Multi-objective Optimization for Surface Roughness in CNC Turning of Brass using Response Surface Method

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Response surface methodology (RSM) has been applied to determine the optimum cutting conditions leading to optimum surface roughness in CNC turning operation on UNS C34000 brass. The second order mathematical models in terms of machining parameters are developed for surface roughness prediction using RSM on the basis of experimental results. The experiments are carried out with coated carbide tool for machining of brass. The model selected for optimization has been validated with F-test. The adequacy of the models of surface roughness has been established with Analysis of Variance (ANOVA). An attempt has also been made to optimize cutting parameters using multi-objective characteristics for the surface roughness prediction models.

Keywords: ANOVA, CNC Turning, Optimization, Surface Roughness.

1. INTRODUCTION

Surface finish is an important attribute of quality in any machining operation. The accuracy of mating surface is directly proportional to the surface finish produced on the machined part. Good surface finish also contributes to the aesthetic appeal of the product. The surface finish is the one of most important quality characteristic in the manufacturing industries which influences the performance of mechanical parts as well as production cost. In recent times, modern industries are keeping in mind to achieve the high quality products in a very short time with less operator input. For that purpose, the computer numerically controlled (CNC) machine tools with automated and flexible manufacturing systems have been implemented. In the manufacturing industries, various manufacturing processes are adopted to remove the material from the work piece for a better product. Turning is one of the most common method for metal cutting because of its ability to remove materials faster with a reasonable good surface quality. In actual practice, there are many factors which affect the surface roughness, e.g. cutting conditions, tool variables and work piece variables. It is very difficult in a study to consider all the parameters that control the surface roughness for a particular manufacturing process. Generally, the desired cutting parameters are selected based on experience or by the hand books. But the modeling of surface roughness and optimization of cutting parameters is essential for increasing productivity.

Several mathematical models based on statistical regression or neural network techniques have been constructed to establish the relationship between the cutting performance and cutting parameters. A brief review of literature on roughness modeling in turning operations is presented here. Palanikumar and Karthikeyan [1] reported that feed rate is the factor which has greater influence on centre line average roughness \( R_a \), followed by cutting speed and % volume fraction of SiC in machining of Al/SiC particulate composites using response surface methodology. Nalbant et al. [2] optimized the cutting parameters for turning of AISI 1030 steel bars using the Taguchi method. They investigated centre line average roughness \( R_a \) exclusively. The use of greater insert radius, low feed rate and low depth of cut are recommended to obtain better surface roughness for the specific test range. Singh and Rao [3] developed a mathematical model for \( R_a \) to optimize the tool geometry and cutting parameters for hard turning using genetic algorithm. Zhong and Khoo [4] predicted surface roughness parameters \( R_a \) and \( R_t \) of turned surface using neural network. Sahin and Motorcu [5] proposed a mathematical model of surface roughness parameter \( R_a \) for turning of mild steel with coated carbide tools using RSM. They concluded that feed rate was the main influencing factor on surface roughness. It was increased with increase in feed rate, but was decreased with increase in cutting speed and depth of cut, respectively. Among these process parameters, depth of cut was found to be more intensive than that of the cutting speed. Noordin et al. [6] described the performance of coated carbide tools using RSM when turning AISI 1040 mild steel. They found that feed rate is the most significant parameter influencing \( R_a \) and tangential force. Taguchi method was used by Yang and Tarang [7] to find the optimal cutting parameters for
turning operations. Choudhury and El Baradie [8] predicted $R_s$ using RSM and factorial design when turning high strength steel. Few researchers have used grey relational analysis to optimize turning operations with multiple performance characteristics.

However, a surface generated by machining is composed of a large number of length scales of superimposed roughness and generally characterized by two different types of parameters, which are amplitude parameters and spacing parameters. Amplitude parameters are measures of the vertical characteristics of the surface deviations, and examples of such parameters are centre line average roughness, root mean square roughness, skewness, kurtosis, peak-to-valley height, etc. Spacing parameters are the measures of the horizontal characteristics of the surface deviations, and examples of such parameters are mean line peak spacing, high spot count, peak count, etc. Thus, consideration of only one parameter like centre line average roughness is not sufficient to describe the surface quality though it is the most commonly considered roughness parameter. The present study aims to consideration of five different roughness parameters, which are centre line average roughness ($R_s$), root mean square roughness ($R_q$), skewness ($R_q$), kurtosis ($R_q$) and mean line peak spacing ($R_{lp}$) for the surface texture generated in turning operation of brass. A rotatable central composite experimental design is used in the present investigation. In addition to direct evaluation of the variables involved in the process, this design allows the study of the interactions among them and the modeling of multifactor response surfaces, thus providing a great deal of information about the behavior of the system with the help of a rather small number of experiments. Statistical models have been developed through response surface methodology based on experimental results [9]. The machining parameters depth of cut ($d$) (mm), spindle speed ($N$) (rpm) and feed rate ($f$) (mm/rev) are considered as independent variables, and surface roughness parameters as response variables. Lastly, an attempt has been made to obtain multi-objective optimization of machining conditions with respect to each of the roughness parameters considered in the present study with the help of response surface technique.

2. RESPONSE SURFACE METHOD

Response surface method (RSM) adopts both mathematical and statistical techniques that are useful for the modeling and analysis of problems in which a response of interest is influenced by several variables. RSM attempts to analyze the influence of the independent variables on a specific dependent variable (response). The purpose of developing mathematical models relating the machining responses and their factors is to facilitate the optimization of the machining process. The mathematical model commonly used for the machining response $Y$ is represented as:

$$Y = \psi (d, N, f) + \varepsilon \quad (1)$$

where, $d$, $N$ and $f$ are depth of cut, spindle speed and feed rate respectively, and $\varepsilon$ is the error which is normally distributed about the observed machining response $Y$. If $\psi (d, N, f) = \eta$, the surface represented by $\eta$ is called response surface.

For a second order polynomial model (quadratic model): $Y_u = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} b_{ij} x_i x_j + \sum_{i=1}^{n} b_{ii} x_i^2 \quad (2)$

where $Y_u = f(Y, \varepsilon)$ is the expected response of higher-order polynomial model, $x_i$ are process variables such as depth of cut, spindle speed and feed rate respectively, and $b_{0}, b_{ij}, b_{ii}$ are regression coefficients that can be calculated by linear multiple regression analysis.

3. EXPERIMENTAL DETAILS

3.1. Design of Experiment

The design of experiments technique is a very powerful tool, which permits to carry out the modeling and analysis of the influence of process variables on the response variables. In a turning operation, there are a large number of factors that can be considered as the machining parameters. But, the review of literature shows that the depth of cut ($d$), spindle speed ($N$) and feed rate ($f$) are the most widespread machining parameters taken by the researchers. In the present study they are selected as design factors while other parameters have been assumed to be constant over the experimental domain. A rotatable central composite design is selected for the experimentation. It is the most widely used experimental design for the modeling of a second order response surface.

For a given number of variables, the $\alpha$ required to achieve rotatability is computed as $\alpha = (n_f)^{1/4}$, where $n_f$ is the number of points in the $2^k$ factorial design (with $k$ as the number of factors). Rotatability refers to the uniformity of prediction error. In rotatable designs, all points at the same radial distance ($r$) from the centre point have the same magnitude of prediction error. A rotatable CCD consists of $2^k$ fractional factorial points (usually coded as $\pm 1$), augmented by $2k$ axial points ($\pm \alpha, 0, \ldots, 0$), $(0, \pm \alpha, 0, \ldots, 0)$, $(0, 0, \pm \alpha)$ and $n_c$ centre points $(0, 0, 0, 0, \ldots, 0)$.

The centre points vary from three to six. With proper choice of $n_c$ the CCD can be made orthogonal or a uniform precision design can be applied. It means that the variance of response at origin is equal to the variance of response.
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at a unit distance from the origin. Hence, a CCD with uniform precision has been selected in this investigation. In this study, eight factorial points \( (2^4) \), six axial points \( (2 \times 3) \) and six centre runs have been considered, which implies a total of 20 experimental runs.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOC ((d))</td>
<td>-0.32</td>
</tr>
<tr>
<td>Spindle speed ((N))</td>
<td>528 800 1200 1600 1872</td>
</tr>
<tr>
<td>Feed ((f))</td>
<td>0.0224</td>
</tr>
</tbody>
</table>

A randomized experimental run has been carried out to minimize the error due to machining set-up. The levels of cutting parameters such as depth of cut, spindle speed and feed rate for the experiments have been listed in Table 1. Experiments were made according to the experimental plan based on central composite rotatable second order design.

3.2. Work Piece Material

The present study is carried out with UNS C34000 medium leaded brass. The chemical composition of brass is 0.095\%Fe, 0.9\%Pb, 34\%Zn and balance Cu, and its mechanical properties are: hardness of 68 HRF, density of 8.47 g/cc, and tensile strength of 340 MPa. All the specimens are of 20 mm diameter and 60 mm length.

3.3. Machining Equipment

The machine used for the turning experiments is a JOBBERXL CNC lathe having the control system FANUC Series Oi Mate-Tc and equipped with maximum spindle speed of 3500 rpm, feed rate of 15-20 mm/rev and KVA rating of 16 KVA.

3.4. Cutting Tool

The tool holder considered is WIDIA PTGNR-25-25 M16 050, and insert is WIDIA TNMG 160404–FL. The tool is coated with titanium nitride coating having hardness, density and transverse rupture strength of 1570 HV, 14.5 g/cc and 3800 N/mm\(^2\) respectively. The compressed coolant WS 50-50 with a ratio of 1:20 is used as cutting fluid.

3.5. Roughness Measurement

Roughness measurement is made using a portable stylus-type profilometer Taylor Hobson Surtronic 3+ Talysurf. The profilometer was set to a cut-off length of 0.8 mm, filter 2CR, traverse speed of 1 mm/sec and traverse length of 4 mm. Roughness measurements in the transverse direction of the work piece are repeated four times, and the average of the four measurements of surface roughness parameter is recorded. The measured profile is digitized and processed through the dedicated advanced surface finish analysis software Talyprofile for evaluation of the roughness parameters.

3.6. Response Variables Selected

The response variables used to accomplish the present study on surface roughness are the following ones:

\( (i) \) Centre Line Average Roughness \((R_a)\)

It is defined as the arithmetic mean deviation of the surface height from the mean line through the profile, while the mean line is defined so as to have equal areas of the profile above and below it. \( R_a \) may be expressed in the form:

\[
R_a = \frac{1}{L} \int |Z(x)| dx
\]

\((3)\)

where \(Z(x)\) is the ordinate of the profile curve, \(x\) are the different positions in the profile direction and \(L\) is the sampling length. It is usually expressed in \(\mu m\), like root mean square roughness \((R_q)\).

\( (ii) \) Root Mean Square Roughness \((R_q)\)

It represents the standard deviation of the distribution of surface heights. It is defined as the root mean square deviation of the profile from the mean line and can be expressed as:

\[
R_q = \sqrt{\frac{1}{L} \int |Z(x)|^2 dx}
\]

\((4)\)

\( (iii) \) Skewness \((R_{sk})\)

It is a measure of the departure of a distribution curve from its symmetry and may be expressed in the form:

\[
R_{sk} = \frac{1}{R_q^3 Q \frac{L}{6}} \int |Z(x)|^3 dx
\]

\((5)\)

It is a non-dimensional number and for a symmetrical distribution like Gaussian distribution, \( R_{sk} = 0 \). A surface with positive skewness has a wider range of peak heights that are higher than the mean. A surface with negative skewness has more peaks with heights close to the mean as compared to a Gaussian distribution.

\( (iv) \) Kurtosis \((R_{ku})\)

It is measure of the bump on a distribution curve and may be expressed in the form:

\[
R_{ku} = \frac{1}{R_q^4 Q \frac{L}{6}} \int |Z(x)|^4 dx
\]

\((6)\)

It is also non-dimensional and \( R_{ku} = 3 \) for Gaussian distribution. \( R_{ku} > 3 \) means that peaks are sharper than Gaussian and vice versa.
(v) **Mean Line Peak Spacing (R\textsubscript{sm})**

It is known as the mean spacing between peaks, with a peak defined relative to the mean line (a peak must cross above the mean line and then back below it). This parameter may be expressed in the form:

\[
R_{sm} = \frac{1}{m} \sum_{n=1}^{m} S_n
\]

where \(m\) is the number of peak spaces and \(S\) is the spacing between two consecutive peaks. It is usually expressed in mm.

### 4. RESULTS AND DISCUSSION

The influence of the CNC turning cutting parameters (depth of cut, spindle speed and feed rate) on the response variables selected have been assessed for brass by conducting experiments as per design of experiments. Table 2 shows measured roughness values obtained in the experiments.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>(R_x)</th>
<th>(R_y)</th>
<th>(R_z)</th>
<th>(R_{yf})</th>
<th>(R_{sm})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.778</td>
<td>0.972</td>
<td>0.542</td>
<td>3.65</td>
<td>0.070</td>
</tr>
<tr>
<td>2</td>
<td>0.829</td>
<td>1.038</td>
<td>0.685</td>
<td>3.64</td>
<td>0.074</td>
</tr>
<tr>
<td>3</td>
<td>1.102</td>
<td>1.387</td>
<td>0.156</td>
<td>3.78</td>
<td>0.086</td>
</tr>
<tr>
<td>4</td>
<td>1.000</td>
<td>1.282</td>
<td>0.213</td>
<td>4.01</td>
<td>0.076</td>
</tr>
<tr>
<td>5</td>
<td>4.027</td>
<td>4.930</td>
<td>0.993</td>
<td>2.98</td>
<td>0.215</td>
</tr>
<tr>
<td>6</td>
<td>4.112</td>
<td>4.997</td>
<td>0.936</td>
<td>2.82</td>
<td>0.216</td>
</tr>
<tr>
<td>7</td>
<td>3.922</td>
<td>4.840</td>
<td>0.905</td>
<td>3.01</td>
<td>0.216</td>
</tr>
<tr>
<td>8</td>
<td>4.030</td>
<td>4.997</td>
<td>0.992</td>
<td>3.12</td>
<td>0.209</td>
</tr>
<tr>
<td>9</td>
<td>1.952</td>
<td>2.455</td>
<td>0.542</td>
<td>3.09</td>
<td>0.133</td>
</tr>
<tr>
<td>10</td>
<td>2.002</td>
<td>2.525</td>
<td>0.619</td>
<td>2.96</td>
<td>0.133</td>
</tr>
<tr>
<td>11</td>
<td>1.915</td>
<td>2.380</td>
<td>0.845</td>
<td>2.94</td>
<td>0.141</td>
</tr>
<tr>
<td>12</td>
<td>2.240</td>
<td>2.795</td>
<td>0.411</td>
<td>2.75</td>
<td>0.139</td>
</tr>
<tr>
<td>13</td>
<td>0.319</td>
<td>0.398</td>
<td>0.690</td>
<td>4.30</td>
<td>0.113</td>
</tr>
<tr>
<td>14</td>
<td>5.237</td>
<td>6.315</td>
<td>0.860</td>
<td>2.77</td>
<td>0.265</td>
</tr>
<tr>
<td>15</td>
<td>1.915</td>
<td>2.390</td>
<td>0.618</td>
<td>2.84</td>
<td>0.133</td>
</tr>
<tr>
<td>16</td>
<td>1.930</td>
<td>2.440</td>
<td>0.540</td>
<td>2.99</td>
<td>0.235</td>
</tr>
<tr>
<td>17</td>
<td>1.887</td>
<td>2.385</td>
<td>0.629</td>
<td>2.96</td>
<td>0.132</td>
</tr>
<tr>
<td>18</td>
<td>2.015</td>
<td>2.535</td>
<td>0.882</td>
<td>2.96</td>
<td>0.143</td>
</tr>
<tr>
<td>19</td>
<td>2.060</td>
<td>2.962</td>
<td>0.763</td>
<td>2.07</td>
<td>0.131</td>
</tr>
<tr>
<td>20</td>
<td>2.055</td>
<td>2.577</td>
<td>0.653</td>
<td>2.91</td>
<td>0.134</td>
</tr>
</tbody>
</table>

The second order response surface equations have been fitted using MINITAB software [10] for all the five response variables (\(R_x\), \(R_y\), \(R_z\), \(R_{yf}\) and \(R_{sm}\)). The equations can be given in terms of the coded values of the independent variables as follows:

\[
R_x = 1.971 + 0.016d + 0.062N + 1.512f - 0.016dN + 0.03df - 0.085Nf + 0.038d^2 + 0.073N^2 + 0.321f^2
\]

\[
R_y = 2.5409 + 0.0222d + 0.0928N + 1.8332f - 0.0101dN + 0.0330df - 0.0937Nf + 0.0275d^2 + 0.0620N^2 + 0.3339f^2
\]

\[
R_z = 0.6807 + 0.0262d - 0.1187N + 0.1843f + 0.0072dN - 0.0211df + 0.1031Nf - 0.0316d^2 - 0.0149N^2 + 0.0370f^2
\]

\[
R_{yf} = 2.7837 - 0.0040d + 0.0378N - 0.4184f + 0.0646dN - 0.0328df - 0.0221Nf + 0.1296d^2 + 0.0678N^2 + 0.3099f^2
\]

\[
R_{sm} = 0.15211 - 0.00091d + 0.00061N + 0.05897f - 0.00268dN - 0.00013df - 0.00301Nf - 0.00841d^2 - 0.00594N^2 + 0.01147f^2
\]

The analysis of variance (ANOVA) and the F-ratio tests have been performed to check the adequacy of the models, as well as the significance of the individual model coefficients. Analysis of variance has been carried out for the second order models of Equations (8-12), proposed for roughness parameters on brass turning. ANOVA shows that the P-value is lower than 0.05, which means that the model is significant at 95% confidence level. Thus, the model is adequate at 95% confidence level to represent the relationship between the machining response and the machining parameters of CNC turning. F-test value of lack-of-fit for \(R_x\) as well as for other roughness parameters appears insignificant. It implies that the lack-of-fit is not significant relative to pure error. Also, the \(R^2\) value is found to be 0.99, 0.99, 0.75, 0.82 and 0.83 for \(R_x\), \(R_y\), \(R_z\), \(R_{yf}\) and \(R_{sm}\) respectively. These high values close to 1 indicate desirability of results. The predicted \(R^2\) is in reasonable agreement with the adjusted \(R^2\).

A summary table for significant parameters for roughness is shown in Table 3. It is clearly seen that feed rate is the most significant one of roughness parameters. The normal probability plots of residuals for \(R_x\), \(R_y\), \(R_z\), \(R_{yf}\) and \(R_{sm}\) are proved to fall on a straight line, which means that the errors are normally distributed. Similarly, the comparison of each experimental value with the prediction obtained from the model for roughness parameters shows a good agreement. It can be concluded that the regression model is fairly well fitted to the observed values for all the responses. Residual error plots depict that there is no obvious pattern or unusual structure, and the models proposed are adequate.
Three dimensional surface and contour plot in Figure 1 shows the effect of depth of cut and feed rate on center line average roughness parameter $R_a$ when keeping spindle speed at its average level, i.e. 200 rpm. The roughness parameter $R_a$ is increased with the increase in feed rate. The non-linear nature of variation of $R_a$ with depth of cut and feed rate has been observed. Figures 2-5 present the surface and contour plots of surface roughness parameters $R_q$, $R_{sk}$, $R_{ku}$ and $R_{sm}$ respectively. In these figures, it is clearly shown that with increasing feed rate, $R_q$, $R_{sk}$, and $R_{sm}$ is increased while $R_{ku}$ decreases. It depicts good agreement with the summary table of ANOVA for roughness parameters.

### 4.1. Multi-objective Optimization for Surface Roughness Parameters

The multi-objective optimization analysis is made to optimize all the five surface roughness parameters in turning of brass using MINITAB software. Based on tribological aspects [11], the objective is set as to minimize...
\( R_a, R_q, R_{sk}\) and \( R_{sm}\) while \( R_{ku}\) is targeted at 3. The cutting parameter combination predicted for multi-objective optimization is: depth of cut of 0.1219 mm, spindle speed of 1189.32 rpm and feed rate of 0.083 mm/rev. The overall desirability is very close to 1 (0.925). The optimization results are represented in Figure 6. A confirmation test has been conducted with optimal parameter setting and the experimental value is in well agreement with the predicted one with % error lying within 2 to 4%. The confirmation test results are shown in Table 4.

5. CONCLUSION

The empirical models for surface roughness parameters are developed in terms of depth of cut, spindle speed and feed rate for CNC turning of brass within the present experimental domain. These models can be utilized to achieve the optimal value of cutting parameters to obtain the required value of roughness parameters. In the present study it is observed that feed rate is the most influencing cutting parameter. With increasing feed rate, the roughness parameters \( R_a, R_q, R_{sk}\) and \( R_{sm}\) increase while \( R_{ku}\) decreases. The multi-objective optimization of roughness parameters gives an optimal setting of cutting parameters which is in good agreement with experimental values.

REFERENCES


