Solving Integer Solutions Based Transportation Problem Under Fuzzy Environment

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Abstract: This paper deals with the transportation problem by considering the passenger capacity of an aircraft as a fuzzy parameter to minimize the operating cost. The passenger capacity of an aircraft is assumed to be fuzzy numbers with trapezoidal membership function. By the graded mean integration representation method, the model will be derived in the fuzzy sense in order to obtain the optimal total cost. A numerical example is provided to illustrate the results of the proposed model.

Keywords: Transportation problem, Fuzzy Modeling, Trapezoidal Membership Function.

1. INTRODUCTION

In today’s highly competitive market the pressure on organizations to find better ways to create and deliver value to customers becomes stronger. How and when to send the products to the customers in the quantities they want in a cost-effective manner become more challenging. Transportation models provide a powerful framework to meet this challenge. They ensure the efficient movement and timely availability of raw materials and finished goods (Liu and Kao, 2004).

The basic transportation problem was originally developed by Hitchcock (1941). Efficient methods of solution derived from the simplex algorithm were developed in 1947. The transportation problem can be modeled as a standard linear programming problem, which can then be solved by the simplex method. However, because of its very special mathematical structure, it was recognized early that the simplex method applied to the transportation problem can be made quite efficient in terms of how to evaluate the necessary simplex-method information (variable to enter the basis, variable to leave the basis and optimality conditions). In practice, the parameters of the transportation problem are not always exactly known and stable. This imprecision may follow from the lack of exact information or may be a consequence of a certain flexibility the given enterprise has in planning its capacities. A frequently used means to express the imprecision are the fuzzy numbers (Dutta and Murthy, 2010).

Slowinski (1986) presented a method for solving a multi-criteria linear program where the coefficients of the objective functions and the constraints are fuzzy numbers of the L-R type. He transformed the original problem into a multi-criteria linear fractional problem by assuming the aspiration levels for particular criteria to be fuzzy and basing on comparison of fuzzy numbers, and then solved the obtained problem by using an interactive technique involving a linear programming procedure in the calculation phase.

2. FUZZY TRANSPORTATION PROBLEM

(LIU AND KAO (2004))

Consider a transportation problem with m supply nodes and n demand nodes, in that sij > 0 units are supplied by supply node i and dji > 0 units are required by demand node j. Associated with each link (i, j) from supply node i to demand node j, there is a unit shipping cost cij for transportation. The problem is to determine a feasible way of shipping the available amount to satisfy the demand that minimizes the total transportation cost.

Let xij denote the number of units to be transported from Supply i to Demand j. The mathematical description of the conventional transportation problem is:

\[ Z = \min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \]

s.t. \[ \sum_{j=1}^{n} x_{ij} \leq s_i, \quad i = 1, ..., m, \]

\[ \sum_{i=1}^{m} x_{ij} \leq d_j, \quad j = 1, ..., n, \]

\[ x_{ij} \geq 0, \quad i = 1, ..., m, \quad j = 1, ..., n. \]

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\[ \sum_{j=1}^{m} x_{ij} \geq d_j, \quad j = 1, \ldots, n, \]
\[ x_{ij} \geq 0, \forall i, j. \]

Naturally, if any of the parameters \( c_{ij}, s_i, \) or \( d_j \) is fuzzy, the total transportation cost \( Z \) becomes fuzzy as well. The conventional transportation problem defined above then turns into the fuzzy transportation problem.

### 3. LITERATURE REVIEW

In this section we will present some relevant literature in the field of transportation problem.

Ringuest and Rinks (1987) proposed interactive algorithms to find more than \( k \) non-dominated and dominated solutions if there are \( k \) objectives. Thus the decision maker has to determine a compromise solution from the set of non-dominated solutions. For the larger problem, it is not easy to find the compromise solution by using the algorithm developed by Ringuest and Rinks (1987) but, using the fuzzy programming method, one can easily find a compromise solution.

Dhingra and Moskowitz (1991) defined some non-linear membership functions like exponential, quadratic and logarithmic, and applied them to an optimal design problem. This procedure is useful in engineering and management design situations where uncertainty or ambiguity arises about the preciseness of permissible parameters, degree of credibility, and correctness of statements and judgements.

Chanas and Kuchta (1998) have solved the fuzzy integer transportation problem. The membership functions of the constraints are defined in different forms; linear, exponential, power shape and rational. The fuzzy integer transportation problem is solved with an iterative algorithm based on \( \alpha \)-cuts. The problem is transformed into a crisp interval transportation problem and solved for different \( \alpha \) values according to proposed iterative algorithm. Hussien (1998) studied the complete set of possibly efficient solutions of multi-objective transportation problem with possibilistic coefficients of the objective functions.

Sakawa et al. (1997) proposed an interactive fuzzy decision making model using linear and non-linear membership functions to solve the multi-objective linear programming problem.

Das et al. (1999) focused on the solution procedure of the multi-objective transportation problem where the cost coefficients of the objective functions, and the source and destination parameters are expressed as interval values by the decision maker. They transformed the problem into a classical multi-objective transportation problem where to minimize the interval objective function. They defined the order relations that represent the decision maker’s preference between interval profits. They converted the constraints with interval source and destination parameters into deterministic ones. Finally, they solved equivalent transformed problem by fuzzy programming technique. Verma et al. (1997) proposed a special type of non-linear (hyperbolic and exponential) membership functions to solve the multi-objective transportation problem and compared the obtained result with the solution obtained by using a linear membership function and shown that the results found to be nearly same.

Li and Lai (2000) presented a fuzzy compromise programming approach to multi-objective transportation problems. Liu and Kao (2004) have solved fuzzy transportation problems in their study. In their model, the cost coefficients, the supply and demand quantities were fuzzy triangular and trapezoidal numbers. The fuzzy model was solved with a method based on extension principle. A pair of crisp parametric problems is obtained for solving the fuzzy model. With this pair of problems, the lower and upper bounds of the objective function at different \( \alpha \) levels are found. From different values of \( \alpha \), the membership function of the objective function is constructed. Some examples of the problems that are modeled and solved using fuzzy mathematical programming for transportation problem [21,12,8,14,10,13,15,16,17,].

### 4. FUZZY PRELIMINARIES

The theory of fuzzy set is based upon the investigation reported by Zadeh [1965], involves a mathematical description of vague (inexact, fuzzy) elements, with the vagueness of information resulting not from the stochastic character of the systems, but from the lack of uniqueness or selectivity thereof. Accordingly, the answer to the question whether an element is associated with a fuzzy set will not be in the form of a YES-OR-NO decision but it will require a carefully graded judgment of its association. The degree of association of defined elements is determined by an association function that must come within the scope of particular mathematical definitions, axioms and operational rules.

Fuzzy sets are used to incorporate knowledge in the solutions of problems, whose formulation is based on imprecise concepts. Let \( Z \) be a set of elements (objects) with a generic element of \( Z \)
denoted as \( z \); that is \( Z = \{ z \} \). This set is called a **universe** or **discourse**. A **fuzzy set** \( A \) in \( Z \) is characterized by a membership function \( \mu_A(z) \) that associates a real number in [0 1] with each element of \( Z \).

The value of \( \mu_A(z) \) at \( z \) is a **grade of membership** of \( z \) in \( A \); the closer it is to one, the higher the grade of membership is.

In **ordinary (crisp) sets**, an element either belongs or does not belong to a set.

In fuzzy sets, however, we say that all members for which \( \mu_A(z) = 1 \) are **full members** of the set. All members for which \( \mu_A(z) = 0 \) are **not members** of the set. The members for which \( \mu_A(z) \) is between 0 and 1 are **partial members** of the set. Therefore, a fuzzy set is an **ordered pair** consisting of values of \( z \) and a corresponding membership function:

\[
A = \{ z, \mu_A(z) \mid z \in Z \}
\]

Now, we present some necessary definitions come from [Saad, 2005].

**Definition 1**

A real fuzzy number is a fuzzy subset from the real line \( R \) with membership function \( \mu_x(a) = 0 \) satisfies the following questions:

1. \( \mu_x(a) \) is a continuous mapping from \( R \) to the closed interval [0,1],
2. \( \mu_x(a) = 0 \) \( \forall a \in (-\infty, a_1] \),
3. \( \mu_x(a) = 0 \) is strictly increasing and continuous on \([a_1, a_2]\),
4. \( \mu_x(a) = 1 \) \( \forall a \in [a_2, a_3) \)
5. \( \mu_x(a) \) is strictly increasing and continuous on \([a_3, a_4]\),
6. \( \mu_x(a) = 0 \) \( \forall a \in [a_4, +\infty] \)

where \( a_1, a_2, a_3, a_4 \) are real numbers and the fuzzy number \( a \) is denoted by \( \bar{a} = [a_1, a_2, a_3, a_4] \).

**Definition 2**

The fuzzy number \( \bar{a} \) is a trapezoidal number, denoted by \([a_1, a_2, a_3, a_4]\), and it is membership function \( \mu_x(a) \) is given by (see Fig. 1)

\[
\mu_x(a) = \begin{cases} 
0, & x \leq a_1, \\
\frac{x-a_2}{a_1-a_2}, & a_1 \leq x \leq a_2, \\
1, & a_2 \leq x \leq a_3, \\
\frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4, \\
0, & \text{otherwise}
\end{cases}
\]

**Fig. 1: Membership Function of a Fuzzy Number \( \bar{a} \).**

**Definition 3**

The \( \alpha \)-level set of the fuzzy number \( \bar{a} \) is defined as the ordinary set \( L_\alpha(\bar{a}) \) for which the degree of the membership function exceeds the level \( \alpha \in [0,1] \):

\[
L_\alpha(\bar{a}) = \{ x \mid \mu_x(a) \geq \alpha \}.
\]

**5. PROBLEM DEFINITION**

In order to propose an alternative solution methodology namely Graded mean integration representation method to solve fuzzy transportation problem we use the problem formulated by Saad (2005). The problem description is as follows:

Given \( m \) routes and \( n \) types of aircrafts, how many aircraft should be assigned to which routes, when the passenger capacity of an aircraft can be considered as a fuzzy parameter, so as to minimize the total operating cost?

When \( a_i \) aircraft of type \( i \) are available then, if \( x_{ij} \) aircraft of type \( i \) are allocated to route \( j \),

\[
\sum_{i=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \ldots, m.
\]

Let \( \bar{p}_{ij} \) be the passenger capacity of an aircraft of type \( i \) allocated to route \( j \) and \( d_j \), the total demand for route \( j \), then the unsatisfied demand for route \( j \) is

\[
P_j = d_j - \sum_{i=1}^{m} \bar{p}_{ij} x_{ij}, \quad j = 1, 2, \ldots, n.
\]

If \( c_{ij} \) is the operating cost of an aircraft of type \( i \) allocated to route \( j \) and \( k_{ij} \), the revenue lost by an unsatisfied demand on route \( j \) then the total cost of the operation (ignoring the cost of idle aircraft) is

\[
\sum_{i,j} c_{ij} x_{ij} + \sum_{i} k_{ij} P_j, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n.
\]
and the fuzzy integer generalized transportation problem (FIGTP) can be formulated mathematically as follows:

\[
\text{(FIGTP): Minimize } \\
\sum_{i,j} c_{ij}x_{ij} + \sum_{j} k_{j}p_{j}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n.
\]

Subject to

\[
\sum_{j} x_{ij} \leq a_{i}, \quad i = 1, 2, \ldots, m.
\]

\[
P_{j} = d_{j} - \sum_{i} \bar{a}_{ij}x_{ij}, \quad j = 1, 2, \ldots, n.
\]

Where the quantities \( c_{ij}, k_{j}, a_{i}, \bar{a}_{ij}, \) and \( d_{j} \) are all positive integers. In addition, it is assumed that \( \bar{a}_{ij} \) the passenger capacity of an aircraft of type \( i \) allocated to route \( j \) is a fuzzy parameter involved in the constraints where its membership function is \( \mu_{x_{ij}}(\bar{a}_{ij}) \).

### 6. FUZZY SOLUTION METHODOLOGY

Fuzzy arithmetic is a successful tool to solve engineering problems with uncertain parameters. The generalized mathematical operations for fuzzy numbers can theoretically be defined making use of Zadeh’s extension principle. Practical real-world applications of fuzzy arithmetic, however, require an appropriate form of implementation for the fuzzy numbers and the fuzzy arithmetic operations. Fuzzy sets are an extension of classical set theory and are used in fuzzy logic. In classical set theory the membership of elements in relation to a set is assessed in binary terms according to a crisp condition, an element either belongs or does not belong to the set. By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in relation to a set; this is described with the aid of a membership function. Fuzzy sets are an extension of classical set theory since, for a certain universe, a membership function may act as an indicator function, mapping all elements to either 1 or 0, as in the classical notion. Graded mean integration representation method based on the integral value of graded mean h-level of generalized fuzzy number for defuzzifying generalized fuzzy number for achieving computational efficiency is proposed by Chen and Hsieh (1999).

Trapezoidal Fuzzy Numbers is described as any fuzzy subset of the real line \( R \), whose membership function satisfies the following conditions:

\[
\mu_{A}(x) = \begin{cases} 
0, & -\infty < x \leq a_{1} \\
L(x) \text{ is strictly increasing on } [a_{1}, a_{2}] \\
w_{A}, & a_{2} < x \leq a_{3} \\
R(x) \text{ is strictly decreasing on } [a_{3}, a_{4}] \\
0, & a_{4} < x < \infty
\end{cases}
\]

Where \( 0 < w_{A} \leq 1 \), and \( a_{1}, a_{2}, a_{3}, a_{4} \) are real numbers. Also this type of generalized fuzzy number be denoted as \( \tilde{A} = (a_{1}, a_{2}, a_{3}, a_{4}; w_{A}) \). When \( w_{A} = 1 \), it can be simplified as \( \tilde{A} = (a_{1}, a_{2}, a_{3}, a_{4}) \).

Let \( L^{-1} \) and \( R^{-1} \) be the inverse functions of \( L \) and \( R \) respectively, the graded mean h-level value of \( \tilde{A} = (a_{1}, a_{2}, a_{3}, a_{4}; w_{A}) \) is

\[
h(L^{-1}(h) + R^{-1}(h))/2
\]

Therefore, the graded mean integration representation of generalized trapezoidal fuzzy number \( P(\tilde{A}) \) with grade \( w_{A} \):

\[
P(\tilde{A}) = \int_{0}^{w_{A}} h \left( \frac{L^{-1}(h) + R^{-1}(h)}{2} \right) dh
\]

Where \( 0 < h \leq w_{A} \) and \( 0 < w_{A} \leq 1 \)

Chen (1985), proposed the Function Principle for the fuzzy arithmetical operations by trapezoidal fuzzy numbers. Suppose \( \tilde{A} = (a_{1}, a_{2}, a_{3}, a_{4}) \) and \( \tilde{B} = (b_{1}, b_{2}, b_{3}, b_{4}) \) are two trapezoidal fuzzy numbers, arithmetical operations are:

1. \( \tilde{A} \oplus \tilde{B} = (a_{1} + b_{1}, a_{2} + b_{2}, a_{3} + b_{3}, a_{4} + b_{4}) \)
2. \( \tilde{A} \odot \tilde{B} = (a_{1}b_{1}, a_{2}b_{2}, a_{3}b_{3}, a_{4}b_{4}) \)
3. \( \tilde{A} \ominus \tilde{B} = (a_{1} - b_{1}, a_{2} - b_{2}, a_{3} - b_{3}, a_{4} - b_{4}) \)
4. \( \tilde{A} \phi \tilde{B} = (a_{1}/b_{1}, a_{2}/b_{2}, a_{3}/b_{3}, a_{4}/b_{4}) \)
5. \( \alpha \odot \tilde{A} = (\alpha a_{1}, \alpha a_{2}, \alpha a_{3}, \alpha a_{4}) \) \( \alpha > 0 \)
6. \( \alpha \odot \tilde{A} = (\alpha a_{1}, \alpha a_{2}, \alpha a_{3}, \alpha a_{4}) \) \( \alpha < 0 \)

### 7. AN ILLUSTRATIVE EXAMPLE (SAAD, 2005)

In this section we present the modified example given by Saad (2005) to validate the proposed alternative solution methodology namely Graded mean integration representation method.

Consider the fuzzy integer generalized transportation problem (FIGTP) specified by the following data:
The fuzzy integer generalized transportation problem (FIGTP) can be written as follows:

Min \[ Z = \sum_{i=1}^{5} \sum_{j=1}^{3} c_{ij} x_{ij} \]

Subject to

- \[ x_{ij} \leq 340, \]
- \[ x_{21} + x_{22} + x_{23} \leq 120, \]
- \[ x_{31} + x_{32} + x_{33} \leq 70, \]
- \[ x_{41} + x_{42} + x_{43} \leq 120, \]
- \[ x_{51} + x_{52} + x_{53} \leq 720, \]
- \[ p_{1} + n_{11} x_{11} + n_{12} x_{12} + n_{13} x_{13} + n_{41} x_{41} + n_{51} x_{51} = 3000, \]
- \[ p_{2} + n_{12} x_{12} + n_{22} x_{22} + n_{32} x_{32} + n_{42} x_{42} + n_{52} x_{52} = 2450, \]
- \[ p_{3} + n_{13} x_{13} + n_{23} x_{23} + n_{33} x_{33} + n_{43} x_{43} + n_{53} x_{53} = 1400, \]
- \[ p_{j} > x_{ij} > 0, \quad (i=1,2,3,4,5; j=1,2,3), \]

Where \( p_{j} > x_{ij} > 0, \) and \( x_{ij} \) are decision variables.

The following results are obtained using LINGO optimization software for optimality by solving the above non fuzzy problem. The optimum objective function value obtained by the proposed technique is \( Z = 1748 \) and by Saad (2005) technique is 1924. The decision variables for the proposed models are shown below:

- \[ x_{11} = 49, \quad x_{12} = 0, \quad x_{13} = 291, \]
- \[ x_{21} = 18, \quad x_{22} = 100, \quad x_{23} = 4, \]
- \[ x_{31} = 70, \quad x_{32} = 0, \quad x_{33} = 0, \]
- \[ x_{41} = 120, \quad x_{42} = 0, \quad x_{43} = 0, \]
- \[ x_{51} = 512, \quad x_{52} = 538, \quad x_{53} = 7, \]

With

- \[ p_{1} = 0, \quad p_{2} = 0, \quad p_{3} = 0. \]

7. CONCLUSIONS

In this paper we have proposed a solution methodology for solving the fuzzy generalization transportation problem; it has been shown that the fuzzy transportation problem could be converted into a non fuzzy problem using Graded mean integration representation method. An illustrative example has been given to validate the proposed solution methodology. The proposed method yield better solution compared to the solution obtained through the methodology given by Saad (2005) in the literature. This paper can be extended for the...
proposed solution methodology for solving the integer/transportation problem having fuzzy demands.

REFERENCES


