APPLICATION OF ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM (ANFIS) FOR PREDICTION OF CONSTANT AMPLITUDE FATIGUE LIFE OF ALUMINUM ALLOYS UNDER THE EFFECT OF R-RATIO


Abstract: The constant amplitude fatigue crack growth life is affected by load ratio which quantifies the influence of mean load. Several research works have been conducted to study the effect of load ratio on crack growth rate through deterministic approach. However, the application of artificial intelligence methods particularly adaptive neuro-fuzzy technique (ANFIS) is lacking. The current research presents a methodology to predict the constant amplitude loading fatigue life under the influence of load ratio by developing an ANFIS model. The model result has been verified on 2024 T3 aluminum alloy and observed that it is better in comparison to simple ANN model.

Key-words: Adaptive neuro-fuzzy inference system (ANFIS); Fatigue crack growth rate; root mean square error (RMSE)

1. INTRODUCTION

The science of fatigue crack growth has received wide attention in fatigue community in order to establish the timely inspection schedule for maintenance of modern sophisticated equipments so as to avoid catastrophic failure. Most of the fatigue crack growth models proposed till date are usually based on fracture mechanics concepts, correlating the instantaneous fatigue crack propagation rate with the corresponding stress intensity factor range ($\Delta K$). However, apparent effectiveness of $\Delta K$ is known to be affected by the load ratio (minimum load / maximum load), crack closure, overload, crack size, environment, geometry, temperature etc [1]. The primary loading parameter affecting the fatigue crack growth is the load ratio $R$, which quantifies the influence of mean load. It is well known that the growth rate either increases or decreases by increasing the value of load ratio under different loading conditions [2-4]. Therefore, the ability to correlate and predict the fatigue crack growth rate for different load ratios is of significant importance.

Several research works [5, 6] have been attempted to account for the $R$-ratio effect on fatigue crack growth. Three important trends could be identified while considering the influence of load ratio on fatigue crack growth rate curves. With the increase in applied $\Delta K$, for higher $R$ values, differences in the corresponding crack growth rates could either increase (‘diverging’ $da/dN - K$ curves), or be constant (‘parallel’ $da/dN - \Delta K$ curves), or decrease (‘converging’ $da/dN - \Delta K$ curves). As a consequence no one interpolation model is able to satisfactorily follow all these different $R$-influences [7]. Therefore, it is important to establish a crack growth law in order to predict fatigue life taking into account the load ratio effect. In the present work an attempt has been made to predict fatigue life of 2024 T3 aluminum alloy under the influence of load ratio by applying adaptive neuro-fuzzy inference system (ANFIS). The predicted result has been compared with experimental results as well as the results obtained from authors’ previously proposed [8] artificial neural network (ANN) model. The results show that ANFIS gives better prediction than ANN.

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2. EXPERIMENTATION AND DATA PREPARATION

2.1. Fatigue Crack Growth Experiment

The material under investigation was 2024 T3 Al alloy in the form of 6.5mm thickness plate. The chemical composition and the mechanical properties of the alloy are given in Tables 1 and 2 respectively.

<table>
<thead>
<tr>
<th>Alloying Elements</th>
<th>Al</th>
<th>Cu</th>
<th>Mg</th>
<th>Mn</th>
<th>Fe</th>
<th>Si</th>
<th>Zn</th>
<th>Cr</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wt. %</td>
<td>90.7–94.7</td>
<td>3.8–4.9</td>
<td>1.2–1.8</td>
<td>0.3–0.9</td>
<td>0.5</td>
<td>0.5</td>
<td>0.25</td>
<td>0.1</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 2

Mechanical Properties of 2024 T3 Al-alloy

<table>
<thead>
<tr>
<th>Tensile strength (σut) MPa</th>
<th>Yield strength (σys) MPa</th>
<th>Young’s modulus (E) MPa</th>
<th>Poisson’s ratio (ν)</th>
<th>Plane Strain Fracture toughness (KIC) MPam</th>
<th>Plane Stress Fracture toughness (KC) MPam</th>
<th>Elongation</th>
</tr>
</thead>
<tbody>
<tr>
<td>469</td>
<td>324</td>
<td>73,100</td>
<td>0.33</td>
<td>37.0</td>
<td>95.31</td>
<td>19% in 12.7 mm</td>
</tr>
</tbody>
</table>

Single-edge notched tension (SENT) specimens were machined in the LT-direction from the plate whose geometry is shown in Fig 1. Both the sides of the specimen were mirror polished in order to facilitate the observation of crack growth.

Figure 1: Single Edge Notched Specimen Geometry
The crack growth experiments were conducted in air at room temperature on a servo-hydraulic test machine (Instron-8502) having a load capacity of 250 kN with a frequency of 6 Hz. Pre-cracking were introduced under mode-I loading with a sinusoidal waveform to an \( a/w \) ratio of 0.3 and were subjected to constant load test (i.e. progressive increase in \( \Delta K \) with crack extension) maintaining different load ratios (\( R \)) of 0, 0.2, 0.4, 0.5, 0.6, 0.8 respectively for both the materials. The crack growth was monitored with the help of a COD gauge mounted on the face of the machined notch. The following equations were used to determine stress intensity factor \( K \) [9].

\[
K = f(g) \cdot \frac{F \sqrt{\pi a}}{wB}
\]

where,

\[
f(g) = 1.12 - 0.231(a/w) + 10.55(a/w)^2 - 21.72(a/w)^3 + 30.39(a/w)^4
\]

2.2. Crack Growth Rate Determination

After the fatigue crack growth rate tests, raw \( a-N \) data were obtained under each load ratio which usually contained scatter. In order to smoothen the test data and to determine the fatigue crack growth rate, the following procedures were adopted by applying the concept of authors’ previously proposed exponential [10]. Observing the exponential nature of growth of crack length with number of cycles, the experimental \( a-N \) data has been fitted with the following exponential equation as per the previous model.

\[
a_j = a_i e^{m_{ij}(N_j - N_i)}
\]

where, \( a_i \) and \( a_j \) = crack length in \( i^{th} \) step and \( j^{th} \) step in ‘mm’ respectively,

\( N_i \) and \( N_j \) = No. of cycles in \( i^{th} \) step and \( j^{th} \) step respectively,

\( m_{ij} \) = specific growth rate in the interval \( i-j \),

\( i \) = No. of experimental steps,

and \( j = i + 1 \)

The values of specific growth rate ‘\( m_{ij} \)’ have been calculated according to the equation (4) and subsequently refined by curve fitting with calculated ‘\( a \)’ values (i.e. crack lengths from initial to final with an increment of 0.005mm). The smoothened values of the number of cycles have been calculated in the excel sheet from the refined ‘\( m_{ij} \)’ values as per the following equation.

\[
N_j = \frac{\ln \left( \frac{a_j}{a_i} \right)}{m_{ij}} + N_i
\]

The crack growth rates (\( da/dN \)) have been calculated directly from the above calculated values of ‘\( N \)’ as follows:

\[
\frac{da}{dN} = \frac{a_j - a_i}{N_j - N_i}
\]
Figures 2 and 3 depict the superimposed $a - N$ and log $(da/dN) - \log(\Delta K)$ curves under different load ratios.

3. ANFIS: METHODOLOGY

The fuzzy inference system (FIS) may be used as tools for approximating ill-defined nonlinear functions. It can implement qualitative aspects of human knowledge and reasoning by using following four functional components as shown in Fig. 4.
A rule base containing a number of fuzzy if-then rules.

- A decision-making unit as the inference engine.
- A fuzzification interface which transforms crisp inputs to linguistic variables.
- A defuzzification interface converting fuzzy outputs to crisp outputs.

Adaptive neuro-fuzzy inference system (ANFIS) is an integrated system of artificial neural network (ANN) and fuzzy inference system (FIS) and utilizes the advantages of both. ANFIS is a class of adaptive networks, whose membership function parameters are tuned (adjusted) using either a back-propagation algorithm or hybrid algorithm based on a combination of back-propagation and least squares estimate (LSE). In the present investigation, type-3 ANFIS [11] topology based on first-order Takagi-Sugeno (TSK) [12] if-then rules has been used.

The structure of proposed ANFIS model consists of a number of interconnected fixed and adjustable nodes corresponding to first-order TSK fuzzy model as shown in Fig. 5. It is composed of five layers having three inputs and one output. Bell-shaped membership function has been chosen for the present investigation because it is the best membership function type [13]. A hybrid-learning algorithm is applied to adapt the premise and consequent parameters to optimize the network. Heuristic rules are used to guarantee fast convergence.
4. APPLICATION DESIGN

It is known that fatigue crack growth life decreases as load ratio increases [3]. Accordingly, the maximum stress intensity factor ($K_{\text{max}}$), and the stress intensity factor range ($\Delta K$) are also affected by load ratio. Therefore, during model formulation load ratio ($R$), maximum stress intensity factor ($K_{\text{max}}$), and stress intensity factor range ($\Delta K$) were selected as linguistic input variables whereas, crack growth rate ($da/dN$) was taken as output variable. A set of linguistic rules formulated in the “If-Then” form were derived from expert observation and experimentation.

The experimental data base consists of six sets of fatigue crack growth data having load ratios ($R$) of 0, 0.2, 0.4, 0.5, 0.6 and 0.8. Each set for a particular load ratio consists of approximately 300 data of both $K_{\text{max}}$ and $\Delta K$ along with their corresponding $da/dN$ (calculated as per the procedure given in sub-section 2.2). The model was applied to simulate the crack growth rate of an unknown input/output data set for load ratio of 0.5 as validation set (VS) by constructing a training set (TS) with five known input/output data sets for load ratios ($R$) of 0, 0.2, 0.4, 0.6 and 0.8 for both the materials respectively. Fig. 6 shows the flow chart of ANFIS hybrid learning algorithm. Before applying ANFIS model, the pre-processing of experimental data is essential in order to achieve optimum modeling results.

![Figure 6: Flow Chart of ANFIS Hybrid Learning Algorithm](image-url)
The input variables i.e. load ratios ($R$), maximum stress intensity factor ($K_{\text{max}}$) and stress intensity factor range ($\Delta K$) were normalized in such a way that their maximum values were normalized to unity. The crack growth rate ($\frac{d a}{d N}$), which constitutes the system output, was also normalized in similar manner. The numbers of membership functions (MF) were chosen to be 5-5-5 corresponding to the inputs $R$, $K_{\text{max}}$ and $\Delta K$ respectively.

The $5 \times 5 \times 5 = 125$ fuzzy ‘if-then’ rules were constituted in which fuzzy variables were connected by T-norm (fuzzy AND) operators. The adjustment of premise and consequent parameters was made in batch mode based on the hybrid-learning algorithm. The model was trained for 4000 epochs until the given tolerance was achieved. Table 3 summarizes all the characteristics of ANFIS network used during training.

<table>
<thead>
<tr>
<th>Type of membership function</th>
<th>Generalized bell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of input nodes ($n$)</td>
<td>3</td>
</tr>
<tr>
<td>Number of fuzzy partitions of each variable ($p$)</td>
<td>5</td>
</tr>
<tr>
<td>Total number of membership functions</td>
<td>15</td>
</tr>
<tr>
<td>Number of rules</td>
<td>125</td>
</tr>
<tr>
<td>Total number of nodes</td>
<td>394</td>
</tr>
<tr>
<td>Total number of parameters</td>
<td>545</td>
</tr>
<tr>
<td>Number of epochs</td>
<td>4000</td>
</tr>
<tr>
<td>Step size for parameter adaptation</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Referring to Fig. 5, layer 1 has 15 $(5 \times 3)$ nodes with 45 parameters. Layers 2, 3 and 4 have $125(5^3)$ nodes each with 500 parameters associated in layer 4. The model performances during training and testing were verified by computing root mean square error (RMSE); coefficient of determination ($R^2$) and mean percent error (MPE) defined by the following equations:

$$\text{RMSE} = \left( \frac{1}{p} \sum_{i=1}^{p} |t_i - o_i| \right)^{1/2}$$  \hspace{1cm} (7)

$$R^2 = 1 - \frac{\sum_{i=1}^{p} (t_i - o_i)^2}{\sum_{i=1}^{p} (o_i)^2}$$  \hspace{1cm} (8)

$$\text{MPE} = \frac{1}{p} \sum_{i=1}^{p} \left( \frac{t_i - o_i}{t_i} \times 100 \right)$$  \hspace{1cm} (9)

where ‘$t$’ is the target value, ‘$o$’ is the output value, and ‘$p$’ is the number of data items.

The model was trained and tested by using MATLAB with Fuzzy Logic Toolbox. The performance of the model during training and testing was verified through three statistical indices (Eqs. 7 to 9) and presented in Table 4.
## Table 4
Performance of ANFIS Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>During Training</th>
<th>During Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>0.001283</td>
<td>0.01285</td>
</tr>
<tr>
<td>MPE</td>
<td>0.28679</td>
<td>0.77895</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.99987</td>
<td>0.99783</td>
</tr>
</tbody>
</table>

5. MODEL VALIDATION AND DISCUSSION

As observed from the performance table, the MPE and RMSE values for the training data were negligible in both the cases. MPE values for testing were found to be slightly higher than those for training. The coefficient of determination was found to be close to 1.0 for training in both the materials. However, its value for testing was slightly less than unit. Based on the above statistical performances, the trained ANFIS model was tested for load ratio of 0.5 for both the materials. The predicted crack growth rates (for $R = 0.5$) obtained from ANFIS model were compared with experimental results in Figs. 7 and found to be in good agreement. The numbers of cycles (fatigue life) were calculated as per the following equation and compared with authors’ previously proposed [8] ANN model (Figs. 8).

$$N_{i+1} = \frac{a_{i+1} - a_i}{\Delta K}/dN + N_i$$

The predicted (ANFIS) numbers of cycles are presented along with ANN and experimental results in Table 5 for quantitative comparison.

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**Figure 7:** Comparison of $da/dN$–$\Delta K$ Curves for $R=0.5$
Table 5
Comparison of Fatigue Lives (ANFIS, ANN and Experimental)

<table>
<thead>
<tr>
<th>$N_f^{AN}$</th>
<th>$N_f^A$</th>
<th>$N_f^E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>K cycle</td>
<td>K cycle</td>
<td>K cycle</td>
</tr>
<tr>
<td>112.391</td>
<td>110.919</td>
<td>113.298</td>
</tr>
</tbody>
</table>

The performances of models were evaluated by comparing the prediction results with the experimental findings by the following criteria:

- Percentage deviation of predicted life from the experimental (measured) life i.e.
  $$\% \text{Dev} = \frac{\text{predicted} - \text{Experimental}}{\text{Experimental}} \times 100$$

- Prediction ratio which is defined as the ratio of actual life (i.e. experimental) to predicted life i.e. Prediction ratio, $P_r = \frac{\text{actual}}{\text{Experimental}}$

- Error bands i.e. the scatter of the predicted life in either side of the experimental life within certain error limits.

Table 6 shows the performance of the model evaluated by the first two criteria. It is observed that the ANFIS model prediction is better in comparison to ANN model. Further, the prediction ratios of both the models are approximately 1.0, which is adequate and also acceptable [14]. Figure 9 illustrates the performance of the alloy evaluated graphically under the third criteria. It is observed that the scatter of the predicted life is within ± 0.025%.
5. CONCLUSION

The focus of this work was to develop an ANFIS model in order to predict crack growth rate and in turn the fatigue life under the effect of load ratio. The performance of the model was verified by comparing its outputs with an ANN model and also with experimental data. It was observed that life prediction from the ANFIS model was superior to earlier ANN model and also compatible with experimental findings. This is because of the fact that for an ANN model, we have to perform a trial and error process to develop the optimal network architecture, while the ANFIS model does not require such a procedure. Another aspect of superiority for ANFIS model in comparison with ANN model is the lower number of epochs which is needed to reach convergence. This advantage reduces the computing time and cost.

REFERENCES


