordinates along and perpendicular to the boundary, respectively, u and v be the corresponding velocity components. The contour of the body of revolution is specified by the radii r(x) of the section perpendicular to the axis (Fig. 1). A magnetic field B_0 is applied in y-direction normal to the body surface and it is assumed that magnetic Reynolds number is small. The blowing rate of the fluid is assumed to be small and it does not affect the inviscid flow at the edge of the boundary layer. It is also assumed that the injected fluid possesses the same physical properties as the boundary layer fluid and has a static temperature equal to the wall temperature.

![Figure 1: Flow Model and Co-Ordinate System](image-url)
Neglecting the effects of transverse curvature, the boundary layer equations governing the flow are given by:

\[
\frac{\partial}{\partial x} (r^j u) + \frac{\partial}{\partial y} (r^j v) = 0 \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{\partial u_e}{\partial x} + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} (u - u_e) \tag{2}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}
\]

With boundary conditions:

\[u(x, 0) = 0, \quad v(x, 0) = v_w(x), \quad T(x, 0) = T_w = \text{constant} \tag{4}\]

\[u(x, \infty) = u_e(x), \quad T(x, \infty) = T_\infty = \text{constant} \]

Applying the following transformations:

\[
\xi = \int_0^x \frac{u_e}{u_\infty} \left( \frac{r}{L} \right)^{2j} d \left( \frac{x}{L} \right); \quad \eta = \frac{u_e}{u_\infty} \left( \frac{Re_L}{2 \xi} \right)^{1/2} \frac{y}{L} \left( \frac{r}{L} \right)^j
\]

\[
\psi(x, y) = u_\infty L \left( \frac{2 \xi}{Re_L} \right)^{1/2} f(\xi; \eta); \quad u = \left( \frac{L}{r} \right)^j \psi_y; \quad v = -\left( \frac{L}{r} \right)^j \psi_x; \quad T = T_\infty + (T_w - T_\infty) G(\xi; \eta) \tag{5}
\]

to Eqns. (1)-(3), we see that the continuity equation (1) is identically satisfied and Eqns. (2) and (3) reduces to non-dimensional form, respectively, as:

\[
F^* + \beta(\xi) (1 - F^2) + fF' - PM (F - 1) = 2\xi (FF_\xi - F'f_\xi) \tag{6}
\]

\[
\Pr^{-1} G^* + fG' = 2\xi (FG_\xi - G'f_\xi) \tag{7}
\]

where

\[
\frac{u}{u_e} = f^* = F; \quad v = -\left( \frac{r}{L} \right)^j u_e (2 \xi Re_L)^{-1/2} (f + 2 \xi f_\xi + F\eta (\beta + \alpha_1 - 1));
\]
\[ \beta(\xi) = \left( \frac{2\xi}{u_e} \right) \left( \frac{du_e}{d\xi} \right) ; \quad \alpha_1 = \left( \frac{2\xi}{r} \right) \left( \frac{dr}{d\xi} \right) ; \quad M = \left( \frac{2}{3} \right) \frac{\sigma B_0^2 L}{\rho u_\infty} ; \]

\[ P = 3\xi \left( \frac{L}{r} \right)^2 \left( \frac{u_\infty}{u_e} \right)^2 ; \quad \text{Re}_L = \frac{\rho u_\infty L}{\mu} ; \quad P_r = \frac{v}{\alpha} ; \quad f = \int_0^\eta F d\eta + f_w \]

and

\[ f_w = -\xi^{-1/2} \left( \frac{\text{Re}_L}{2} \right)^{1/2} \int_0^x \left( \frac{v_w}{u_\infty} \right) \left( \frac{r}{L} \right)^i d \left( \frac{x}{L} \right) . \] (8)

Here, \( \xi \) and \( \eta \) are transformed co-ordinates; \( \psi \) and \( f \) are the dimensional and dimensionless stream functions, respectively; \( \text{Pr} \) is the Prandtl number; \( \rho, v, \alpha \) are respectively density, kinetic viscosity and thermal diffusivity; \( F \) is the dimensionless velocity; \( T \) and \( G \) are dimensional and dimensionless temperature, respectively; \( L \) is the characteristic length; \( R \) is the radius of the cylinder, \( \text{Re}_L \) is the Reynolds number; \( f_w \) is the surface mass transfer; \( \alpha_i \) is a dimensionless parameter; \( \beta \) is the pressure gradient parameter; \( M \) is the nondimensional magnetic parameter. The subscripts \( \infty \), \( e \) and \( w \) denote the conditions at the free stream, edge of the boundary layer and at the wall, respectively, and \( j = 0 \) for two-dimensional flow. \( u_e(x) \) is the potential flow velocity and \( v_w(x) \) denotes the surface mass transfer distribution. \( T_\infty \) is the constant temperature of the fluid maintained at the edge of the boundary layer and \( T_w \) is the uniform temperature of the body. The subscript \( \xi \) denote partial derivatives with respect to \( \xi \) and prime (‘) denote derivates with respect to \( \eta \).

The transformed boundary conditions are

\[ F(\xi, 0) = 0, \quad G(\xi, 0) = 1, \quad F(\xi, \infty) = 1, \quad G(\xi, \infty) = 0 . \] (9)

The skin friction coefficient and, the heat transfer coefficient in the form of Nusselt number, can be expressed respectively as

\[ C_f = 2 \left( \frac{u_e}{u_\infty} \right)^2 \left( \frac{r}{L} \right)^i \left( 2\xi \text{Re}_L \right)^{-1/2} F_w' \quad \text{and} \quad Nu = \left( \frac{\text{Re}_L}{2\xi} \right)^{1/2} \left( \frac{r}{L} \right)^j \left( \frac{u_e}{u_\infty} \right) \left( -G_w' \right) . \] (10)

Eqns. (6) and (7) under conditions (9) can be solved numerically if \( \beta \) and \( u_e \), which depend on the shape of the body, are prescribed. In particular, we have analyzed the effect of non-uniform slot injection (suction) into boundary layer flows over a cylinder.
The free stream velocity distribution for the case of circular cylinder and the distance from the axis of the body are given by [9].

\[
\frac{u_x}{u_\infty} = 2 \sin (\overline{x}); \quad \overline{x} = \frac{x}{R}; \quad j = 0; \quad L = R.
\]  

(11)

where \(\overline{x}\) is the dimensionless distance along the surface which give rise non similarity in the flow. Consequently, the expressions for \(\xi, \beta, \alpha_1, f_w\) are respectively given by

\[
\xi = 2P_1; \quad \beta = 2 \cos (\overline{x}) P_2^{-1}; \quad P = \frac{3}{2(1 + \cos \overline{x})}; \quad \alpha_1 = 0;
\]  

(12)

\[
f_w = \begin{cases} 
0, & \overline{x} \leq \overline{x}_0 \\
A(2P_1)^{-1/2} C(\overline{x}, \overline{x}_0), & \overline{x}_0 \leq \overline{x} \leq \overline{x}_0^* \\
A(2P_1)^{-1/2} C(\overline{x}_0^*, \overline{x}_0), & \overline{x} \geq \overline{x}_0^*
\end{cases}
\]  

(13)

where the function

\[
C(\overline{x}, \overline{x}_0) = 1 - \cos \{w^*(\overline{x} - \overline{x}_0)\}; \quad P_1 = 1 - \cos \overline{x}, \quad P_2 = 1 + \cos \overline{x}
\]  

(14)

Here \(v_w(x)\) in expression (8) is taken as

\[
v_w = \begin{cases} 
-\frac{u_\infty}{2} \left(\frac{Re_L}{2}\right)^{-1/2} A w^* \sin \{w^*(\overline{x} - \overline{x}_0)\}, & \overline{x}_0 \leq \overline{x} \leq \overline{x}_0^* \\
0, & \overline{x} \leq \overline{x}_0 \text{ or } \overline{x} \geq \overline{x}_0^*
\end{cases}
\]  

(15)

where \(w^*\) and \(\overline{x}_0\) are the two free parameters which determine the slot length and slot location. The function \(v_w\) is continuous for all values of \(\overline{x}\) and it has a non-zero value only in the interval \((\overline{x}_0, \overline{x}_0^*)\). The reason for taking such a type of function is that it allows the mass transfer to change slowly in the neighbourhood of the leading and trailing edges of the slot. The mass transfer parameter \(A > 0\) for suction and \(A < 0\) for injection.

It is convenient to express Eqns (6) and (7) in terms of instead of \(\xi\). Eqn. (12) gives the relation between \(\xi\) and \(\overline{x}\) as

\[
\frac{\partial \xi}{\partial \overline{\xi}} = S(\overline{x}) \frac{\partial}{\partial \overline{x}} \quad \text{where} \quad S(\overline{x}) = \tan \left(\frac{\overline{x}}{2}\right).
\]  

(16)
Substituting (12) and (16) into Eqns. (6) and (7) we obtain

\[ F'' + \beta(\overline{x})(1 - F^2) + fF' - PM(F - 1) = 2S(\overline{x})(FF_x - F'f_x). \]  

(17)

\[ Pr^{-1}G'' + fG' = 2S(\overline{x})(FG_x - G'f_x). \]  

(18)

The boundary conditions (9) reduce to

\[ F(\overline{x}, 0) = 0, \quad G(\overline{x}, 0) = 1, \quad F(\overline{x}, \infty) = 1, \quad G(\overline{x}, \infty) = 0. \]  

(19)

The skin friction coefficient and, the heat transfer coefficient in the form of Nusselt number can be expressed as

\[ C_f(Re_L)^{1/2} = 4P_r^{1/2}F'_w; \quad Nu(Re_L)^{-1/2} = 2^{1/2} \cos \left( \frac{\theta}{2} \right) (-G'_w). \]  

(20)

It is worth mentioning here that Kao and Elrod [2] have studied the non similar boundary layer flow, for two dimensional bodies, considering the Eqns. (6) and (7) without slot suction (injection) as well as, without magnetic field. Further, when \( M \neq 0 \) the Eqns. (6) and (7) are exactly same as those of Meena and Nath [4], without slot suction (injection).

### 3. Results and Discussion

The set of partial differential equations (17) and (18) under the boundary conditions (19) have been solved numerically using an implicit finite difference scheme in combination with the quasilinearization method. Since the method is described in detail in [10], its details are not presented here for brevity. Computation have been carried out for various values of \( M (0 \leq M \leq 1.0) \) and \( \overline{x} \) \( (1.0 \leq \overline{x} \leq 1.5) \). In order to

![Figure 2: Comparision of Skin Friction and Heat Transfer Results with Those of (a) Kao and Elrod [2] (b) Meena and Nath [4]](image-url)
validate the accuracy of the numerical method used, our skin friction coefficient and heat transfer coefficient results are compared, without slot suction (injection), with those of Kao and Elrod [2] when $M = 0$, and Meena and Nath [4] when $M \neq 0$ [See Fig. 2(a) and 2(b)]. It is observed that the computed results are in good agreement, with the above studies [2, 4]. In the following paragraphs, we have discussed the effect of slot injection (suction) on skin friction and heat transfer coefficients in presence of an applied magnetic field.

The effect of magnetic field $M (0 \leq M \leq 1.0)$ on skin friction coefficient $[C_f (Re_L)^{1/2}]$ and heat transfer coefficient $[Nu (Re_L)^{-1/2}]$ is shown in Fig. 3, in the absence of slot injection(suction). It is observed that both skin friction and heat transfer coefficient increases as magnetic field increases. The percentage of increase of skin friction coefficient is about 233% and percentage of increase of heat transfer coefficient is about 3.27% in the range $0.0 \leq M \leq 1.0$ at an arbitrary value of $ar{x}$ ($\bar{x} = 1.0$)

![Figure 3: Effect of Magnetic Field on (a) Skin Friction Coefficient and (b) Heat Transfer Coefficient](image)

Figure 4 and 5 show the effect of non-uniform slot suction (or injection) parameter ($A > 0$ or $A < 0$) and $\bar{x}_0$ (which fixes the slot location) on the skin friction and heat transfer coefficients $[C_f (Re_L)^{1/2}, Nu (Re_L)^{-1/2}]$. In the case of slot suction ($A > 0$), both $C_f (Re_L)^{1/2}$ and $Nu (Re_L)^{-1/2}$ increase and attain their maximum values before the trailing edge of the slot. The percentage of increase in skin friction and heat transfer coefficients are about 312.4% and 27.44% respectively at $\bar{x} = 1.2$. Further, both skin friction and heat transfer coefficients decrease from their maximum values and $C_f (Re_L)^{1/2}$ reaches zero, but $Nu (Re_L)^{-1/2}$ remains finite. Thus it is confirmed that laminar boundary layer separation is delayed by slot suction ($A > 0$). To be more precise, the point of separation moves downstream approximately by 2% as the rate of suction
But slot injection on the body surface has the reverse effect as evident in the Fig. 5, which results in early separation of the laminar boundary layer. The percentage of decrease in skin friction and heat transfer coefficients during slot injection are about 256.4% and 22.37% respectively at $\bar{x} = 1.2$ as the rate of injection ($A < 0$) increases from 0.0 to 0.5.

4. CONCLUSION

The effect of non-uniform slot suction (injection) on steady non-similar MHD boundary layers in two dimensional flow with an applied magnetic field has been studied. Numerical solutions have been obtained by a stable implicit finite-difference method along with a quasilinearization technique. The results of the present study reveal that the magnetic field enhances both skin friction and heat transfer coefficients.
Further it is found that slot suction helps delay in the boundary layer separation, whereas slot injection does the opposite.

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